Examination System Identification and Parameter Estimation WB2301

Friday June 30, 2006
9.00 - 12.00 hours

Instructions:
The answers to the questions should be written on separate sheets. Write your name and student number on each sheet. To this exam a separate sheet is added to draw the diagrams. You can use all the lecture material that has been handed out during the course. Please answer the questions shortly and precise (in English or Dutch). With each question the number of points is indicated between the brackets. The total number of points is 100.

Question 1 (12/100):
For an experiment a deterministic input signal u was created by designing a multi-sine signal in the frequency domain (Figure 1) and then transforming it back into time domain by inverse Fourier transform. The resulting signal, which is 5000 samples long, consists of sinusoids in the range 1-20 Hz. In this range (1-20Hz), the full frequency resolution was used, which resulted in a total number of 381 sinusoids in the signal.

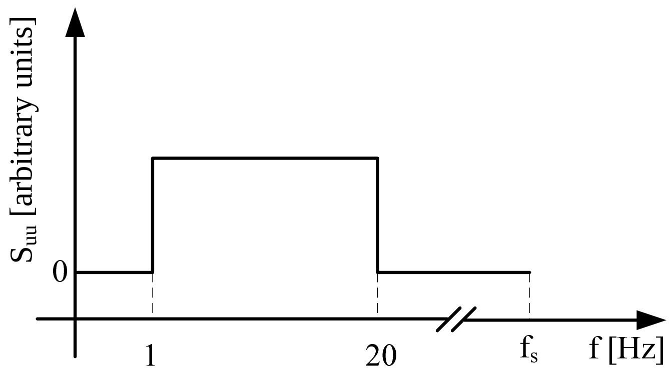


Figure 1. Power spectrum of multisine signal u.

a) Determine the sample frequency f_s [Hz]
b) Determine the signal length T [s].
c) Draw the autospectral density $S_{uu}(f)$ if f_s is doubled (use answer sheet 1, provided at the end of this exam). Explain the differences and similarities.
d) Draw $S_{uu}(f)$ if signal u is resampled to 30 Hz (use answer sheet 1, provided at the end of this exam). Explain the differences and similarities.

Question 2 (15/100):
From the cross-spectral density and the auto-spectral densities of input and output the coherence can be estimated.
a) Give the expression of the coherence function.
b) Name two causes that may explain a low coherence?
c) The coherence function is always overestimated. Explain why.
d) Consider a highly under-damped system with output noise. What is the signal to noise ratio at the oscillation frequency compared to the other frequencies?
e) Following d): is the coherence larger or smaller and what is the effect of frequency averaging at the oscillation frequency?

Question 3 (15/100):
Consider the open loop system H(s) in Figure 2.

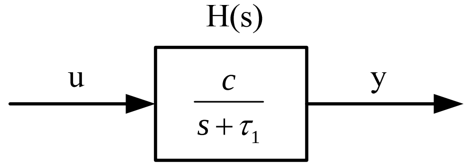


Figure 2. Open loop system H(s).

H(s) is identified by applying ideal white noise to the input and recording both input and output. The noise signal has equal power at all non-zero frequencies: In our case the power is 2 at all non-zero frequencies:

$$S_{uu}(f) = \begin{cases} 2 & f < 0 \\ s_0 & f = 0 \\ 2 & f > 0 \end{cases}$$

a) What is the meaning of s_0 and what is its value?
b) Give the expression of the spectral estimator for H(s).
c) On the double-logarithmic axis (provided on the last page of the exam), draw the cross-spectral density function $S_{uy}(f)$ of signals u(t) and y(t). Specify its cut-off frequency, its static value for the lower frequencies and the slope for higher frequencies. (Only draw for positive frequencies.)
d) On the second double-logarithmic axis (provided on the last page of the exam), draw the auto-spectral density function $S_{yy}(f)$. Again, specify its cut-off frequency and its static value for the lower frequencies. (Only draw for positive frequencies.)
e) Assume that signal y contains additional output noise: y'(t)=y(t)+n(t). Output noise n(t) is white and the power is substantially smaller than k.u(t) (<10%). Sketch the coherence function for this case.

Question 4 (9/100):
The model in Figure 3 is to be identified using a parametric model in the time domain. It has an input u, an output y and noise source e.

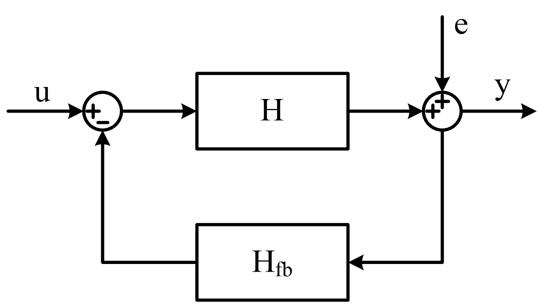


Figure 3. Model to be identified.

a) Determine the model structure (ARX / ARMAX / OE / FIR / BJ) that should be used for identifying this system.
b) What is the major advantage of an ARX model over a Box-Jenkins (BJ) model?

c) Identification has shown that the effect of feedback in this system is negligible (i.e. $H_{fb} << H$). Which model structure can be used in future experiments?

