Examination System Identification and Parameter Estimation WB2301

Wednesday July 2, 2008
9.00 - 12.00 hours

Instructions

Please answer the questions briefly and precise (in English or Dutch). **Do not forget to write your name and student number on each paper**. You can use all lecture materials. Points are indicated between brackets for each question. The total number of points is 100.

Question 1 (12)

Consider the following linear second order mass -spring-damper system

$$H(s) = \frac{1}{ms^2 + bs + k}$$

a. Express the static gain, eigenfrequency and relative damping in system parameters (m, b, k) and schematically draw the frequency response function in a Bode diagram (gain and phase).
b. You have performed a system identification to identify H(s) from input-output measurements. The gain is as you expected from a 2nd order system but the phase lag gradually increases with frequency compared to the expected phase (for H(s)). Explain this result.
c. Assume the gain at the eigenfrequency is huge (i.e. the system has very little damping). What is required to achieve an accurate estimation of the system around this frequency? Motivate.

Question 2 (12)

When using a gradient search optimization method for fitting model parameters to a dataset, it is generally unknown in advance whether the criterion function has one or more local minima.

a. Describe a way to decrease the likelihood that your final estimate is a local minimum instead of the global minimum (using the gradient search method).
b. State two alternative optimization procedures that suffer less from local minima than gradient search methods. Describe briefly <u>and in your own words</u> how these optimization procedures work.

A common criterion function (L) for fitting the frequency response function (FRF) of the model onto the experimentally obtained FRF is:

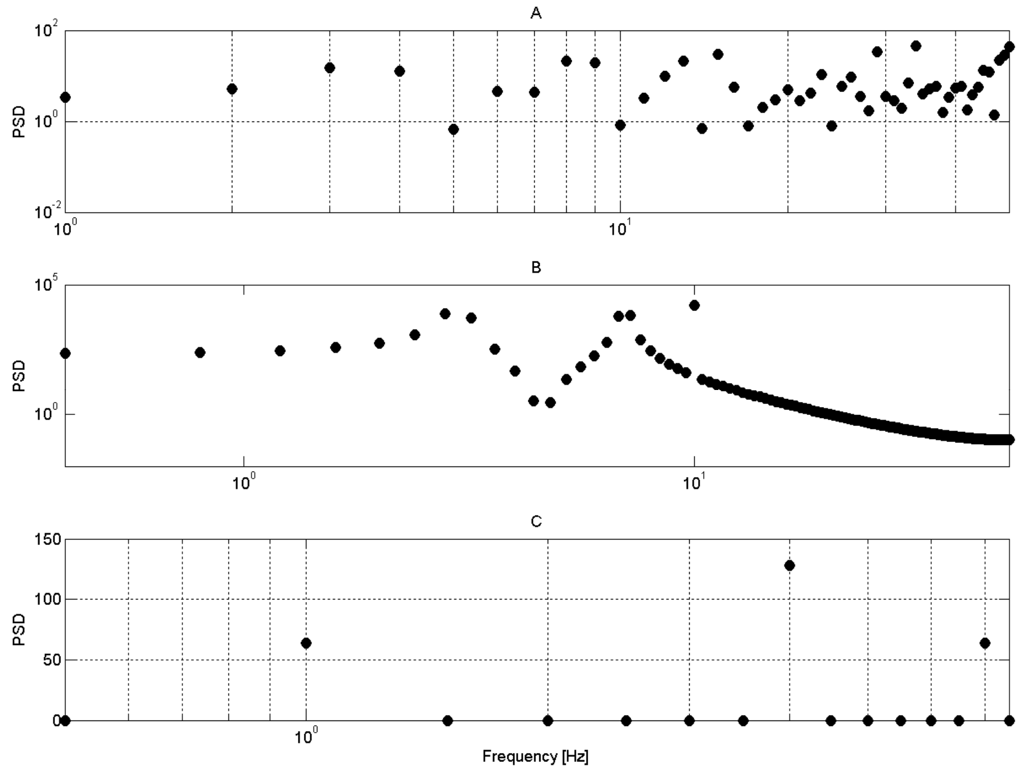
$$L = \sum e \qquad \text{with } e = \sum_{k=1}^{N} \frac{1}{f_k} \Gamma_k \left(\ln \frac{H_k^{mod}}{\hat{H}_k} \right)^2$$

where f is the frequency vector (indexed with k), Γ the coherence vector, H^{mod} the FRF of the parametric model and $\hat{H}$ the experimentally obtained FRF.

c. Could you indicate why the frequency vector is used as a weighing factor in this criterion function?
d. Why is the coherence vector used as a weighing factor in this criterion function?

Question 3 (12)

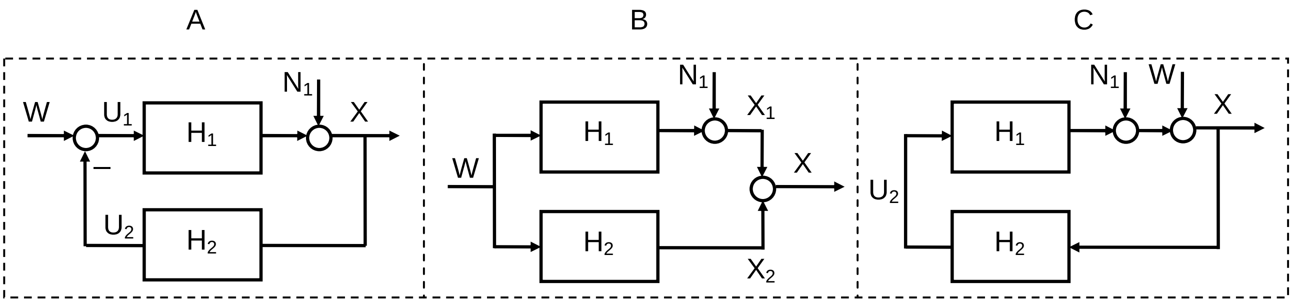
Figures A to C show the three raw (i.e. without frequency averaging) auto-spectral densities of signals a, b and c. Answer the questions below and explain how you deducted your answer from the graphs.



