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(1)

$$\omega = 3^\circ s^{-1} = 3 \cdot \frac{\pi}{180} \, \text{rad}/s = 0.052 \, \text{rad}/s$$

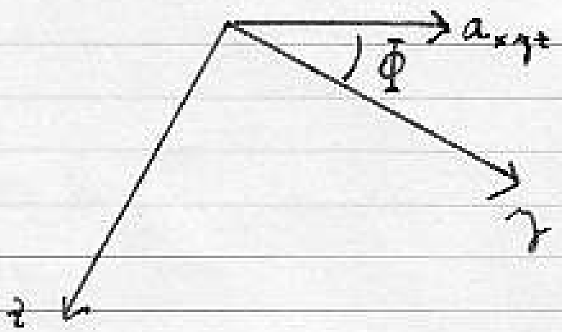
$$v = 900 \, \text{km}/h = 900 \cdot \frac{1000 \, m}{1 \, km} \cdot \frac{1 \, h}{3600 \, s} = 250 \, m/s$$

$$g = 9{,}81 \, m/s^2$$

a)

$$R = v/\omega = 250/0.052 = 4775 \, m$$

b)



$$a_{xyz} = \omega^2 R = \frac{v^2}{R} = \omega v = 13 \, m/s^2$$

$$\underline{a}_{xyz} = a_{xyz} \cos \Phi \, \underline{j} - a_{xyz} \sin \Phi \, \underline{k}$$

$$= 13 \cos \Phi \, \underline{j} - 13 \sin \Phi \, \underline{k}$$

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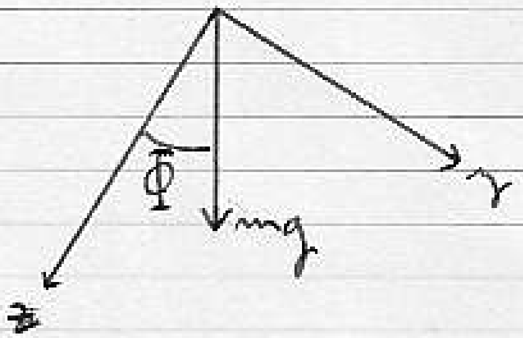
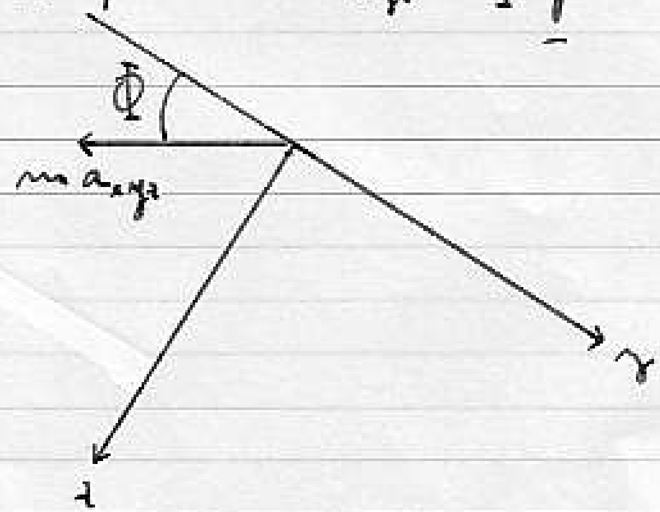
c) FBD:



$$\underline{F}_{fict} = -m\,\underline{a}_{xyz} = -m a_{xyz}\cos\Phi\,\underline{j} + m a_{xyz}\sin\Phi\,\underline{k}$$