Question 5 (12/100):

A mass-spring-damper system is to be identified. This is done in two ways. In the first experiment, a force input is provided to mass of the system. This force, together with the output position, are recorded and used for estimating a parametric model in the time domain. This experiment is illustrated in Figure 4-A.

a) Which model structure (ARX / ARMAX / OE / FIR / BJ) would you choose for this experiment?

In a second experiment a strong servo-motor is used to enforce a position to the mass. The resulting reaction force is measured. So in this situation, the input is the position and output is the reaction force. This experiment is illustrated in Figure 4-B.

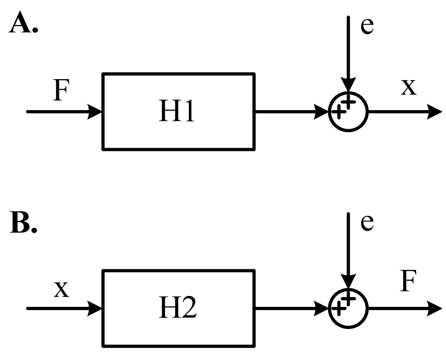


Figure 4. A: System model with force as input and position as output. B: System model with position as input and force as output.

b) Give the expressions of both transfer functions (H1 & H2) in the Laplace domain including the mass, spring and damper. Which model is causal? Explain.
c) Which model structure would you use for experiment B?
d) Assume an equal rectangular input spectrum and equal noise (e) in both cases. Which experiment would yield the best signal-to-noise ratio at the output for the higher frequencies? Explain.

Question 6 (15/100):

Consider the non-linear system below, which performs a so-called logical AND (&) operation:

$$y(u_1, u_2) = \begin{cases} 1, & u_1 = u_2 = 1 \\ 0, & u_1 = 0 \vee u_2 = 0 \end{cases}$$

$$u_1, u_2 \in \{0, 1\}$$

A neural network (Figure 5) will be trained to mimic the behavior of this system. It contains an input layer (2 neurons) and an output layer (one neuron). There is no hidden layer.

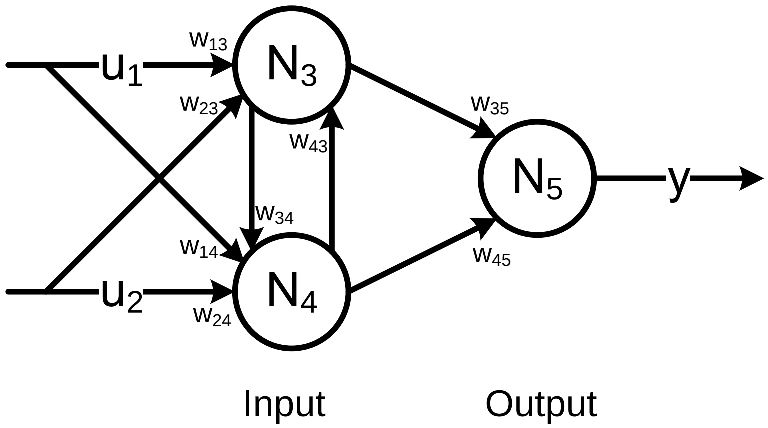


Figure 5. Neural network used for fitting the AND operator system.

a) Would you classify this training problem as supervised or unsupervised learning?

b) Would you use a step function (f_1) or a linear relation (f_2) for the activation function of the neurons in the input layer? And for the neuron in the output layer? Explain.

$$f_1(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$f_2(x) = x$$

c) How many parameters need to be optimized in total? Explain.

d) The network is trained using a back-propagation algorithm, until the error between training signal y and network output y_N does not decrease anymore. What is the best obtainable remaining error in this network? Explain.

e) Give one solution (=values for the parameters you mentioned in part c.) that could have resulted from the training.

Question 7 (12/100)

In the first stage of an identification procedure, the following FRFs were obtained for four different systems. What can you say about the structure and properties of these systems? Which one(s) is (are) anticausal? Explain.

