Flight and Orbital Mechanics

Exams

TU Delft

Exam AE2104 – Flight and Orbital Mechanics - 29 October

Remarks:

Please put your name and ALL YOUR INITIALS on your work. Answer all questions and put your name on each page of your exam.

This exam consists of questions: **1a-d, 2, 3a-d and 4a-d, 5a-e, 6a-g and 7a-e**

Derive the expressions for each required calculation.

The way the answer is obtained should be clearly indicated by visibly substituting the numbers in the formulas. Only mentioning the final answer will **NOT result in any credits** use of pencils to write the exam is **NOT permitted**. Scrap paper may not be added to your exam work (please take the scrap paper with you after the exam). It is **not permitted** to have any pre-programmed information on your calculator. The memory of your calculator should be erased prior to the start of the exam. Failure to do so will be seen as fraud.

In total 100 points can be earned. Good luck!

Question 1 – Flight mechanics (20 points)

The following data are given for a single engine propeller aircraft:

Aircraft weight:	$W = 14$ [kN]
Wing surface area:	$S = 13$ [m^2]
Maximum power available at 1000 [m] ISA:	$P_a = 85$ [kW]

Power available can be assumed independent of airspeed.

A steady symmetric climb is conducted at 2000m altitude in the international standard atmosphere ($\rho = 1.0065\ kg/m^3$). The climb is performed at the airspeed which results in the maximum (steady) rate of climb. The maximum rate of climb of this aircraft is 4 m/s.

a. (5 points) Draw clear free body diagrams and kinetic diagrams for at least two views of the aircraft when it is flying a steady, horizontal and coordinated turn. These diagrams should contain all relevant forces and accelerations.
b. (2 points) Based on the diagrams of question a, derive the 3D equations of motion for horizontal, steady and coordinated flight.

c. (10 points) Calculate the maximum bank angle of this aircraft when it is flying at the same power setting ($P_{a,max}$), the same altitude and the same angle of attack as in the climbing flight described above.
d. (3 points) Make one clear qualitative drawing of the performance diagram of this aircraft with the power curves for both the climbing flight and the turning flight. Clearly indicate both flight conditions described above (maximum rate of climb in symmetric flight and maximum bank angle in turning flight) in the same diagram.

Question 2 – Flight mechanics (5 points)

A typical propeller aircraft is performing a climb in the international standard atmosphere at constant equivalent airspeed and with the maximum power setting. The power available of this aircraft can be assumed independent of airspeed and decreases with altitude according to the following relation:

$$P_{a,\max} = P_{a,\max,sea-level}\left(\frac{\rho}{\rho_0}\right)^{0.72}$$

Fill in the correct answer in the following sentence:
It can be said that the aircraft is performing a ………. flight

A: Straight (quasi-rectilinear) and steady
B: Straight (quasi-rectilinear) and unsteady
C: Curved and steady
D: Curved and unsteady

(For this question it is sufficient to answer with A, B, C or D without explaining your answer)

Question 3 – Flight mechanics (25 points)

A propeller aircraft used for reconnaissance of pollution in the Pacific Ocean has to be designed to be able to fly a total distance (range) of 4000km at the airspeed $1.1V_{Dmin}$. (V_{Dmin} is the airspeed at minimum drag). The flight altitude which is most suitable for monitoring is 5000m. This altitude is therefore kept constant. At 5000m altitude, the fuel flow of this aircraft can be represented with the following equation.

$$F = c_P P_{br}$$
$$c_P = 0.93 \cdot 10^{-7} \text{ [kg/s W]}$$

The specific fuel consumption c_p at this altitude can be assumed constant. The relation between power available P_a and shaft power P_{br} is given with the following equation:

$$P_a = \eta P_{br}$$

The propeller efficiency η equals 35% and can be assumed constant. Power available can be assumed independent of airspeed.

The fuel weight equals 40% of the total aircraft weight at the start of the cruise flight. Additionally, the lift drag polar of the aircraft is given:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$
$$C_{D_0} = 0.018$$
$$e = 0.82$$

Note that the aspect ratio A is unknown.

a. (7 points) Derive an equation for the lift coefficient C_L that is needed to fly at $1.1V_{Dmin}$. The lift coefficient should be a function of C_{D0}, e and A. (Hint: first determine the lift coefficient needed at the minimum drag airspeed V_{Dmin}) Note: include a full derivation in your answer!
b. (7 points) Derive an equation for the range of this propeller aircraft. Clearly state the assumptions that you make during the derivation, if any. The equation should be a function of specific fuel consumption c_P, efficiency η lift coefficient C_L, drag coefficient C_D and aircraft weight W.
c. (8 points) Calculate the minimum aspect ratio A $(=b^2/S)$ that the aircraft must have such that it is able to achieve the specified range of 4000km.
d. (3 points) When the whole flight is considered; is the (a) aircraft slowly accelerating, (b) slowly decelerating or (c) flying at a constant airspeed? Give an explanation with your answer.

