Delft University of Technology, Faculty of Aerospace Engineering

Exam AE3-914: Dynamics and Stability
Date: August 23, 2010, Time: 9:00 – 12:00

Question 1 (3.0 points)

Figure 1 shows a pendulum of mass *m* and length *l*, which is placed on a frictionless inclined plane. The inclined plane rotates at (varying) rate $\dot{\beta}(t)$ about the *Y*-axis of a fixed, inertial frame of reference *X*-*Y*-*Z*, such that at all times the contact between the mass and the inclined plane is preserved. The pendulum is attached at the origin *0* of the moving frame of reference *x*-*y*-*z* that is connected to the inclined plane. The distance between the origin of this frame of reference and the bottom of the inclined plane is *c*. The rotation of the pendulum about the *z*-axis is $\theta(t)$. The gravitational acceleration is *g*.

a) Derive the absolute velocity of the mass *m* in terms of the base vectors of the moving frame of reference *x*-*y*-*z*.

b) Construct the kinetic energy of the system.

c) Construct the Lagrangian.

d) Find the equations of motion.

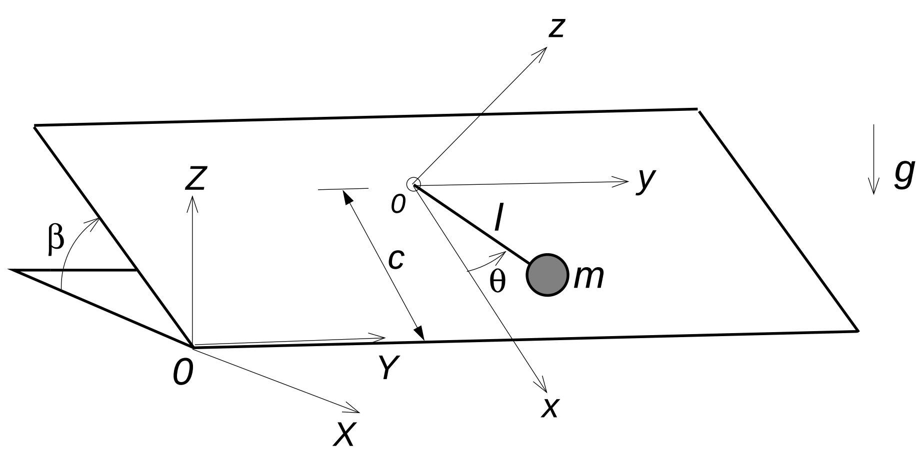

Figure 1: A pendulum of mass *m* and length *l* is placed on a frictionless, inclined plane. The inclined plane rotates at (varying) rate $\dot{\beta}(t)$.

Question 2 (2.0 points)

A wheel of mass m and radius a is attached to a massless rod of length l, see Figure 2. The rod rotates with a *constant* angular velocity Ω about a fixed Z-axis, as a result of which the wheel rolls without slipping on a horizontal plane. The rolling motion occurs about the x-axis of a moving frame of reference x-y-z that is connected to the rod. The thickness of the wheel may be ignored, so that it can be assumed that the reaction forces on the wheel occur at the *contact point c* in the horizontal plane. The centre of mass of the wheel is indicated by G. The gravitational acceleration is g.

a) Formulate the angular momentum with respect to origin *0* of the frame of reference x-y-z.

Hint: The mass moments of inertia of the wheel are $\frac{1}{4}ma^2$ about a centroidal axis in the plane of the wheel and $\frac{1}{2}ma^2$ about the centroidal axis perpendicular to the plane of the wheel.

b) Use the expression for the angular momentum to derive the corresponding equation of motion about the origin *0* of the frame of reference x-y-z.