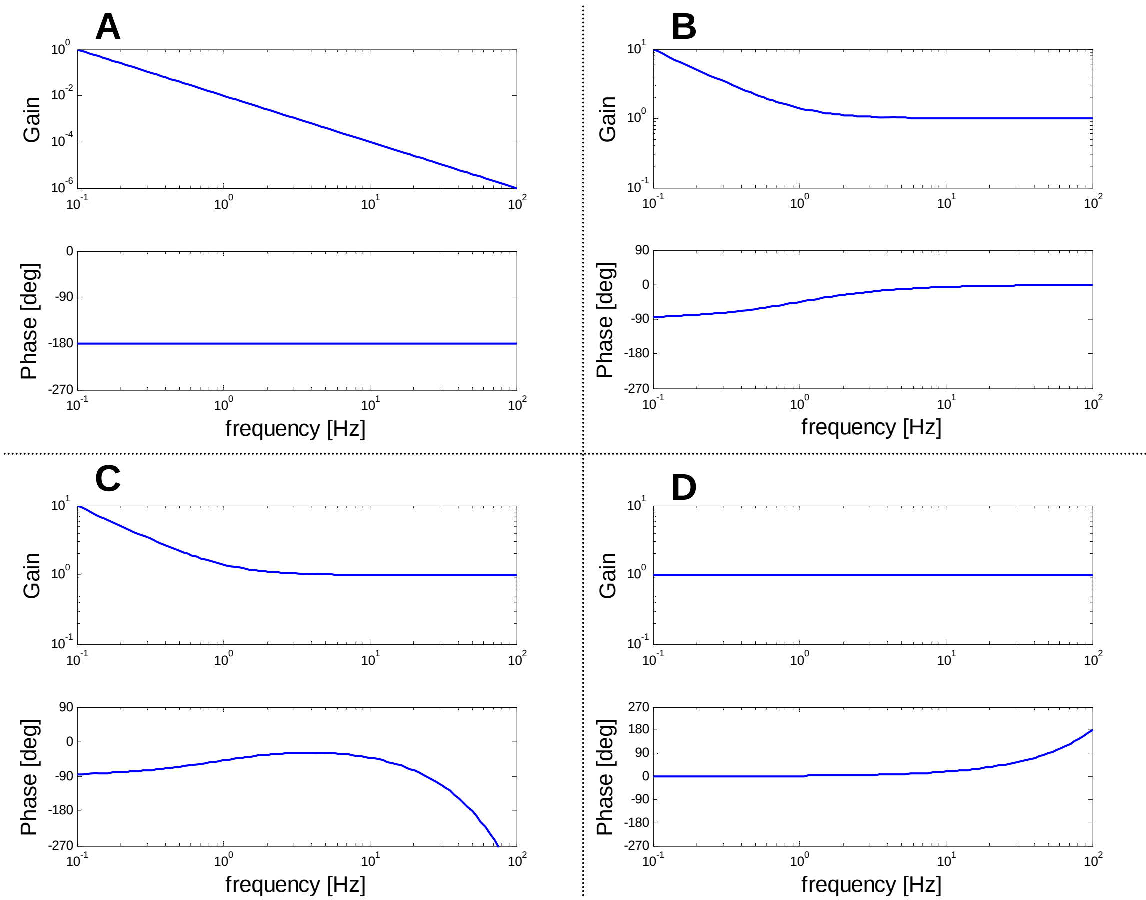


Figure 6 A-D. Bode plots of the four systems in question 7.

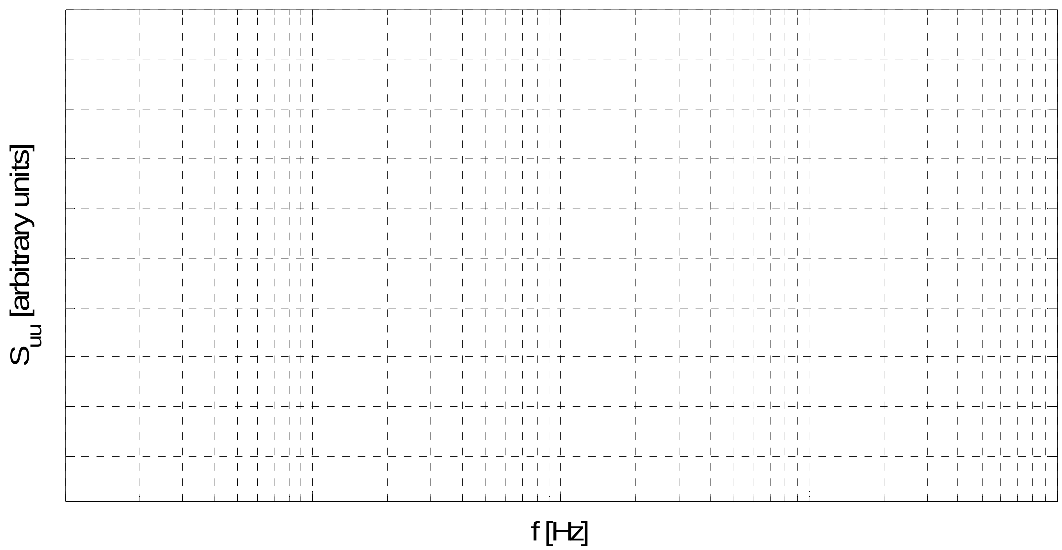
Question 8 (10/100):

Iets over David's experiment met twee inputs En gelijk iets met VAFs?