FBD$_{fict}$:



$$\Sigma F_y + \Sigma F_{y\,fict} = mg\sin\Phi - m a_{xyz}\cos\Phi = 0$$

$$\tan\Phi = \frac{a_{xyz}}{g} = \frac{13}{9{,}81}$$

$$\Phi = 53^\circ$$

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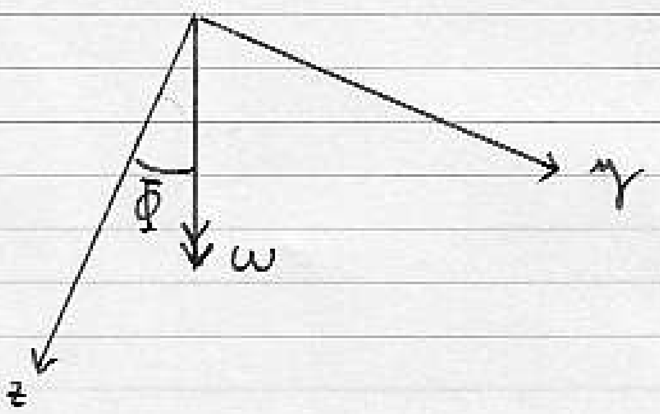
d. $v_{rel} = 4\ \text{km}/\text{h} = 4 \cdot \frac{1000\ \text{m}}{1\ \text{km}} \cdot \frac{1\ \text{h}}{3600\ \text{s}} = 1{,}1\ \text{m}/\text{s}$

$\underline{v}_{rel} = -v_{rel}\ \underline{i}$



$\underline{\omega} = \omega \sin\Phi\ \underline{j} + \omega \cos\Phi\ \underline{k}$

Coriolis term:

$$\underline{a}_{cor} = 2\ \underline{\omega} \times \underline{v}_{rel} = 2 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & \omega\sin\Phi & \omega\cos\Phi \\ -v_{rel} & 0 & 0 \end{vmatrix}$$

$$= -v_{rel}\,\omega\cos\Phi\ \underline{j} + v_{rel}\,\omega\sin\Phi\ \underline{k}$$

As experienced by the person:

$$\underline{a} = -\underline{a}_{cor} = v_{rel}\,\omega\cos\Phi\ \underline{j} - v_{rel}\,\omega\sin\Phi\ \underline{k}$$

$$= 0{,}034\ \underline{j} - 0{,}046\ \underline{k}\quad [\text{m}/\text{s}^2]$$

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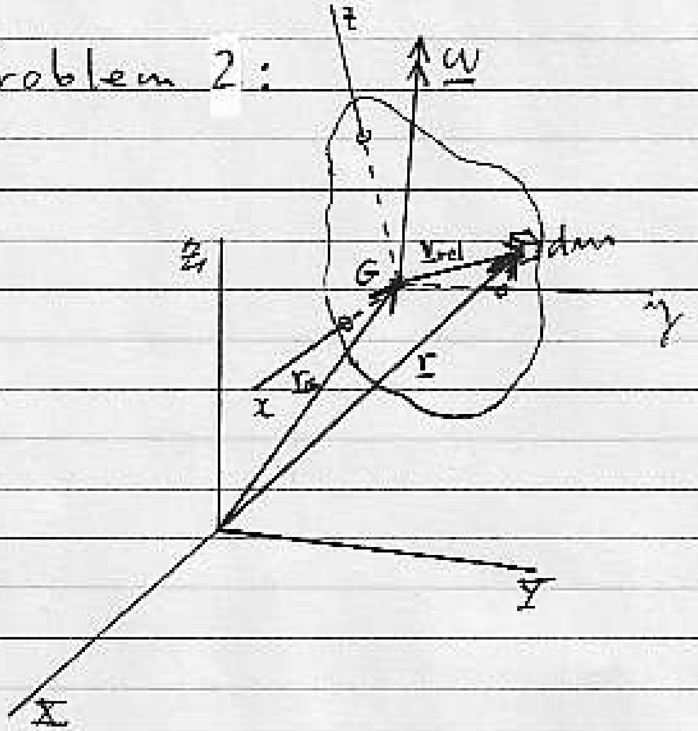
Problem 2:



$$\underline{r} = \underline{r}_G + \underline{r}_{rel}$$

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{d\underline{r}_G}{dt} + \frac{d\underline{r}_{rel}}{dt} = \underline{v}_G + \underline{\omega} \times \underline{r}_{rel}$$

