Delft University of Technology, Faculty of Aerospace Engineering

Exam AE3-914: Dynamics and Stability
Date: November 1, 2010, Time: 18:30 – 21:30

Question 1 (2.0 points)

Figure 1 shows a compound system of a car of mass *m* and a pendulum of mass *m* and length *l* that runs along a rail and is connected to a discrete spring of stiffness *k*. The horizontal displacement of the car is *x(t)* and the rotation of the pendulum is $\theta(t)$. The mass moment of inertia of the pendulum about its centre of mass is $I_c = ml^2/12$. The friction between the car wheels and the rail may be ignored. The gravitational acceleration is *g*.

a) Determine the kinetic energy of the system.

b) Determine the potential energy of the system.

c) Derive the equations of motion from the Lagrangian.

d) Use the “integral of motion” concept to demonstrate whether or not the total energy of the system is conserved.

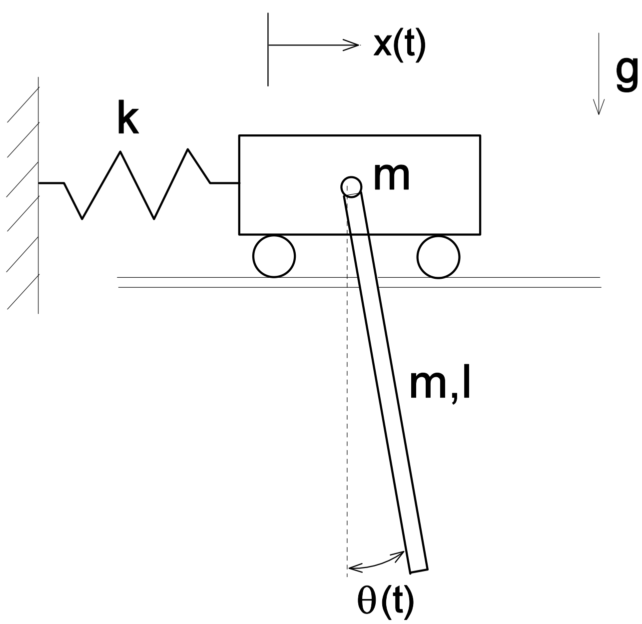

Figure 1: A compound system of a car of mass *m* and a pendulum of mass *m* and length *l* is connected to a discrete spring of stiffness *k*.

Question 2 (2.5 points)

A circular disk is connected to a massless rod of length *l.* The mass of the disk is *m*, and the mass moments of inertia of the disk are I about a centroidal axis in the plane of the disk, and J about the centroidal axis perpendicular to the plane of the disk. The rod is attached to a non-inertial frame of reference *x-y-z*. The disk spins at *constant* angular velocity Ω about the *z*-axis. The orientation of the rod with respect to the vertical axis is indicated by a *fixed* angle θ about the *x*-axis. The precession of the system about the vertical *Z*-axis of an inertial frame of reference is represented by a *constant* angular velocity ω_p (i.e., a *steady* motion occurs). The gravity acceleration is *g*.

a) Determine the angular velocity $\boldsymbol{\omega}$ of the system in terms of the base vectors of the non-inertial frame of reference *x-y-z*.

b) Determine the angular acceleration $\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$.

c) Derive an expression for the angular momentum ***L*** about the origin *0* of the frame of reference *x-y-z*.

d) Use the expression for the angular momentum to derive the corresponding equation of motion about the origin *0*.

e) Derive expressions for the two angular velocities ω_p at which the steady precession can take place.

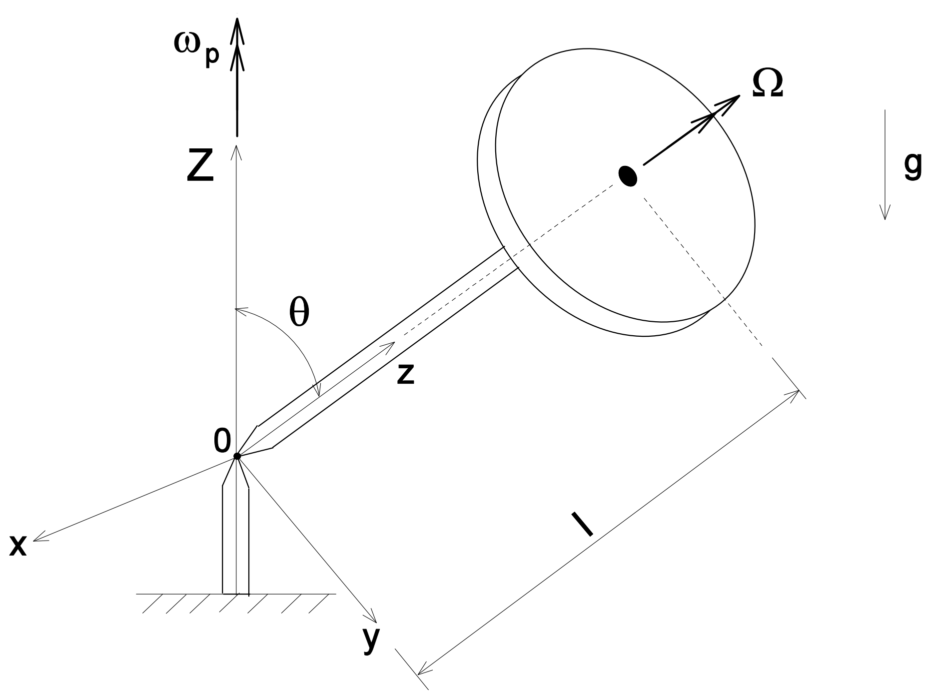

Figure 2: A circular disk connected to a massless rod of length *l* rotates at *constant* spin Ω and *constant* precession ω_p.