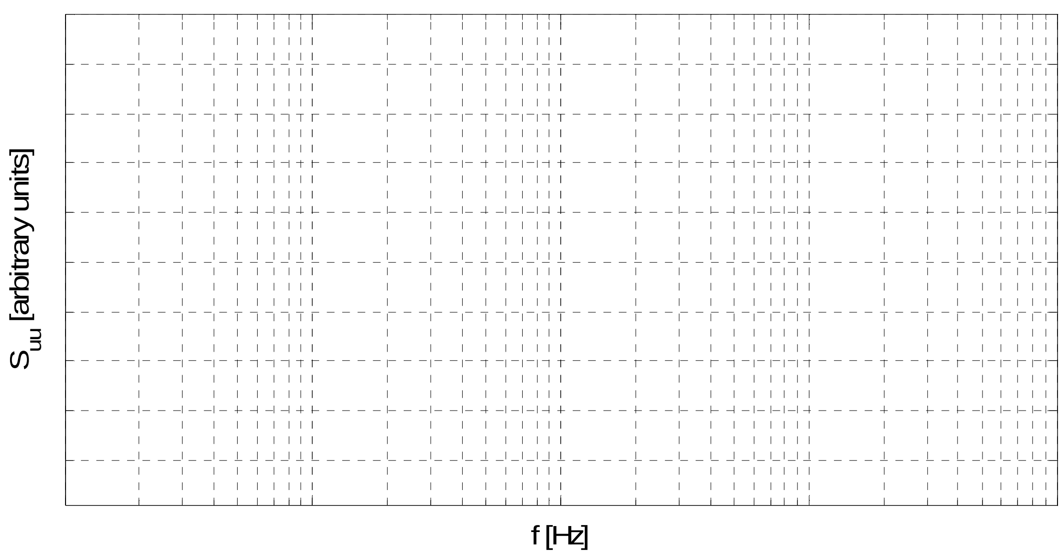
Answer sheet for question 1c and 1d.

NAME: STUDENT NR:

Question 1c:



Question 1d:



Answer sheet for question 2c and 2d.

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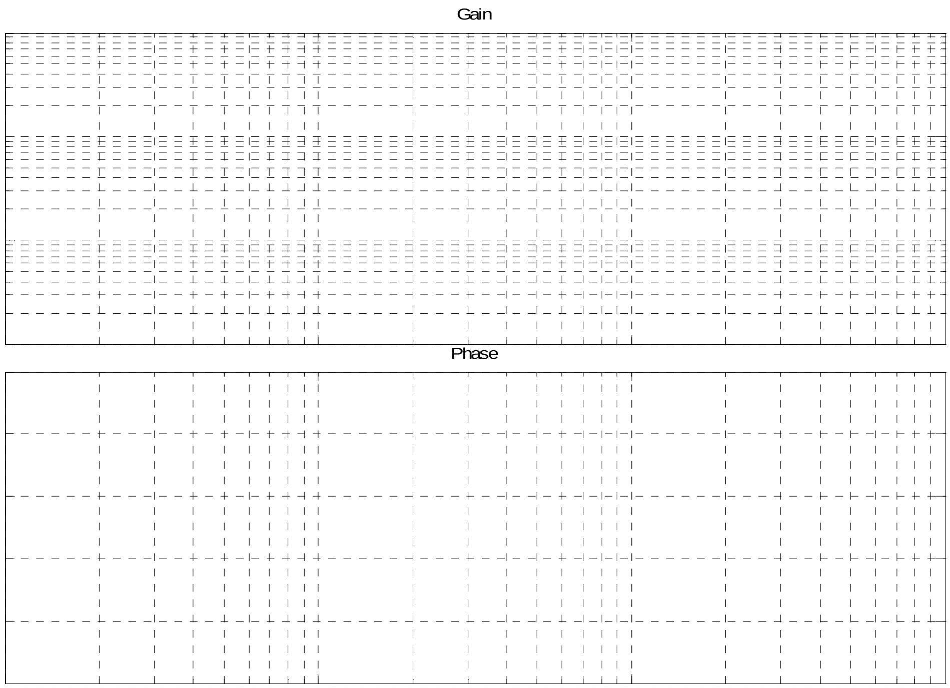
Question 2c:



Question 2d: