Delft University of Technology, Faculty of Aerospace Engineering

Exam AE3-914 / AE4-914: Dynamics and Stability
Date: April 14, 2011, Time: 18:30 – 21:30

Question 1 (2.5 points)

Figure 1 shows a cylinder of mass *m* and radius *R*, which rolls *without slipping* in a groove with radius *3R* of a block of mass *3m*. The rotational velocity of the cylinder is $\dot{\phi}(t)$. The block moves *frictionless* in the vertical direction along a guide, and is supported by a spring with stiffness *k*. The locations of the block and the cylinder are described by the vertical position of the centre of mass $z_b(t)$ of the block and the orientation $\theta(t)$ of the cylinder (measured in the inertial frame of reference *X-Y-Z)*. Note that the centre of mass $z_b(t)$ of the block is located at the intersection between the centerline of the block and the surface of the groove. The spring *k is unstretched* when the centre of mass is located at a vertical distance z_0 from the origin of the inertial frame of reference. The gravitational acceleration *g* acts in the vertical, downward direction.

a) Determine the translational velocity of the cylinder in terms of the generalized coordinates $z_b(t)$ and $\theta(t)$.

b) Construct the non-holonomic constraint for the rotational velocity $\dot{\phi}(t)$ of the cylinder.

c) Determine the kinetic energy of the system.

d) Determine the potential energy of the system.

e) Derive the equations of motion from the Lagrangian.

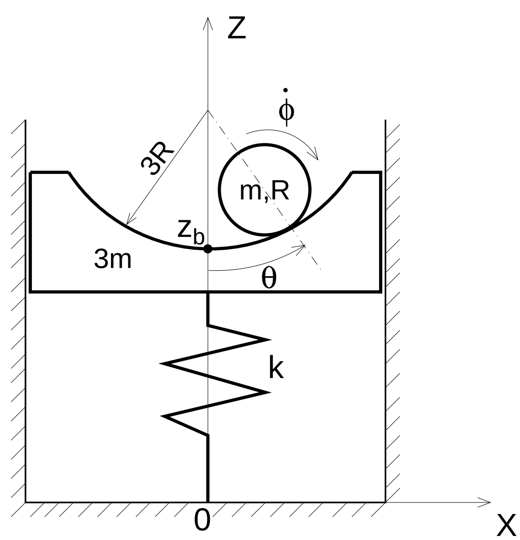

Figure 1: A cylinder of mass *m* rolls without slipping in a groove of a vertically moving block of mass *3m*.

Question 2 (2.5 points)

Figure 2 shows an antenna used for tracing satellites. As a result of the external driving moment *M(t),* the antenna rotates with a rotational velocity ω and a rotational acceleration $\alpha = d\omega / dt$ about *Z*-axis of the inertial frame of reference *X-Y-Z* (which is attached to the fixed base). The reaction moments at the fixed base generated by the antenna motion are M_x and M_y. The antenna is inclined at a *constant* angle θ, as measured between the *x*-axis of the non-inertial frame of reference *x-y-z* (connected to the rotating antenna) and the *Z*-axis of the inertial frame of reference. The mass moments of inertia of the antenna are *J* about the *z*-axis (i.e., the symmetry axis) and *I* about the *x*- and *y*-axes.

Note from Figure 2 that the *y*-axis of the non-inertial frame of reference and the *Y*-axis of the inertial frame of reference run parallel.

a) Derive an expression for the angular momentum $\boldsymbol{L}_0$ about the origin *0* of the non-inertial frame of reference *x-y-z* (in terms of the base vectors of this non-inertial frame of reference).

b) Use the expression for the angular momentum to derive the corresponding equations of motion about the origin *0*.

c) Determine expressions (in terms of the externally applied moment *M*) for *(i)* the angular acceleration α and *(ii)* the reaction moment M_x.

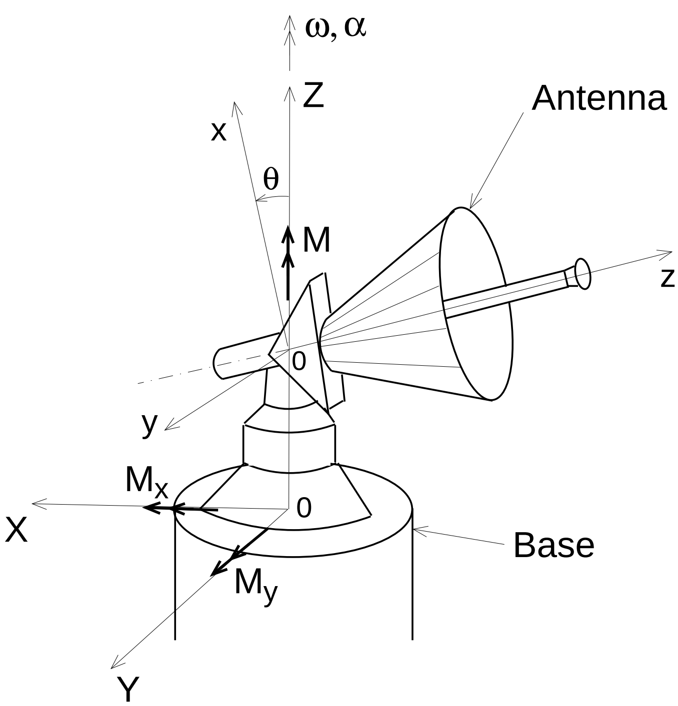

Figure 2: An antenna is connected through a rotational hinge (i.e., rotation about the *Z*-axis) to a fixed base.