a. Signal a in Figure A was obtained by measuring a white noise source. The frequency axis ends at the Nyquist frequency and the mean component is not shown. How long was the duration T of the measurement?
b. Signal b in Figure B is the sum of three cosines with frequency 3, 7 and 10 Hz. The frequency axis ends at the Nyquist frequency and the mean component is not shown. Was the duration T of the measurement 1s, 2s or 2.5s? Explain your answer.
c. Signal c in Figure C was designed as a sum of two sinusoids with frequencies of 1 and 4 Hz respectively and with equal amplitude. The frequency axis ends at the sampling frequency and the mean component is not shown. However, the auto-spectral density is not equal at the corresponding frequencies. What has happened here? How can this problem be prevented next time?

Question 4 (20)

Two systems (H_1 and H_2) are presented in three system configurations with input W and output X (configuration A, B and C). All indicated signals are measured, except for the noise N_1 which represent the noise within system H_1. Input signal W is applied to identify the system(s). The input signal W is uncorrelated with noise N_1.



a. Give the correct spectral estimators of the total system $H_{wx}(f)$, and of the subsystem $H_1(f)$ for every configuration.
b. Give a correct expression for the coherence in configuration B.

Assume H_2 from configuration A is estimated with the following spectral estimator: $\hat{H}_2 = \frac{S_{xu_2}}{S_{xx}}$

c. Express the estimate of H_2 as a function of the auto-spectral density of W (S_{ww}) and the auto-spectral density of N_1 (S_{nn}).
d. Does the estimator give an accurate result for H_2? Is this estimator biased or unbiased?
e. If we want to estimate system H_2 of configuration A with a time domain model (ARX/ARMAX/OE/BJ). Explain which model structure we should use (ARX/ARMAX/OE/BJ) and how can we estimate subsystem H_2?

Question 5 (12)

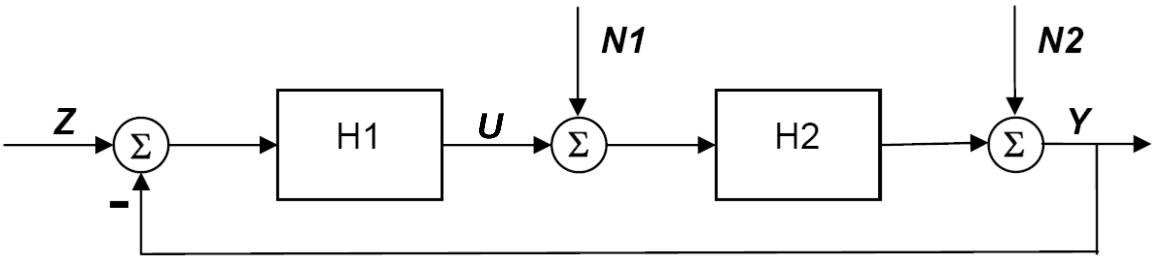
a. What is the effect of a zero order hold (ZOH) circuit in the reconstruction of a discrete signal to an analogous signal? Draw the filter function of a ZOH.
b. Give two drawbacks of an overly high sample frequency when using parametric models for system identification (i.e. ARX).
c. Why does Akaike's error function continue to decrease with increasing number of model order? Secondly, is this decrease of error an indication of increased model accuracy?
d. Consider a closed loop system configuration that is driven by a known input and an unknown noise, the latter enters somewhere in the system. Note that we do not concern measurement noise here. What can you say in general about the poles and zeros of the noise model in comparison with those of the system? Is that important for the choice of the discrete model to use for identification? Explain.

Question 6 (9)

a. To determine the variance of parameters from an optimization procedure, the Jacobian matrix is used for (co)variance calculations. The Jacobian is a derivative. Describe what kind of derivation is represented by the Jacobian. Second, is the Jacobian low or high at the minimum of the error function.
b. Given an optimal set of parameters of a linear model. What is the quality of the optimal model in the case the VAF appeared to be low while the estimated system's coherence was high?
c. Generally spoken, does it implicate that your system's model is incorrect when the VAF is low? Explain.

Question 7 (15)

Given the closed loop system shown below.



Closed loop configuration of two linear systems H1 and H2. ***Z***: external input; ***U***: measured output of H1, ***Y***: measured output of H2, ***N1***: measurement noise, ***N2***: output noise of H2.

a) Give the correct spectral estimator of H1.
b) Now assume that the external input signal Z is not available from measurements. Can you still use the estimator of a) for estimation of H1? How would you estimate H1 and what are the emerging problems?
c) What is the influence of both noise sources on the chosen estimator of b)?
d) Under what conditions are the estimators of a) and b) unbiased given that the observation time goes to infinite?
e) Give the closed loop spectral estimator for H2.

Question 8 (8)

a) Give two advantages and two disadvantages of discrete time domain models (ARX, etc) compared to spectral models (FRFs)
b) Give the solution equation of ARX model parameters. Describe what each part in this equation (matrices, vectors) contains.
c) Is it possible to derive a similar solution equation as in b) in case of an OE model structure? Explain.
d) What is the main difference between a Fourier and a wavelet transform?