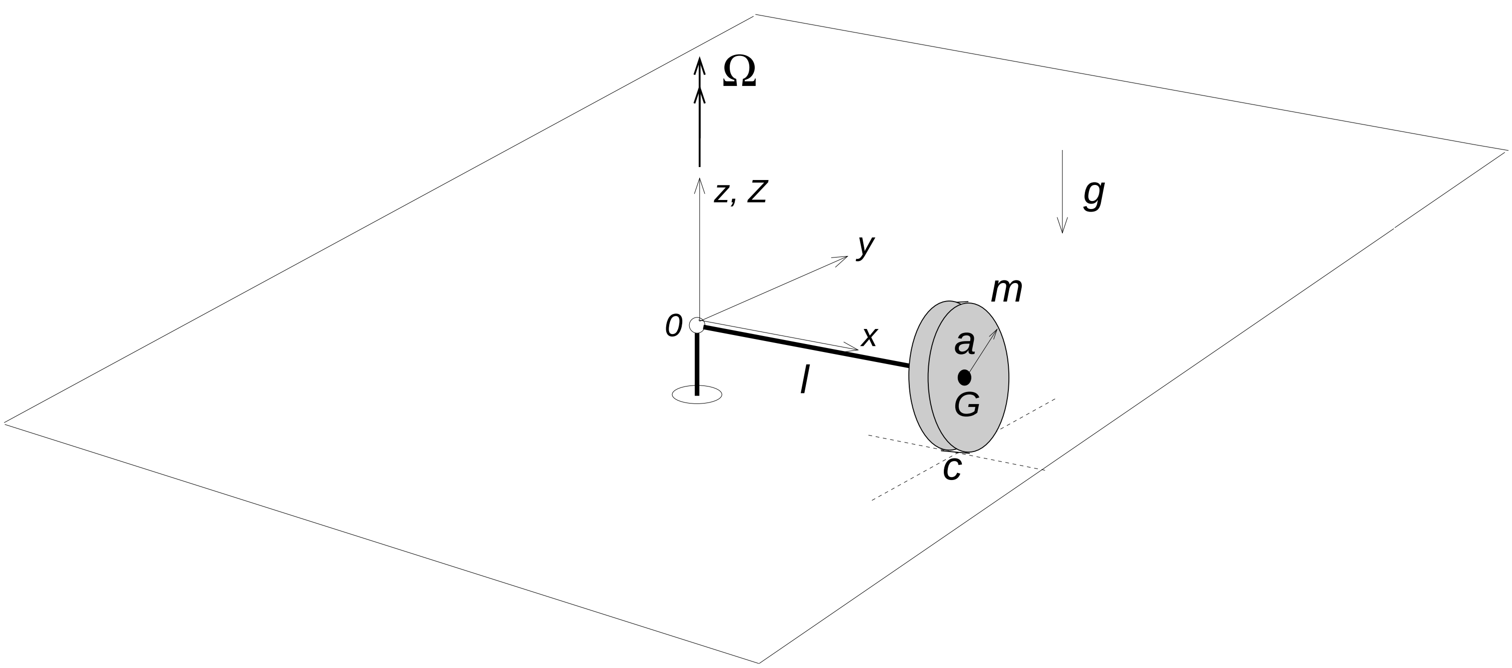

Figure 2: A wheel of mass m and radius a is connected to a rotating, massless rod of length l.

Question 3 (2.5 points)

A shaft of length *L* is fully clamped at its left end *x=0,* and is subjected to a torsional moment *M* at its right end *x=L*, with *x* the (horizontal) coordinate measured along the axis of the shaft, see Figure 3. The shaft is further subjected to a *distributed* torsional moment *m* over its full length $0 \le x \le L$. The rotation of the shaft about the *x*-axis (which uses a right-handed coordinate system) is $\phi(x)$.

The energy functional of the system loaded under torsion is given by

$$V = \int_0^L \left(-m\phi + \frac{1}{2} G I_p \phi_x^2 \right) dx - M\phi(L),$$

where G is the shear modulus of the shaft and I_p is its polar moment of inertia.

a) Find the differential equation for the rotation $\phi(x)$ of the shaft (under equilibrium conditions).

b) Identify the boundary conditions and indicate whether the boundary conditions are natural or essential.

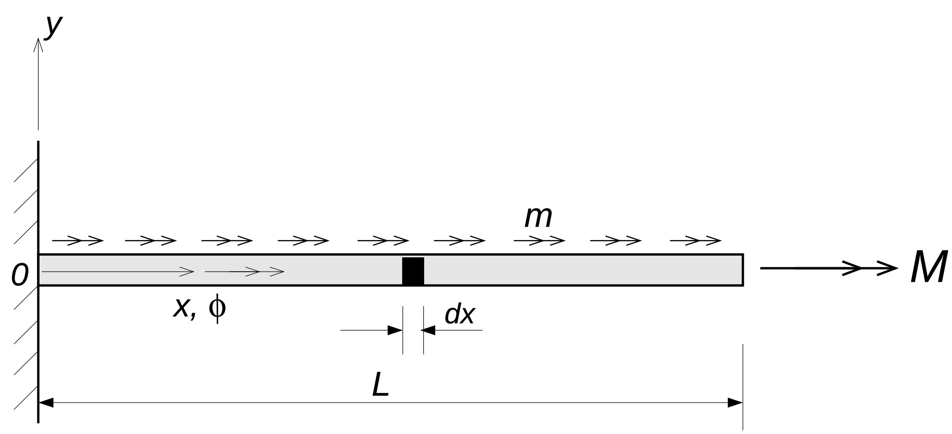

Figure 3: A clamped shaft is subjected to a *distributed* torsional moment *m* over its full length, and a torsional moment *M* at its right end.

Question 4 (2.5 points)

An inverted pendulum composed of a mass *m* and a massless bar of length *l* is connected to two discrete springs of stiffness *k*, see Figure 4. The springs are positioned at a distance *a* from the hinge support at the bottom of the inverted pendulum. The orientation of the springs remains horizontal during deformation. The rotation of the inverted pendulum with respect to its vertical position is $\theta(t)$. The gravitational acceleration is *g*.

a) Construct the Lagrangian of the system.

b) Find the equation of motion in terms of the generalized coordinate θ.

c) Investigate the stability of the system about its vertical position using the linearization method.

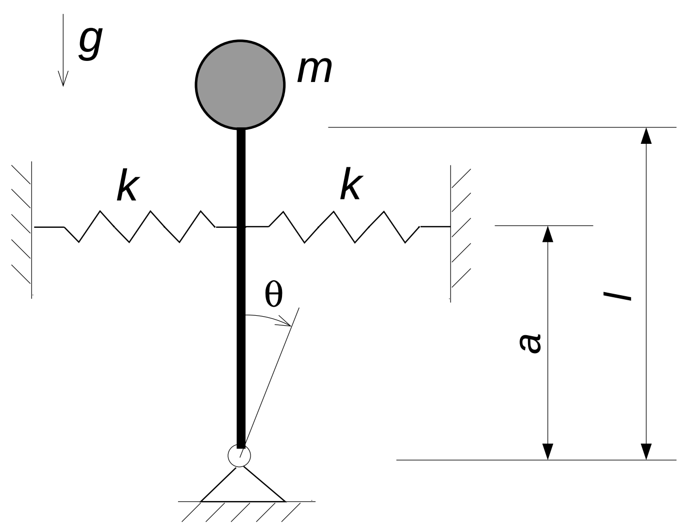

Figure 4: An inverted pendulum is composed of a mass *m* and a massless bar of length *l* and two discrete springs of stiffness *k*.