

# Flight and Orbital Mechanics

Exams

## **Exam AE2104-11: Flight and Orbital Mechanics (23 January 2013, 09.00 – 12.00)**

Please put your name, student number and ALL YOUR INITIALS on your work. Answer all questions and put your name on each page of your exam.

This exam consists of questions: 1, 2, 3, 4, 5a-h, 6a-d, 7, 8a-f

Derive the expressions for each required calculation (unless mentioned in the file '*equations by heart*', for the orbital mechanics part).

For the open questions, the way the answer is obtained should be clearly indicated by visibly substituting the numbers in the formulas. Only mentioning the final answer will NOT result in any credits. For the multiple choice questions, only the final answer has to be provided.

Use of pencils to write the exam is NOT permitted. Scrap paper may not be added to your exam work (please take the scrap paper with you after the exam). It is not permitted to have any pre-programmed information on your calculator. The memory of your calculator should be erased prior to the start of the exam. Failure to do so will be seen as fraud.

In total 100 points can be earned. (50 points for flight mechanics and 50 points for orbital mechanics). At least 58 points are required to pass the exam. (57 points results in a grade 5.5 and 58 points results in a grade 6)

Good luck!

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### **Question 1 (Flight Mechanics - Take-off) [5 points, multiple choice]**

The decision speed  $V_1$  is an important speed during take-off. For a given multi-engine aircraft, this airspeed is \_\_\_\_\_.

Select the best answer from the following options:

- independent of the balanced field length.
- a function of weight, altitude and temperature.
- the lowest speed to ensure an adequate and safe climb out with the critical engine inoperative and the live engine(s) at full take-off thrust.
- a function of aircraft weight.
- generally much higher than the rotation speed  $V_r$ .

**Question 2 (Flight Mechanics - Landing) [5 points, multiple choice]**

Lift dumpers are used to reduce the landing ground run distance \_\_\_\_\_.

Select the best answer from the following options:

- a. by decreasing lift on the wing, this makes it easier for the pilot to put the aircraft on the ground.
- b. by decreasing lift on the wing, this creates a larger normal force on the wheels and thus a larger ground drag when braking.
- c. by disturbing the airflow on the wing, this creates a lot of turbulence and thus aerodynamic drag.
- d. None of the above answers is correct

**Question 3 (Flight Mechanics – Turning performance) [5 points, multiple choice]**

An aircraft is in turn with the bank angle of 45 degrees and it is flying with a true airspeed of 136 m/s. In addition following data is given:

<b>Variable</b>	<b>Value</b>
Wing span (b)	46 [m]
Wing surface area (S)	260 [m <sup>2</sup> ]
Oswald factor (e)	0.75
Zero lift drag coefficient ( $C_{D0}$ )	0.03
Maximum thrust ( $T_{max}$ )	298 [kN]
Flight altitude (h)	1500 [m]
Maximum lift coefficient ( $C_{Lmax}$ )	0.8
Drag coefficient ( $C_D$ )	0.039
Air density ( $\rho$ )	1.0581 [kg/m <sup>3</sup> ]
Aircraft weight (W)	150 [kN]
Gravitational acceleration (g)	9.80665 [m/s <sup>2</sup> ]

Calculate the corresponding load factor and turn radius.

- a.  $n=1.41$  and  $R=1885.4$  m
- b.  $n=2.5$  and  $R=1523.6$  m
- c.  $n=1.41$  and  $R= 4114.3$  m
- d.  $n=2$  and  $R=1088.5$
- e.  $n = 2.5$  and  $R = 1090.1$

**Question 4 (Flight Mechanics - Cruise) [5 points, multiple choice]**

What is the most efficient flying strategy to achieve **maximum range** for a **battery powered UAV** with an electrically driven propeller.