$$T = \frac{1}{2}\int_v \underline{v}\cdot\underline{v}\, dm = \frac{1}{2}\int_v (\underline{v}_G + \underline{\omega}\times\underline{r}_{rel})\cdot(\underline{v}_G + \underline{\omega}\times\underline{r}_{rel})\, dm$$

$$= \frac{1}{2}\underline{v}_G\cdot\underline{v}_G\int_v dm + \underline{v}_G\cdot\left[\underline{\omega}\times\int_v \underline{r}_{rel}\, dm\right] + \frac{1}{2}\int_v (\underline{\omega}\times\underline{r}_{rel})\cdot(\underline{\omega}\times\underline{r}_{rel})\, dm$$

$$= \frac{1}{2} m v_G^2 + \underline{v}_G\cdot(\underline{\omega}\times\underline{Q}_G) + \frac{1}{2}\int_v (\underline{\omega}\times\underline{r}_{rel})\cdot(\underline{\omega}\times\underline{r}_{rel})\, dm$$

$\underline{Q}_G = \underline{0}$ (First moment of mass about G)

$$\frac{1}{2}\int_v (\underline{\omega}\times\underline{r}_{rel})\cdot(\underline{\omega}\times\underline{r}_{rel})\, dm = \longrightarrow$$

$$= \frac{1}{2} \int_v \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}^2 dm =$$

$$= \frac{1}{2} \int_v \left[(\omega_y z - \omega_z y)\, \underline{i} + (\omega_z x - \omega_x z)\, \underline{j} + (\omega_x y - \omega_y x)\, \underline{k} \right]^2 dm$$

$$= \frac{1}{2} \int_v \left[(\omega_y z - \omega_z y)^2 + (\omega_z x - \omega_x z)^2 + (\omega_x y - \omega_y x)^2 \right] dm$$

$$= \frac{1}{2} \int_v \Big[(y^2 + z^2)\, \omega_x^2 + (x^2 + z^2)\, \omega_y^2 + (x^2 + y^2)\, \omega_z^2$$
$$- 2xy\, \omega_x \omega_y - 2xz\, \omega_x \omega_z - 2yz\, \omega_y \omega_z \Big]\, dm$$

$$= \frac{1}{2} (\omega_x \quad \omega_y \quad \omega_z) \int_v \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -xy & x^2 + z^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{bmatrix} dm \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$= \frac{1}{2}\, \underline{\omega}^T \underline{\underline{I}}_G\, \underline{\omega}$$

$$T = \frac{1}{2} m v_G^2 + \underline{v}_G \cdot (\underline{\omega} \times \underline{0}) + \frac{1}{2}\, \underline{\omega}^T \underline{\underline{I}}_G\, \underline{\omega}$$

$$= \frac{1}{2} m v_G^2 + \frac{1}{2}\, \underline{\omega}^T \underline{\underline{I}}_G\, \underline{\omega}$$

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3

$$I[y] = \int_0^1 \left[y + y' + y y' + (y')^2 \right] dx$$

$$y = \alpha h_1(x) + \beta h_2(x) + \gamma h_3(x)$$
$$= \alpha + \beta x + \gamma x^2$$

$$y(0) = 0 \Rightarrow \alpha = 0$$

$$y = \beta x + \gamma x^2$$
$$y' = \beta + 2\gamma x$$

$$I[y] = \Phi(\beta, \gamma) = \int_0^1 \Big[(\beta x + \gamma x^2) + (\beta + 2\gamma x)$$
$$+ (\beta x + \gamma x^2)(\beta + 2\gamma x) + (\beta + 2\gamma x)^2 \Big] dx$$

$$= \int_0^1 \Big[\beta x + \gamma x^2 + \beta + 2\gamma x + \beta^2 x + 2\beta\gamma x^2$$
$$+ \beta\gamma x^2 + 2\gamma^2 x^3 + \beta^2 + 4\beta\gamma x + 4\gamma^2 x^2 \Big] dx$$