Question 4 – Orbital mechanics (10 points)

The gravity potential of the Earth is given by the following equation:

$$U = -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda - \lambda_{n,m}))\right]$$

Here, $P_n(\sin\delta)$ and $P_{n,m}(\sin\delta)$ represent the Legendre polynomials and functions, respectively:

$$P_n(x) = \frac{1}{(-2)^n\, n!}\frac{d^n}{dx^n}(1-x^2)^n$$
$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

a. (2 points) Give the general expression to derive the North-South acceleration from the potential formulation for the gravity field.

b. (4 points) Derive the general equation for the North-South acceleration due to the term J_2 for an arbitrary satellite.
c. (2 points) What is the equation for the North-South acceleration due to J_2 for a satellite at 500 km altitude (expressed in numbers, still for arbitrary latitude and longitude)?
d. (2 points) Make a sketch of this acceleration as a function of latitude ($-90° \le \delta \le 90°$).

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_2 = 1082\times10^{-6}$

Question 5 – Orbital mechanics (10 points)

The parameters of an Earth-repeat orbit have to satisfy the following equation:

$$j\ \left|\Delta L_1 + \Delta L_2\right| \quad = \quad k\ 2\,\pi$$

where

$$\Delta L_1 \quad = \quad -2\,\pi\,\frac{T}{T_E} \qquad \text{[rad/rev]}$$

$$\Delta L_2 \quad = \quad -\frac{3\,\pi\ J_2\ R_e^2\ \cos i}{a^2\ (1-e^2)^2} \qquad \text{[rad/rev]}$$

a. (1 point) What is the main characteristic of an Earth-repeat orbit?
b. (1 point) What is the main characteristic of a Sun-synchronous orbit?
c. (4 points) Derive a general equation for the orbital period of a satellite that satisfies the requirements on Earth-repeat and Sun-synchronous orbits simultaneously.
d. (2 points) Compute the value for the semi-major axis for such an orbit for the Earth-repeat conditions (43,3).
e. (2 points) Compute the corresponding orbital inclination.

Data: $T_E = 23^h56^m4^s$; T_{ES} = 365.25 days; μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_2 = 1082\times10^{-6}$

Question 6 – Orbital mechanics (20 points)

Consider a transfer from a circular parking orbit at 185 km and i =29.8° (i.e., launch from Kennedy Space Center) to the International Space Station (h=400 km, e=0, i=55.6°).

a. (3 points) When is an in-plane maneuver (i.e., ΔV) most efficient?
b. (3 points) When is an out-of-plane maneuver most efficient?
c. (2 points) Compute the velocities in the original orbit and in the target orbit.
d. (6 points) Compute the total ΔV that would be required for the orbit raising, assuming that the two orbits are coplanar (Hohmann transfer).
e. (2 points) Compute the ΔV that would be required to only change the inclination of the original parking orbit to that of the ISS orbit.

f. (2 points) Compute the total ΔV if the sequence of maneuvers was (1) dog-leg maneuver (only) in initial orbit, and (2) Hohmann transfer.
g. (2 points) Compute the total ΔV if the Hohmann transfer to 400 km altitude is done first, and the inclination change is done next (i.e., at 400 km) in an independent maneuver.

Data: μ_{Earth} = 398600.4415 km^3/s^2, R_{Earth} = 6378.137 km

Question 7 – Orbital mechanics (10 points)

Consider a Hohmann transfer from an inner planet 1 to an outer planet 2.

a. (2 points) Derive the following general equations for the epoch of departure t_1 and the epoch of arrival t_2:

$$
\begin{aligned}
t_1 &= t_0 + \frac{\theta_2(t_0) - \theta_1(t_0) + n_2\, T_H - \pi}{n_1 - n_2} \\
t_2 &= t_1 + T_H
\end{aligned}
$$

Here, t_0 is a common reference epoch, T_H is the transfer time in a Hohmann orbit, n_1 and n_2 are the mean motion of the two planets, and θ_1 and θ_2 are the true anomalies of the planetary positions, respectively. Assume circular orbits for both planets.

b. (2 points) Consider a Hohmann transfer from Earth to Neptune. What is the transfer period?
c. (2 points) Assuming that on January 1, 2010, θ_{Earth}=70° and $\theta_{Neptune}$=240°, what would be the epoch of departure (expressed in days w.r.t. this January 1)?
d. (2 points) What would be the arrival epoch?
e. (2 points) Can we change the launch window? If so, how? A qualitative answer is sufficient.

Data: μ_{Sun}=1.3271×10^{11} km^3/s^2; distance Earth-Sun = 1 AU; distance Neptune-Sun = 30.1 AU; 1 AU = 149.6×10^6 km.