- a. Climbing flight, constant angle of attack
- b. Climbing flight, constant thrust setting
- c. Horizontal flight, decreasing airspeed
- d. Horizontal flight, constant airspeed
- e. Descending flight, constant angle of attack
- f. Descending flight , constant thrust setting

**Question 5 (Flight Mechanics - Descent) [30 points, open question]**

A commercial passenger aircraft has a fuel shortage and has to perform a **gliding flight** towards the nearest airport. It is performing a **symmetric descent** with a **constant equivalent airspeed** of 150 [m/s] in the troposphere of the international standard atmosphere. In this emergency situation, the distance flown is critical for the survival of the passengers. In order to make an accurate calculation of the gliding performance, it **cannot be assumed** that the aircraft is performing a **straight flight**. The following data are given to describe the atmospheric conditions at sea level:

$$\rho_0 = 1.225 \text{ [kg/m}^3\text{]}, \quad p_0 = 101325 \text{ [N/m}^2\text{]}, \quad T_0 = 288.15 \text{ [K]}$$

- [questions a, b, c combined 10 points] Draw the Free Body Diagram (FBD) and the Kinetic Diagram (KD) visualizing all forces and accelerations that act on the aircraft for this particular flight condition. Draw the aircraft with a certain pitch angle  $\theta$ , flight path angle  $\gamma$  and angle of attack  $\alpha$ . Also indicate the direction of the velocity vector.
- [questions a, b, c combined 10 points] Derive the corresponding equations of motion for this flight condition using the FBD and KD. Clearly state the assumptions that you make (if any).
- [questions a, b, c combined 10 points] Derive the power equation by multiplying the equation of motion with the airspeed.
- [5 points] Using the power equation, derive the general relation between the rate of descent in unsteady flight and the rate of descent in steady flight  $RD/RD_{st}$  in terms of the quantity  $dV/dH$ .
- [3 points] If the climb is performed with **constant equivalent airspeed**, what is the resulting expression for  $RD/RD_{st}$ ?
- [2 points] The aircraft is flying at 5000 m. Calculate the ratio  $RD/RD_{st}$  for this condition. You can assume the following value for the variation of air density with altitude:

$$\left. \frac{d(1/\rho)}{dH} \right|_{H=5000m} = 1.45 \cdot 10^{-4} \text{ [kg/m}^4\text{]}$$

The horizontal distance  $s$  covered during the descent follows from a simple geometrical relation (as shown below) and can be calculated using the following expression:

$$s = \int_{H_1}^{H_2} \frac{dH}{\tan \gamma} \approx \int_{H_1}^{H_2} \frac{dH}{\sin \gamma}$$


Furthermore, the flight path angle  $\gamma$  and rate of descent  $RD$  are closely related:

$$\frac{\sin \gamma}{\sin \gamma_s} = \frac{RD}{RD_s}$$

- [5 points] Using the equations stated above and the **energy height principle**, derive the following

$$\text{equation: } s = \int_{H_{e1}}^{H_{e2}} \frac{dH_e}{\sin \gamma_s}$$

- [5 points] Is the **horizontal distance**  $s$  travelled during a **steady symmetric** descent larger, smaller or equal to a **symmetric** descent with **constant equivalent airspeed**? Give a thorough (physical) explanation in your answer. (Hint, make use of the equation derived in g.)

**Question 6 (Orbital Mechanics) [18 points, open question]**

The following equation describes an arbitrary Earth-repeat orbit:

$$j \left| -2\pi \frac{2\pi \sqrt{a^3 / \mu}}{T_E} - \frac{3\pi J_2 R_e^2 \cos(i)}{a^2 (1-e^2)^2} \right| = k 2\pi$$