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$$= \int_0^1 \Big[(\beta + \beta^2) + (\beta + 2\gamma + \beta^2 + 4\beta\gamma)\, x + (\gamma + 3\beta\gamma + 4\gamma^2)\, x^2 + 2\gamma^2 x^3 \Big]\, dx$$

$$= \beta + \beta^2 + \frac{1}{2}(\beta + 2\gamma + \beta^2 + 4\beta\gamma) + \frac{1}{3}(\gamma + 3\beta\gamma + 4\gamma^2) + \frac{1}{4}\, 2\gamma^2$$

$$= \frac{3}{2}\beta + \frac{4}{3}\gamma + \frac{3}{2}\beta^2 + \frac{11}{6}\gamma^2 + 3\beta\gamma$$

Stationary if $\dfrac{\partial \Phi}{\partial \beta} = \dfrac{\partial \Phi}{\partial \gamma} = 0$

$$\frac{\partial \Phi}{\partial \beta} = \frac{3}{2} + 3\beta + 3\gamma = 0$$

$$\frac{\partial \Phi}{\partial \gamma} = \frac{4}{3} + \frac{11}{3}\gamma + 3\beta = 0$$

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$$3\beta + 3\gamma = -\frac{3}{2}$$

$$3\beta + \frac{11}{3}\gamma = -\frac{4}{3}$$

$$\beta = \frac{\begin{vmatrix} -3/2 & 3 \\ -4/3 & 11/3 \end{vmatrix}}{\begin{vmatrix} 3 & 3 \\ 3 & 11/3 \end{vmatrix}} = \frac{-\frac{11}{2} + 4}{11 - 9} = -\frac{3}{4}$$

$$\gamma = \frac{\begin{vmatrix} 3 & -3/2 \\ 3 & -4/3 \end{vmatrix}}{\begin{vmatrix} 3 & 3 \\ 3 & 11/3 \end{vmatrix}} = \frac{-4 + \frac{9}{2}}{11 - 9} = \frac{1}{4}$$

$$y = -\frac{3}{4}x + \frac{1}{4}x^2$$

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(4)

a. $I_{xx} = \frac{1}{3} m l^2$

$I_{yy} = \frac{1}{12} m (4l)^2 = \frac{4}{3} m l^2$

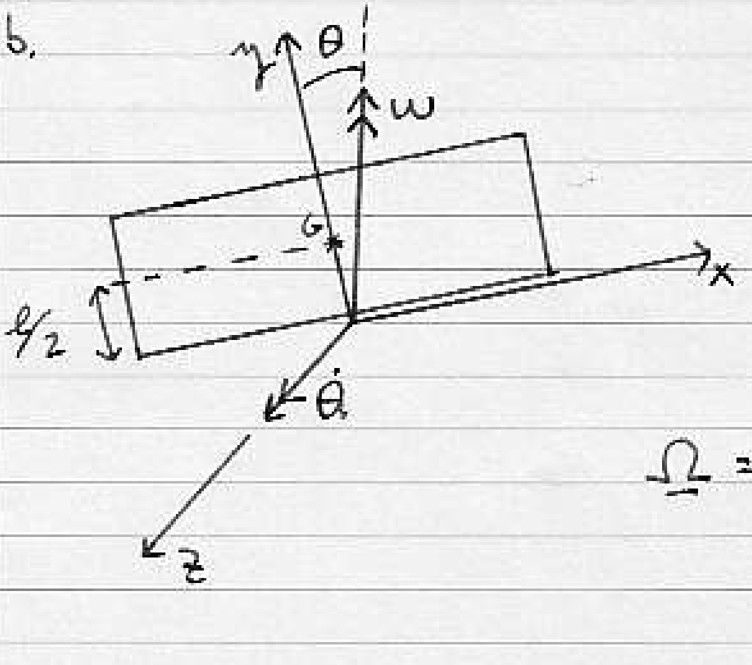
$I_{zz} = I_{xx} + I_{yy} = \frac{5}{3} m l^2$ (Planar body)