Question 3 (1.5 points)

Examine the stability of *all* equilibrium points in the domain $x \in [0,\ 2\pi]$ for a dynamical system with the equation of motion given by

$$\ddot{x} + 3\dot{x} + 4\cos x = 0.$$

Question 4 (2.0 points)

A beam of length *L* is fully clamped at its left end *x=0,* and is subjected to a point load *F* at its right end *x=L*, with *x* the horizontal coordinate along the beam axis. The vertical displacement of the beam during equilibrium is *w(x)*. The strain energy of the beam is given by

$$V^e = \frac{1}{2}\int_0^L EI\ w_{xx}^2 dx,$$

where E is Young's modulus and I is the area moment of inertia.

a) Formulate the generalised potential energy of the beam loaded by the force *F*.

b) Derive the Euler-Lagrange equation for the elastic deformation of the beam, and the corresponding boundary conditions.

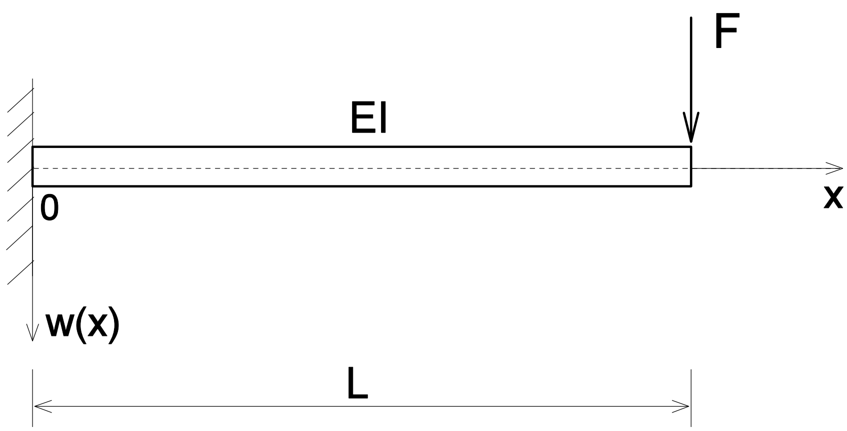

Figure 3: A clamped beam of length *L* is subjected to a point load *F* at its right end.

Question 5 (2.0 points)

A four-blade nose propeller rotates at *constant* angular velocity p about the x-axis of an x-y-z frame of reference attached to the airplane nose. The mass moment of inertia of an individual propeller blade about the x-axis is I_b. At a certain moment in time the airplane turns at *constant* angular velocity Ω about the (vertical) Z-axis of an inertial frame of reference.

a) Compute the principal mass moments of inertia of the four-blade propeller in terms of I_b. Provide the answer with a brief argumentation.

b) Compute the reaction moment $\boldsymbol{M}$ the propeller generates on the airplane during turning, in terms of the base vectors of the moving frame of reference x-y-z.

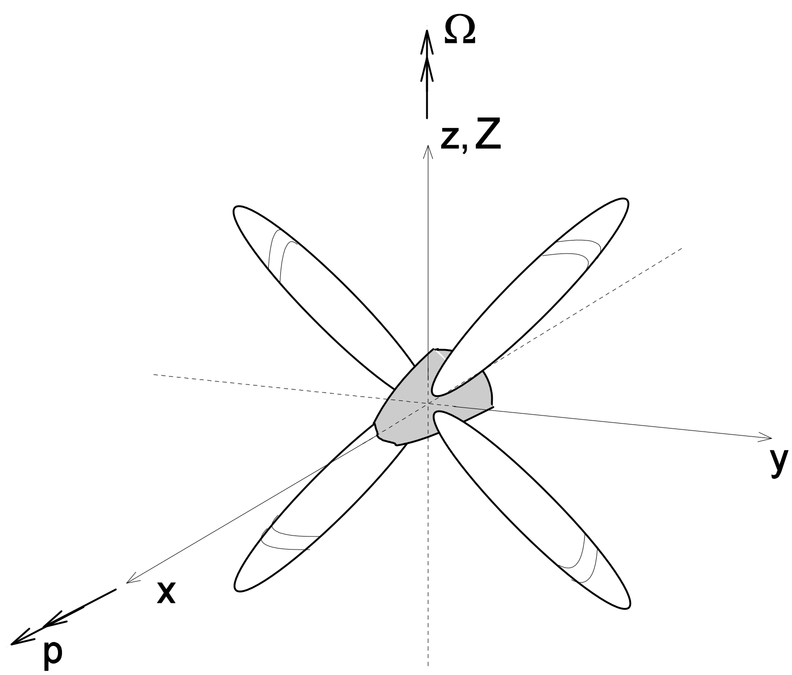

Figure 4: A four-blade nose propeller of an airplane rotates at *constant* angular velocity p. The airplane turns at *constant* angular velocity Ω about the (vertical) Z-axis of an inertial frame of reference.