Flight and Orbital Mechanics

Exams

Exam AE2104: Flight and Orbital Mechanics (4 November 2011)

Remarks:

Please put your name, student number and ALL YOUR INITIALS on your work. Answer all questions and put your name on each page of your exam.

This exam consists of questions: 1, 2, 3a-e, 4a-b, 5a-f, 6a-g, 7a-d

Derive the expressions for each required calculation (unless mentioned in the file '*equations by heart*', for the space part).

The way the answer is obtained should be clearly indicated by visibly substituting the numbers in the formulas. Only mentioning the final answer will NOT result in any credits. Use of pencils to write the exam is NOT permitted. Scrap paper may not be added to your exam work (please take the scrap paper with you after the exam). It is not permitted to have any pre-programmed information on your calculator. The memory of your calculator should be erased prior to the start of the exam. Failure to do so will be seen as fraud.

In total 100 points can be earned. (50 points for flight mechanics and 50 points for orbital mechanics). At least 55 points are required to pass the exam.

Good luck!

Question 1 (Flight Mechanics) [5 points, Multiple choice]

To describe the motion of an airplane, four coordinate systems are used. In order to describe the attitude of an airplane (**body axes**), with respect to the **moving earth axis system**, three Euler angles are used. The sequence in which these angles is very important. Give the correct sequence of these angles to obtain the orientation of the body axes, starting from the moving earth axes.

A. Angle of roll (ϕ), Angle of pitch (θ), Angle of yaw (ψ)
B. Angle of roll (ϕ), Angle of yaw (ψ), Angle of pitch (θ)
C. Angle of pitch (θ),Angle of roll (ϕ), Angle of yaw (ψ)
D. Angle of pitch (θ), Angle of yaw (ψ), Angle of roll (ϕ)
E. Angle of yaw (ψ), Angle of roll (ϕ), Angle of pitch (θ),
F. Angle of yaw (ψ), Angle of pitch (θ), Angle of roll (ϕ)

Question 2 (Flight Mechanics) Turn [5 points, Multiple choice]

A pilot wants to perform a steady coordinated turn. He/she initiates the turn by banking the aircraft. After banking the aircraft he/she must:

A. increase the pitch attitude
B. increase the thrust
C. none of the above
D. increase the pitch attitude and the thrust

Question 3 (Flight Mechanics) [31$^2/_3$ points]

A subsonic turboprop airplane is flying a turn. The following characteristics apply to this aircraft:

Aircraft Weight (*W*):	100 [kN]
Wing area (*S*):	23.3 [m^2]
Wing span (*b*):	14.6 [m]
Zero lift drag coefficient (C_{D0}):	0.04
Oswald factor (*e*):	0.6
Parabolic lift drag polar:	$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$
Maximum power available ($P_{a,max}$)	1000 [kW]

The lift drag polar was estimated at *M* = 0.25 and is valid for low subsonic flight. **Maximum power available** can be assumed **independent of airspeed**

a. (a+b 11$^2/_3$ points) Draw clear free body diagrams and kinetic diagrams for at least two views of the aircraft when it is flying a **steady, horizontal and coordinated turn**. These diagrams should contain all relevant forces and accelerations.

b. Based on the diagrams of question a, derive the 3D equations of motion for **horizontal, steady** and **coordinated** flight.

The maximum load factor that can be attained in this flight condition can be represented with the following equation:

$$n_{max} = \frac{C_L}{C_D}\frac{T_{max}}{W}$$

c. (3$^1/_3$ points) Derive this equation, starting with the equations of motion from question b.

The aircraft is flying at Mach 0.25 at sea level conditions in the international standard atmosphere (ISA) for which the following conditions apply

Air density at sea level (ρ_0):	1.225 [kg/m^3]
Acceleration of gravity at sea level (g_0):	9.80665 [m/s^2]
Sea level temperature (T_0):	288.15 [K]
Sea level pressure (p_0):	101325 [N/m^2]
Specific gas constant of air (*R*):	287.05 [m^2/s^2 K]
Ratio of specific heats of air:	$\gamma = \frac{c_p}{c_v} = 1.4$

d. (8 $^1/_3$ points) Demonstrate that the lift coefficient (C_L) for the maximum load factor condition at Mach 0.25 equals 1.13

e. (8 $^1/_3$ points) Calculate the **minimum time to turn (180 degrees)** at Mach 0.25

Question 4 (Flight Mechanics) [8$^1/_3$ points]

The fuel consumption of a jet aircraft can be represented with the following equation:

$$F = c_T T$$

The thrust specific fuel consumption (c_T) can be assumed constant.

a. (5 points) Derive an equation for the range of a jet aircraft when flying at constant angle of attack and constant airspeed.