$I_{xy} = 0$ (Symmetry)

$I_{xz} = I_{yz} = 0$ (Planar body)

$$\underline{\underline{I}}_0 = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{5}{3} \end{bmatrix} m l^2$$

b.



$$\underline{\Omega} = \begin{bmatrix} \omega \sin\theta \\ \omega \cos\theta \\ \dot{\theta} \end{bmatrix}$$

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		datum / opl.

$$T = \frac{1}{2}\, \underline{\Omega}^T\, \underline{\underline{I}}_0\, \underline{\Omega} =$$

$$= \frac{1}{2} \begin{bmatrix} \omega\sin\theta & \omega\cos\theta & \dot{\theta} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{5}{3} \end{bmatrix} \begin{bmatrix} \omega\sin\theta \\ \omega\cos\theta \\ \dot{\theta} \end{bmatrix} ml^2$$

$$= \frac{1}{2} \begin{bmatrix} \frac{1}{3}\omega\sin\theta & \frac{4}{3}\omega\cos\theta & \frac{5}{3}\dot{\theta} \end{bmatrix} \begin{bmatrix} \omega\sin\theta \\ \omega\cos\theta \\ \dot{\theta} \end{bmatrix} ml^2$$

$$= \left(\frac{1}{6}\omega^2\sin^2\theta + \frac{4}{6}\omega^2\cos^2\theta + \frac{5}{6}\dot{\theta}^2 \right) ml^2$$

$$= ml^2 \left[\frac{5}{6}\dot{\theta}^2 + \frac{1}{6}\omega^2\sin^2\theta + \frac{1}{6}\omega^2\cos^2\theta + \frac{3}{6}\omega^2\cos^2\theta \right]$$

$$= ml^2 \left[\frac{5}{6}\dot{\theta}^2 + \left(\frac{1}{6} + \frac{1}{2}\cos^2\theta \right)\omega^2 \right]$$

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c.

$$L = T - V$$

$$V = mg\frac{l}{2}\cos\theta$$

$$L = ml^2\left[\frac{5}{6}\dot{\theta}^2 + \left(\frac{1}{6} + \frac{1}{2}\cos^2\theta\right)\omega^2\right] - mg\frac{l}{2}\cos\theta$$

$$h = \frac{\partial L}{\partial \dot{\theta}}\dot{\theta} - L = ml^2\frac{5}{3}\dot{\theta}^2$$

$$- ml^2\left[\frac{5}{6}\dot{\theta}^2 + \left(\frac{1}{6} + \frac{1}{2}\cos^2\theta\right)\omega^2\right] + mg\frac{l}{2}\cos\theta$$

$$= \underbrace{ml^2\frac{5}{6}\dot{\theta}^2}_{T_2} \underbrace{- ml^2\left(\frac{1}{6} + \frac{1}{2}\cos^2\theta\right)\omega^2 + mg\frac{l}{2}\cos\theta}_{V_{eff}}$$

$$V_{eff} = mg\frac{l}{2}\cos\theta - ml^2\left(\frac{1}{6} + \frac{1}{2}\cos^2\theta\right)\omega^2$$

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d

$$V'_{eff} = -mg\frac{l}{2}\sin\theta + ml^2\cos\theta\sin\theta\,\omega^2$$

$V'_{eff}(0) = 0 \longrightarrow \theta = 0$ is equilibrium point

$$V''_{eff} = -mg\frac{l}{2}\cos\theta + ml^2\omega^2(-\sin^2\theta + \cos^2\theta)$$

