Flight and Orbital Mechanics

Exams

Exam AE2104-11: Flight and Orbital Mechanics (2 November 2012, 14.00 – 17.00)

Please put your name, student number and ALL YOUR INITIALS on your work. Answer all questions and put your name on each page of your exam.

This exam consists of questions: 1a-b, 2a-g, 3a-e, 4a-e, 5a-h

Derive the expressions for each required calculation (unless mentioned in the file 'equations by heart', for the orbital mechanics part).

The way the answer is obtained should be clearly indicated by visibly substituting the numbers in the formulas. Only mentioning the final answer will NOT result in any credits. Use of pencils to write the exam is NOT permitted. Scrap paper may not be added to your exam work (please take the scrap paper with you after the exam). It is not permitted to have any pre-programmed information on your calculator. The memory of your calculator should be erased prior to the start of the exam. Failure to do so will be seen as fraud.

In total 100 points can be earned. (50 points for flight mechanics and 50 points for orbital mechanics). At least 55 points are required to pass the exam.

Good luck!

Question 1 (Flight Mechanics - Cruise) [18 points]

A large transport aircraft with four engines is performing a cruise flight at 11km altitude. At the start of the cruise flight, the following data are available for this aircraft:

Aircraft Weight (W_{start}):	3500 [kN]
Fuel weight (W_f):	1340 [kN]
Wing surface area (S):	520 [m^2]
True airspeed (V_{start}) :	936 [km/hr]
Air density at 11km altitude (ρ_{11000}) :	0.3648 [kg/m^3]
Temperature at 11km altitude (T_{11000}):	216.7 [K]
Thrust specific fuel consumption (c_T):	0.65 [N/N hr] (constant and independent of airspeed)
Gas constant of air (R):	287.05 [m^2 / s^2 K]

The aircraft is performing the cruise flight at **constant altitude** and **constant angle of attack**.

a. (12 points) Calculate the minimum aspect ratio (A) that will allow this aircraft to fly a distance (range) of 10000km in the cruise phase with the amount of fuel given above. The lift drag polar of the aircraft is the following:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

$$C_{D_0} = 0.018$$

$$e = 0.85$$

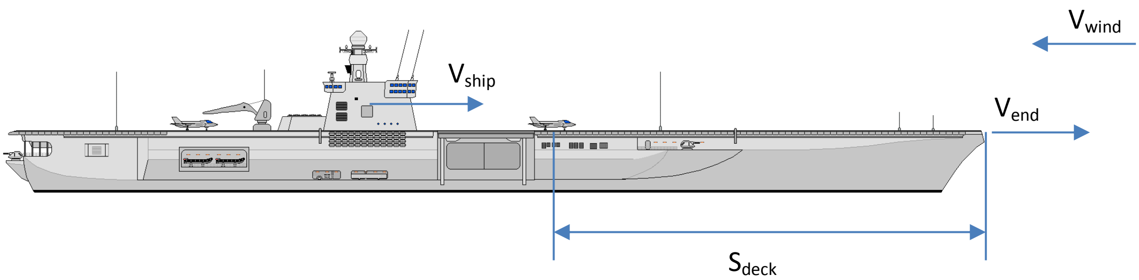
$$A = unknown$$

b. (6 points) Next, calculate the required span of this aircraft and the Mach number at the start and end of the cruise flight.

Question 2 (Flight Mechanics - Take-off, Equations of motion) [32 points]

Aircraft carriers make use of catapult systems to launch aircraft from the limited distance available on the deck. During the launch, maximum thrust is also applied by the aircraft. In general, the ship will have a forward speed into the direction of the wind (as indicated in the picture), to improve the take-off performance. The following data are available:

Wind speed (V_{wind})	10 [knots] (= 5.1 m/s)
Ship speed (V_{ship})	35 [knots] (= 18 m/s)
Distance of runway (s_{deck})	100 [m]
Catapult force ($T_{catapult}$):	100 [kN] (**constant force**)
Aircraft weight (W):	200 [kN]
Maximum aircraft thrust (T_{max}):	120 [kN] (**independent of airspeed**)
Wing surface area (S):	46.5 [m^2]
Lift coefficient in ground run attitude:	0.1
Zero lift drag coefficient (C_{D0})	0.024
Lift drag polar:	$C_D = C_{D_0} + kC_L^2$
Factor k in lift drag polar:	0.095
Coefficient of landing gear friction (μ):	0.04
Air density at sea level (ρ):	1.225 [kg/m^3]
Gravitational acceleration (g)	9.80665 [m/s^2]



a. (a + b total 9 points) Draw a clear free body diagram (FBD) and kinetic diagram (KD) in which all the relevant forces, accelerations, angles and velocities are indicated
b. Derive the equations of motion for the aircraft during the acceleration over the ship deck

You can assume for this catapult assisted take-off that the ground run is a uniformly accelerated motion. The mean acceleration must be calculated at the characteristic **airspeed** $V_{end,air}/\sqrt{2}$

c. (3 points) Derive an expression for the ground run distance s_{deck} in terms of a mean acceleration <u>a</u> and the speed at the moment the aircraft leaves the deck (V_{end}). Clearly indicate if the velocity in the equation is expressed relative to the air or relative to the ship.
d. (8 points) Derive an equation for the average acceleration as a function of the speed at the moment the aircraft leaves the deck (V_{end}). Clearly indicate if the velocity in the equation is expressed relative to the air or relative to the ship.