b. ($3^1/_3$ points) Make a qualitative drawing of a payload range diagram

Question 5 (Orbital Mechanics) [18 points]

The ESA satellite GOCE (altitude 250 km, circular orbit) observes the gravity field of the Earth at this very moment. One of the elements of interest is the term $J_{3,1}$. The gravity potential of the Earth is given by the following equation:

$$U = -\frac{\mu}{r}[1-\sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda-\lambda_{n,m}))]$$

Here, $P_n(\sin\delta)$ and $P_{n,m}(\sin\delta)$ represent the Legendre polynomials and functions, respectively:

$$P_n(x) = \frac{1}{(-2)^n\, n!}\frac{d^n}{dx^n}(1-x^2)^n$$

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

a. (2 points) What do the parameters J_n, $J_{n,m}$ and $\lambda_{n,m}$ represent?

b. (2 points) How are the parameters r, δ and λ defined?

c. (2 points) Give the general expression to derive the radial acceleration from the potential formulation for the gravity field.

d. (6 points) Derive the general equation for the radial acceleration due to the term $J_{3,1}$ for an arbitrary satellite (i.e., without substituting any numbers).

e. (3 points) What is the equation for the radial acceleration due to $J_{3,1}$ as experienced at GOCE' altitude (expressed in numbers, still for arbitrary latitude and longitude)?

f. (3 points) What are the values for δ for which this radial acceleration changes sign?

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_{3,1} = -1.72\times10^{-6}$; $\lambda_{3,1} = -1.0°$.

Question 6 (Orbital Mechanics) [16 points]

Consider a sample return mission to an asteroid (i.e., from Earth to the asteroid and then back to Earth again).

a. (3 points) Compute the 1-way travel time for a Hohmann transfer from Earth to the asteroid.

b. (2 points) The synodic period can be computed with the following equation:

$$\frac{1}{T_{syn}} = \left|\frac{1}{T_1}-\frac{1}{T_2}\right|$$

What is the definition of a synodic period?

c. (2 points) Compute the synodic period for the system Sun-Earth-asteroid.

d. (2 points) Compute the mean motion of Earth around the Sun (normally labeled "*n*", here labeled ω_E), and of the asteroid around the Sun (labeled ω_{ast}) (both in [rad/s]).

e. (2 points) The following general equations give the total round-trip time T and the time to be spent at the asteroid t_{stay} (provided we do both transfers by means of a Hohmann trajectory, and the departure and target objects orbit the Sun in circular orbits):

$$t_{stay} = T - 2T_H = \frac{2\pi(N+1) - 2\omega_E T_H}{\omega_E - \omega_{ast}}$$

$$T = \frac{2\pi(N+1) - 2\omega_{ast} T_H}{\omega_E - \omega_{ast}}$$

Discuss the meaning of the two equations and the parameter *N* (i.e., give a physical interpretation).

f. (3 points) What are the values for the parameter *N*, the stay time, and the total trip time for a mission to this asteroid?

g. (2 points) Can we reduce the total trip time? If so, how? A qualitative answer is sufficient.

Data: μ_{Sun}=1.3271×10^{11} km^3/s^2; distance Earth-Sun = 1 AU; distance asteroid-Sun = 0.95 AU; 1 AU = 149.6×10^6 km.

Question 7 (Orbital Mechanics) [16 points]

a. (6 points) Consider a single-stage rocket. Using the definitions of the payload fraction p ($p=M_{payload}/M_{begin}$) and the structural mass fraction σ ($\sigma=M_{structure}/M_{propellant}$), derive the following relation:

$$\frac{M_{begin}}{M_{end}} = \frac{1+\sigma}{p+\sigma}$$

Here, M_{begin} is the total mass of the launcher before ignition, and M_{end} is the total mass after engine burnout.

b. (4 points) Now consider a multi-stage launcher, where the payload fractions of the individual stages are identical (i.e. $p_{tot} = p_i^N$, where p_{tot} is the payload fraction of the entire launcher, p_i is the payload fraction of an individual stage *i*, and *N* is the total number of stages), the specific impulse is identical for the rocket engines of all stages, and the structural mass fraction σ is also the same for all stages. Derive the following equation for the total velocity gain provided by the launcher:

$$\Delta V_{tot} = I_{sp}\, g_0\, N\,\{\ln(1+\sigma) - \ln(\sigma + \sqrt[N]{p_{tot}})\}$$

c. (3 points) Compute the total velocity gain for a rocket with I_{sp} = 400 s, p_{tot} = 0.02 and σ=0.08, for *N*=1, 2 and 3.

d. (3 points) Discuss the advantages and disadvantages of replacing a single-stage launcher with a multi-stage one. What number of stages would you choose?

Data: g_0 = 9.81 m/s^2.