$$V''_{eff}(0) = -mg\frac{l}{2} + ml^2\omega^2$$

For stability $V''_{eff}(0) > 0$

$$-mg\frac{l}{2} + ml^2\omega^2 > 0$$

$$\cancel{m}l^{\cancel{2}}\omega^2 > \cancel{m}g\frac{\cancel{l}}{2}$$

$$\omega^2 > \frac{g}{2l}$$

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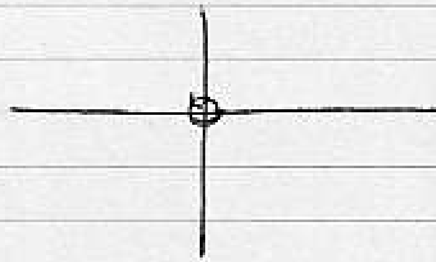
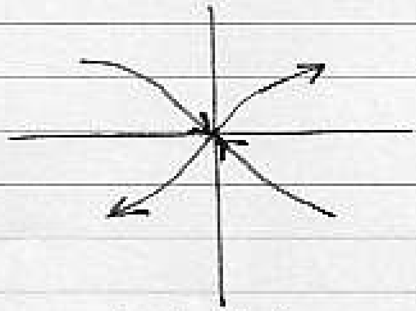
5 a.

$x = 0 \rightarrow$ stable



small deviations from equilibrium will lead to small oscillations.

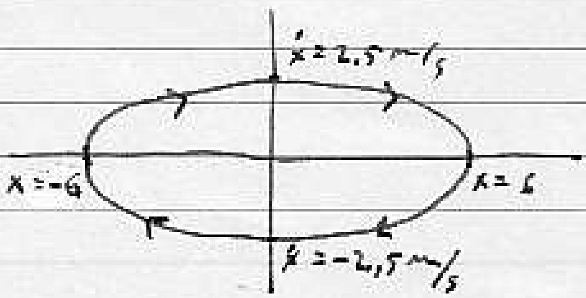
$x = 7.5 \rightarrow$ unstable



small deviations will lead to further separation from equilibrium

b.

Any combination laying on the phase line going through $x = 6$ m will be valid, for example:



$$\left.\begin{array}{l} x(0) = 0 \\ \dot{x}(0) = 2,5\ m/s \end{array}\right\}$$

or

$$\left.\begin{array}{l} x(0) = -6\ m \\ \dot{x}(0) = 0 \end{array}\right\}$$

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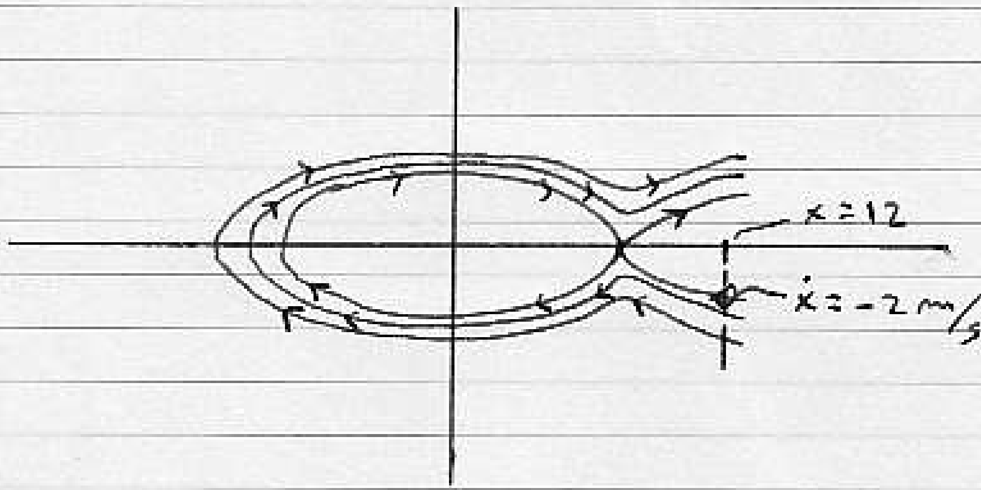
c.



Only if $\dot{x}(0) < -2$ m/s it can be ensured that x will become negative. If $\dot{x}(0) = -2$ m/s, the behaviour at the unstable equilibrium point x = 7.5 m is unpredictable. So, one possible initial value fulfilling the condition is

$$\dot{x}(0) = -3 \text{ m/s}$$

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