e. (3 points) Write an expression that gives the relation between V_{end} relative to the air (**airspeed**) and V_{end} relative to the ship deck.
f. (4 points) Calculate the both the **airspeed** and the **speed of the aircraft relative to the ship** at the moment the aircraft reaches the end of the ship deck.
g. (5 points) The minimum **airspeed** of this fighter aircraft for the given aircraft weight is 55 m/s. Is the catapult force still sufficient to launch this aircraft in case the ship is not moving and when there is no wind present? Give a thorough explanation with your answer.

Question 3 (Orbital Mechanics) [18 points]

An essential aspect of a satellite mission is the occurrence of solar eclipses.

a. (5 points) For what (3) subsystems plays the presence (and absence) of solar radiation a role? Discuss each situation briefly (2-3 lines each).
b. (4 points) For an eclipse to occur, 2 conditions have to be satisfied. What are they? Illustrate in a sketch.
c. (4 points) Consider a 2-dimensional situation, where the Sun is located in the orbital plane of the satellite. What is the length of the eclipse period for a satellite in a circular orbit at 800 km altitude (expressed as percentage of the orbital period)? What is it for a geostationary satellite?
d. (2 points) Give the definition of a sun-synchronous orbit.
e. (3 points) Is it possible to select a satellite orbit such that the satellite is in full sunlight throughout its mission lifetime (*e.g.,* 10 years)? If so, what are the (3) conditions for this? Is this solution applied?

Data: R_e = 6378.137 km, μ_{earth} = 398600.4415 km^3/s^2, h_{GEO} = 35786 km.

Question 4 (Orbital Mechanics) [17 points]

The gravity field of the Earth is dominant for the motion of spacecraft orbiting Earth. It is given by the following equation:

$$U = -\frac{\mu}{r}\left[1-\sum_{n=2}^{\infty} J_n\left(\frac{R_e}{r}\right)^n P_n(\sin\delta)+\sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda-\lambda_{n,m}))\right]$$

Where

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

$$P_n(x) = \frac{1}{(-2)^n\, n!}\frac{d^n}{dx^n}(1-x^2)^n$$

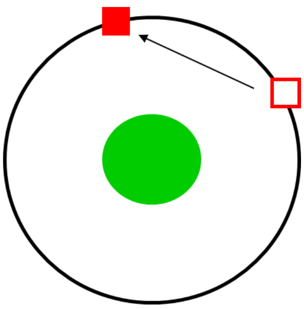
a. (2 points) What is the general definition for the East-West acceleration, based on the potential formulation for the gravity field of the Earth?
b. (7 points) Derive the general equation for the East-West acceleration due to the term (2,2) for an arbitrary satellite.

c. (2 points) Compute the orbit radius of a geostationary satellite.
d. (3 points) What is the (numerical) expression for the East-West acceleration due to $J_{2,2}$ for a geostationary satellite?
e. (3 points) The resulting equation has the following form: $acc_{EW;2,2}$ = constant × $\sin(2(\lambda-\lambda_{2,2}))$. What are the locations of the equilibrium points?

Data: μ_{Earth} = 398600.4415 km^3/s^2; $T_E = 23^h56^m4^s$; $J_{2,2} = 1.816\times10^{-6}$; $\lambda_{2,2}$ = -14.9°; R_e = 6378.137 km.

Question 5 (Orbital Mechanics) [15 points]

Consider the situation where the cargo vehicle Dragon is to dock with the International Space Station (ISS). Both are orbiting Earth in a circular (coplanar) orbit at 400 km altitude. The ISS is 90° ahead of the Dragon vehicle. You are responsible for designing the transfer between the two, where Dragon is the active vehicle and the ISS the (passive) target. In order to catch up with the ISS, Dragon will be taken to an elliptical (*i.e*. non-circular) transfer orbit with a different orbital period.



a. (1 point) In order to rendezvous with the ISS, would you lower or raise the orbit of the Dragon vehicle? Give an argumentation on your choice.
b. (1 point) What is the orbital period at an altitude of 400 km?
c. (2 points) What is the circular velocity at an altitude of 400 km?
d. (2 points) Consider the situation that one wants to have Dragon complete 15 revolutions in its elliptical transfer orbit, before it does the rendezvous with the ISS. What is the required shift per revolution of Dragon in its transfer orbit, w.r.t. the ISS?
e. (3 points) What is the orbital period of this transfer orbit, where the total transfer is to be completed after exactly 15 revolutions? If you were unable to answer question (b), use a value of 100 minutes for the orbital period in the original orbit.
f. (2 points) What is the semi-major axis of this transfer orbit?
g. (2 points) What is the velocity in the transfer orbit at the original altitude of 400 km? In case you could not compute an answer for question (f), use a value of the semi-major axis of 6700.0 km.
h. (2 points) What is the required total velocity change for this transfer scenario?

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km.