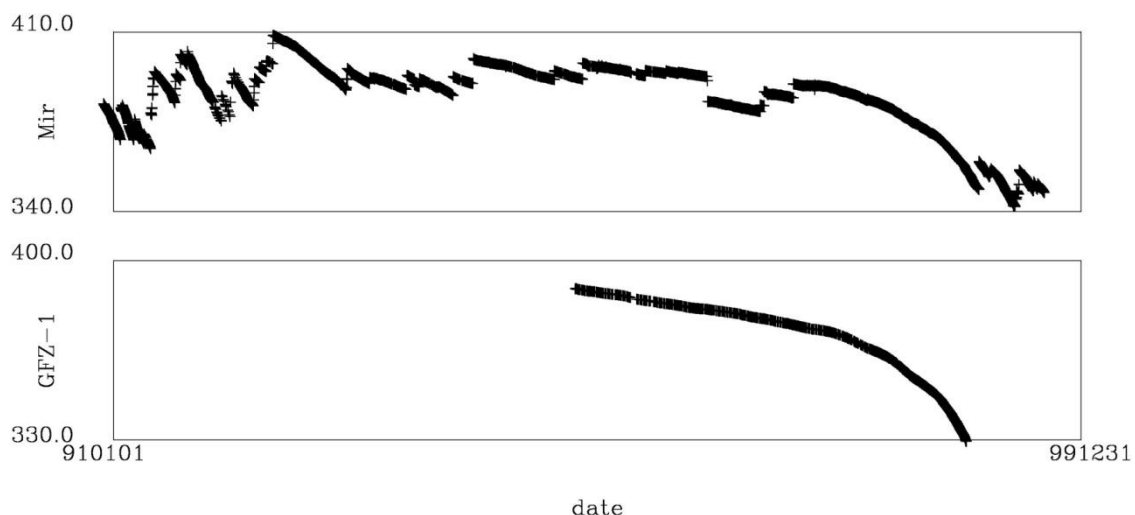
- [5 points] Consider the situation where the semi-major axis has a value of 7300 km,  $j = 478$  and  $k = 35$ . What is the required inclination for the satellite to be in a circular Earth-repeat orbit?
- [5 points] Consider the situation where the semi-major axis has a value of 7300 km,  $j = 493$  and  $k = 35$ . What is the required inclination for the satellite to be in a circular Earth-repeat orbit?
- [3 points] What is the orbital period of a satellite with a semi-major axis of 7300 km?
- [5 points] What is the repeat period of the situation of question (a)? What is it of question (b)? How do you explain the difference? In case you could not answer question (c), use a value of 100.0 minutes as the answer of that question.

Data:  $\mu_{\text{Earth}} = 398600.4415 \text{ km}^3/\text{s}^2$ ;  $T_E = 23\text{h}56\text{m}4\text{s}$ ;  $R_e = 6378.137 \text{ km}$ ;  $J_2 = 1082 \times 10^{-6}$ .

**Question 7 (Orbital Mechanics) [12 points, open question]**

The picture below shows the behavior of the orbital altitude of the space station Mir (top) and the German spherical satellite GFZ-1 (bottom). Mention 4 characteristic aspects of these two plots (individually and/or with respect to each other), and discuss each one briefly.

Data: cross-sectional area Mir = 250 m<sup>2</sup>; mass Mir = 140,000 kg; diameter GFZ-1 = 22 cm; mass GFZ-1 = 22 kg.



**Question 8 (Orbital Mechanics) [20 points, open question]**

Consider a Hohmann transfer from Earth to Mercury. Begin and end of the transfer is in a parking orbit at 500 km altitude, for both cases. Assume that both Earth and Mercury orbit the Sun in perfectly circular orbits.

- a. [3 points] What are the semi-major axis and the eccentricity of the transfer orbit?
- b. [2 points] What is the trip time?  
If you could not answer question (a), use a value of 0.5 AU as the answer of that question.
- c. [2 points] What are the velocities of Earth and Mercury?
- d. [6 points] What are the excess velocities at Earth and at Mercury (i.e., heliocentric)?
- e. [2 points] What are the circular velocities in the parking orbit around Earth and Mercury (i.e., planetocentric)?
- f. [5 points] What are the  $\Delta V$ 's of the maneuvers to be executed in the pericenter of the hyperbola's at Earth and Mercury? What is the total  $\Delta V$ ? If you could not answer question (d), use values of 7.0 and 9.0 km/s for the excess velocities at Earth and Mercury, respectively.  
If you could not answer question (e), use values of 7.0 and 2.0 km/s for the circular velocity around Earth and Mercury, respectively.

Data:  $\mu_{\text{Sun}}=1.3271 \times 10^{11} \text{ km}^3/\text{s}^2$ ;  $\mu_{\text{Earth}}=398600 \text{ km}^3/\text{s}^2$ ;  $\mu_{\text{Mercury}}=22034 \text{ km}^3/\text{s}^2$ ;  $R_{\text{Earth}}=6378.137 \text{ km}$ ;  
 $R_{\text{Mercury}}=2440 \text{ km}$ ; distance Earth-Sun = 1 AU; distance Mercury-Sun = 0.387 AU; 1 AU =  $149.6 \times 10^6 \text{ km}$ .