Delft University of Technology, Faculty of Aerospace Engineering

Exam AE3-914: Dynamics and Stability
Date: April 16, 2010, Time: 14:00 – 17:00

Question 1 (3.5 points)

A *spherical pendulum* composed of a mass m and a spring k experiences a rotation $\phi(t)$ about the vertical axis and a rotation $\theta(t)$ about the z-axis, where the right-handed x-y-z reference frame is attached to the pendulum. The position of the mass with respect to the origin of the x-y-z system is $r(t)$, as measured along the x-axis. The spring is unstretched when $r = 0$. The gravity acceleration is g.

a) Derive the absolute velocity of the mass in terms of the base vectors of the x-y-z reference system.

b) Construct the kinetic energy of the system, using the result of the above question.

c) Construct the Lagrangian.

d) Find the equations of motion.

e) Formulate two different integrals of motion.

f) Construct the Routhian.

g) Identify the effective potential from the Routhian.

h) Establish the conditions for steady motion.

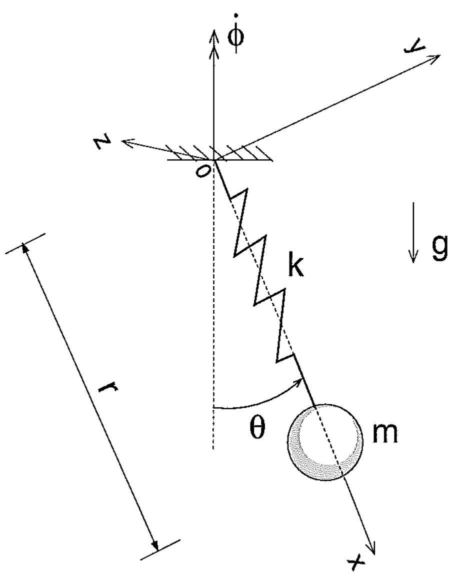

Figure 1: A spherical pendulum composed of mass m and spring k.

Question 2 (1.5 points)

A *homogeneous cube* of mass *m* and edges of length *a* rotates along one edge about the *z*-axis with a rotation $\theta(t)$. The mass moment of inertia of the cube about the *z*-axis, measured with respect to its centre of mass *C*, is $\overline{I}_{z_c z_c} = \frac{1}{12} m a^2$. The gravity acceleration is *g*.

a) Formulate the angular momentum with respect to the edge along which the cube rotates.

b) Use the expression for the angular momentum to derive the corresponding equation of motion.

c) Give the expression for the eigen frequency of the rotating cube.

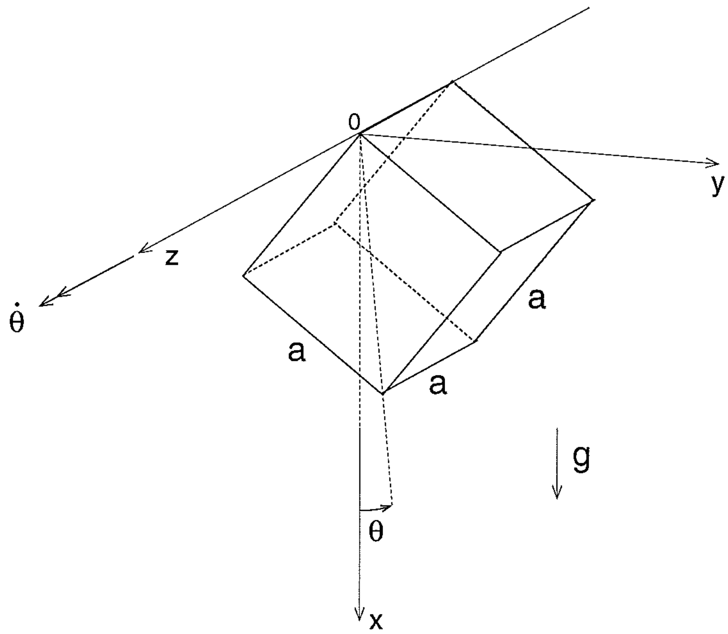

Figure 2: A rotating cube with mass *m* and edges of length *a*.

Question 3 (3.0 points)

A bar with length *l*, mass density ρ, stiffness *E* and cross-section *A* is subjected to a distributed load *q(x)* in the vertical *x*-direction. The vertical displacement measured along the bar is *u(x,t)*. The effect of gravity can be ignored.

a) Derive the generalised potential density related to the distributed load *q(x)*, and add this to the *Lagrangian density.* (*Hint:* The elastic potential energy density of the bar is $\tilde{\mathcal{V}} = \frac{1}{2} EA\, u_x^2$.)

b) Using *Hamilton's principle* for conservative systems, derive the equation of motion for the bar, and identify the natural and essential boundary conditions.

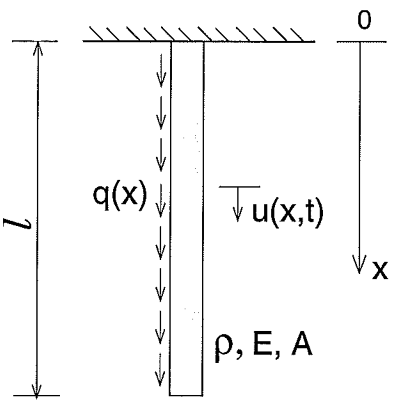

Figure 3: A bar subjected to a distributed load *q(x)*.

Question 4 (2.0 points)

A structure composed of two rigid bars of length *l* is subjected to a force *F*. At the left support the structure has resistance against rotation through the rotational spring k_r. The rotational spring is unstretched when the angle $\theta = 0$. The hinges in the middle of the structure and at the right support can freely rotate. In addition, the right support can move in de horizontal direction. The effect of gravity may be ignored.

a) Find the generalized force related to the coordinate θ. What is the physical meaning of this force?

b) Construct the generalized potential related to the generalized force derived in the previous question.

c) Compute the acceptable range of values of *F* for which the horizontal position $\theta = 0$ is in stable equilibrium.

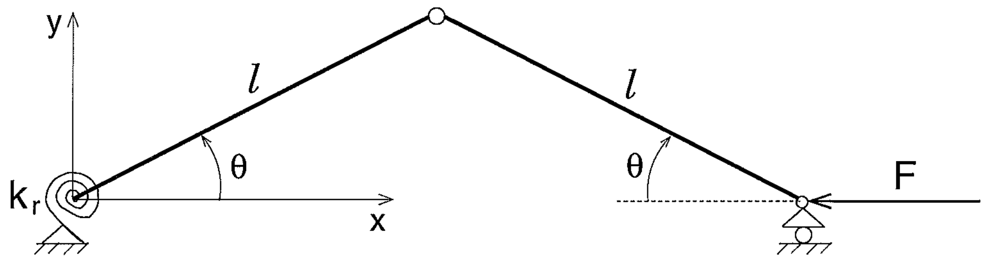

Figure 4: A system composed of two rigid bars of length *l* and a rotational spring k_r is subjected to a force *F*.