Question 3 (2.5 points)

A string of length *L* with mass density $\rho(x) = \rho =$ constant (in mass/unit length) and tensile force $T(x) = T =$ constant is clamped at its left and right ends, and subjected to a distributed vertical load $q(x,t)$. The vertical displacement of the string is $y(x,t)$. The energy potential of the system is given by

$$V = \int_0^L \left(\frac{1}{2} T y_x^2 - qy \right) dx$$

where $y_x = \dfrac{\partial y(x,t)}{\partial x}$.

a) Formulate the kinetic energy of the string, and apply Hamilton's principle to derive the equation of motion.

b) Identify the boundary conditions and indicate whether these are natural and/or essential.

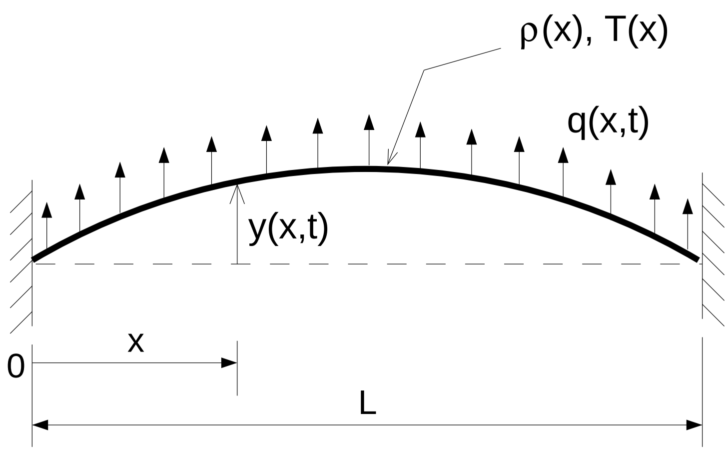

Figure 3: A clamped string of length *L* is subjected to distributed load $q(x,t)$.

Question 4 (2.5 points)

A block of mass *4m* is subjected at its center to a vertical load *F*. Two pendulums of mass *m* and length *l* vertically support the block. The rotation of the pendulums is $\theta(t)$. The resistance of the system against horizontal motion occurs by means of a discrete spring of stiffness *k*. The discrete spring is *unstretched* when $\theta=0$. The width of the block is *l* and the height is *h*. The gravitational acceleration is *g*.

a) Construct the Lagrangian of the system in terms of the generalized coordinate θ.

b) Find the corresponding equation of motion.

c) Investigate the stability of the system about its vertical equilibrium position $\theta = 0$, and express a stability requirement for the load *F*. The analysis method can be chosen freely.

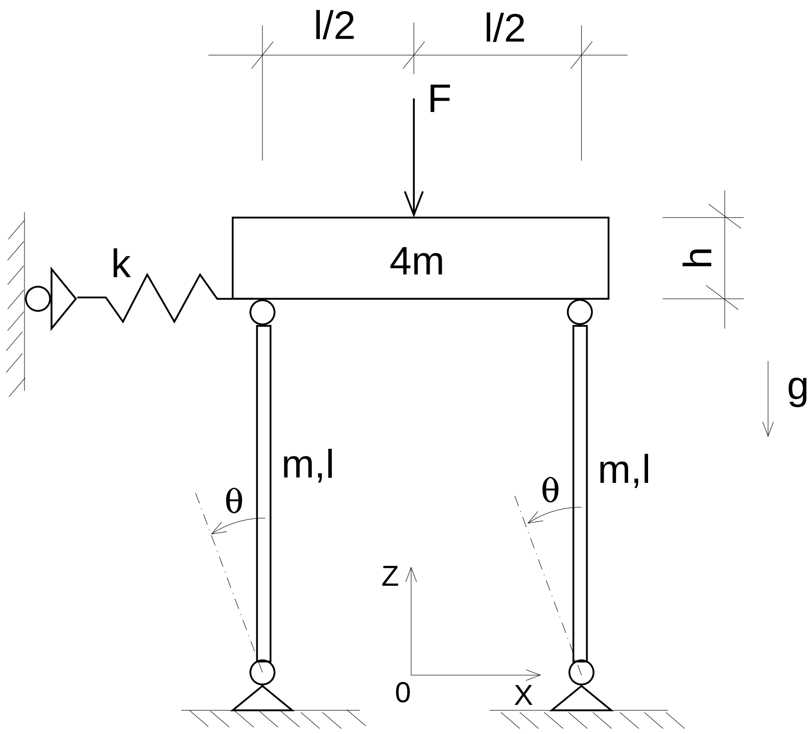

Figure 4: A block of mass *4m* is loaded by a vertical force *F* and is vertically supported by two pendulums of mass *m* and length *l*. The resistance against horizontal motion occurs through a spring of stiffness *k*.