AE4520: Advanced Structural Analysis

Food for thought (Basics)

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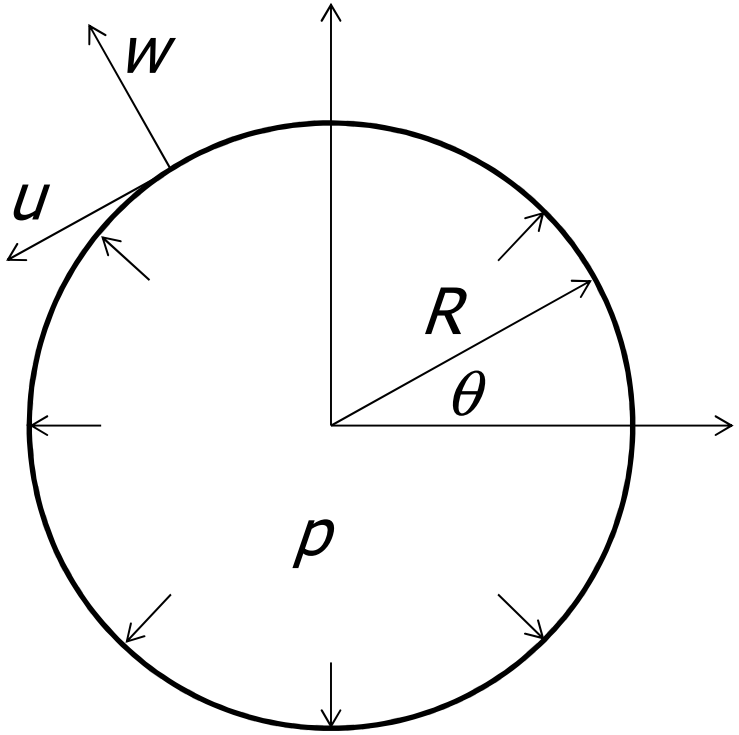
Problem 1

- The circular ring shown is subject to internal pressure *p*
- If the displacement component along the tangent direction is *u*, and along the normal is *w:*
- Show that for small strains:

$$\varepsilon = \frac{u_{,\theta} + w}{R}$$



- Use the PVW (see problem 3) to show that the tensile normal force in the ring is given by

$$N = pR$$

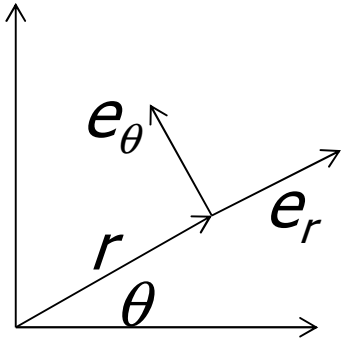
Problem 2

- Consider the use of polar coordinates (r, θ, z)
  - Unit vectors in the radial, circumferential and vertical are

$$\mathbf{e}_r = \begin{pmatrix} \cos\theta & \sin\theta & 0 \end{pmatrix}^t,$$

$$\mathbf{e}_\theta = \begin{pmatrix} -\sin\theta & \cos\theta & 0 \end{pmatrix}^t,$$

$$\mathbf{e}_r = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^t$$



- Show the position vector of a particle is given by

$$\mathbf{r} = r\mathbf{e}_r + z\mathbf{e}_z$$

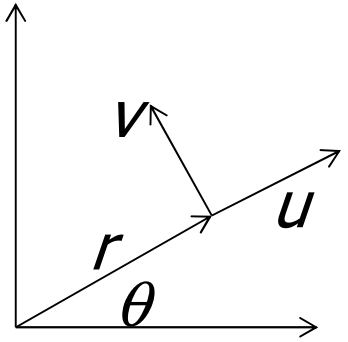
- Show that the relative position of two infinitesimally close particles is given by

$$d\mathbf{r} = dr\ \mathbf{e}_r + rd\theta\ \mathbf{e}_\theta + dz\ \mathbf{e}_z$$

Problem 2 (cont.)

- If the displacement components are (*u, v, w*) in polar coordinates, show that the change in displacement of two infinitesimally close particles is:

$$d\mathbf{u} = \begin{pmatrix} u_{,r} & \dfrac{u_{,\theta} - v}{r} & u_{,z} \\ v_{,r} & \dfrac{v_{,\theta} + u}{r} & v_{,z} \\ w_{,r} & \dfrac{w_{,\theta}}{r} & w_{,z} \end{pmatrix} d\mathbf{r}$$

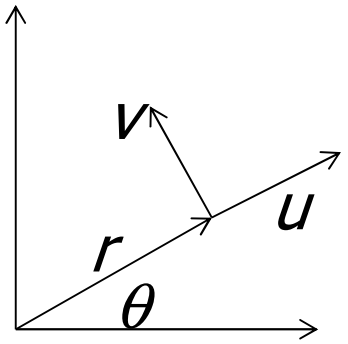


- From this derive the components of "small" strains in polar coordinates

Problem 3

- Show that the virtual work of internal normal forces in a bar is given by:

$$W_{in} = -\int_0^L N\ \delta\varepsilon\ dx$$



- N is the normal force given by:

$$N = \int_{section} \sigma_x\ dA$$

- Use PVW to show that in the absence of distributed loading:

$$N_{,x} = 0$$