4.
MODELING OF STRATIFIED MIXTURE FLOWS
(Heterogeneous flows)

Almost 50 years ago, substantial progress in the exploitation of the hydraulic transport of solids in pipelines initiated systematic investigations in this field. With the design of new industrial pipelines, some of which were of considerable length, the demand for the reliable models capable of predicting slurry flow behaviour grew. Over the years, as experimental, theoretical and computational techniques have progressed, the predictive models have been gradually improved.

The first predictive tools were developed in the 1950's and 1960's using methods of *empirical modeling*. The tools were empirical correlations constructed to predict the basic slurry pipeline characteristics - the frictional head loss and the deposition-limit velocity - for various slurry flow conditions in a pipeline. The correlations were based on the experimental measurements of integral parameters of slurry flow in pipelines. Usually these parameters were mean slurry velocity, volumetric delivered concentration and pressure in flows of slurry containing particles of certain diameter. Some of the models have become popular and are still used in practice (e.g. Durand & Condolios, 1952; Führböter, 1961; Jufin & Lopatin, 1966). They are simple to use and easy to modify to the user's own data. Recently, a semi-empirical model for a heterogeneous flow in slurry pipelines which is calibrated by using the integral-parameter data has been introduced by Wilson et al. (1992-6).

Since the mid 1980's attempts have been made to construct a general model for solid-liquid flow in pipelines by using a *microscopic approach*. A microscopic model defines the laws governing a slurry flow for an infinitesimal control volume of slurry. A slurry flow mechanism is described by using a set of differential equations for conservation of mass, momentum and energy in the solid-liquid flow. A microscopic model provides a numerical solution to the equations in local positions of a pipeline cross section. As a result, the model predicts the concentration and velocity profiles in a pipeline cross section, together with the pressure drop over a pipeline length section. Despite progress in the development of sophisticated experimental techniques which enable reasonably accurate measurements of the internal structure of the flow (concentration and velocity profiles) in a slurry pipeline, there is still not enough information on the slurry flow mechanism at microscopic level.

A suitable compromise between the microscopic and empirical approaches is an approach using the principles of *macroscopic modeling* (called also *physical modeling*). This approach applies the balance (conservation) equations to a larger control volume of slurry given, for instance, by a pipeline cross sectional area of approximately uniform concentration of solids in a unit length of a pipeline. In a

chosen control volume, the balance equations are formulated by using mean quantities, i.e. quantities averaged in the control volume.
Newitt et al. (1955) were the first to apply the balance formulations to a macroscopic control volume to obtain the friction loss equations for different slurry flow regimes in a slurry pipeline. Wilson (1970) introduced the concept of a mechanistic force-balance model to predict the velocity at the limit of stationary deposition in a fully-stratified flow. Wilson (1976) developed the model further to provide a unified predictive tool, called a two-layer model, to predict both the limiting deposition velocity and the frictional head losses in fully-stratified and partially-stratified flows in a horizontal slurry pipeline.

4.1 EMPIRICAL MODELING

Different approaches to empirical modeling of slurry flow in pipelines are described by using several widely used models:
- the model of Laboratoire Dauphinois d'Hydraulique (called the Durand model)
- the Führböter model
- the Jufin - Lopatin model
- the Wilson - GIW model.

4.1.1 Model of Laboratoire Dauphinois d'Hydraulique (Durand model)

FRICTIONAL HEAD LOSS

The empirical model to predict the pressure drop due to friction in the pipeline flow of slurry was constructed by using techniques for dimensional analysis. Durand and his co-workers sought an empirical relationship among the dimensionless groups of quantities anticipated to be of major importance for a description of slurry flow in a pipeline.

A. Experimental observations:

Experimental data were collected for a reasonably wide range of slurry flow conditions including several pipeline sizes and sorts of sand and gravel. Based on the experimental results (for low concentration slurries with delivered concentration C_{vd} up to 22%), the following issues were proposed for the pressure loss, represented by the hydraulic gradient I_m, in the heterogeneous slurry flow characterised by constant particle size d and pipeline size D:

- the solids effect I_m-I_f decreases gradually with increasing mean slurry velocity V_m in flow of constant delivered volumetric concentration of solids C_{vd}

- the solids effect I_m-I_f increases approximately linearly with increasing C_{vd} at constant V_m.

B. Construction of the Durand correlation and its ultimate version:

The latter condition was written as I_m - I_f = const.C_{vd}. To eliminate the direct influence of the properties specific to one experiment (such as the pipe roughness and the slurry temperature) the ratio of the solids effect and the hydraulic gradient for liquid flow, I_f, was introduced in place of the solids effect alone. Then the condition was described by the dimensionless group, marked Φ,

$$\Phi = \frac{I_m - I_f}{I_f C_{vd}} = \text{const.} \quad (4.1).$$

I_m	hydraulic gradient for mixture flow	[-]
I_f	hydraulic gradient for liquid flow	[-]
C_{vd}	delivered volumetric solids concentration	[-]

The flow coefficient Φ is not constant for slurry flows of different pipeline size D, solids size d, or slurry flow rate $V_m A$.

An effect of these parameters was introduced by correlating Φ with the Froude number for mixture flow $Fr^2 = \frac{V_m^2}{gD}$ and the Froude number for a solid particle $Fr_{vt}^2 = \frac{v_t^2}{gd}$. The Froude number is a criterion of dynamic similarity for flows with a dominant effect of inertia and gravity in different flow conditions.
The new dimensionless group was marked Ψ:

$$\Psi = Fr^2 Fr_{vt}^{-1} = \frac{V_m^2}{gD}\frac{\sqrt{gd}}{v_t} \quad (4.2).$$

Fr^2	Froude number for mixture flow	[-]
Fr_{vt}^{-1}	particle Froude number	[-]
V_m	mean mixture velocity in a pipe cross section	[m/s]
g	gravitational acceleration	[m/s^2]
D	pipe diameter	[m]
d	particle diameter	[m]
v_t	terminal settling velocity of a particle	[m/s]

A general empirical relationship was established for a resistance due to friction in a heterogeneous slurry flow as

$$\Phi = K\Psi^n \quad (4.3)$$

with the empirical coefficients K and n. The Φ - Ψ relationship might be determined using the hyperbolic curve in the Φ - Ψ plot (Fig. 4.1) proposed by Durand & Condolios or using the curve approximation giving K = 180 and n = -1.5. A plot Φ versus Ψ was proposed as a unified pattern for the evaluation of experimental data for solids of d = 0.18 - 22.5 mm and pipelines of D = 40 - 580 mm.

The ultimate correlation obtained by Durand et al. is

$$\frac{I_m - I_f}{I_f C_{vd}} = 180\left(\frac{V_m^2}{gD}\frac{\sqrt{gd}}{v_t}\right)^{-1.5} \tag{4.4}$$

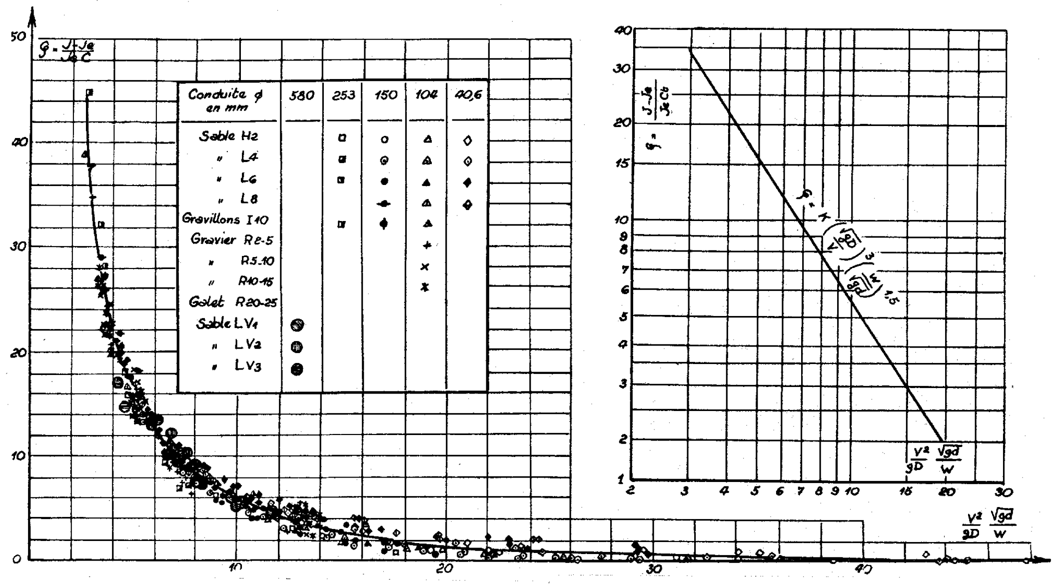

Figure 4.1. The Φ = KΨ^n relationship of the Durand model (Φ on the vertical axis and Ψ on the horizontal axis).

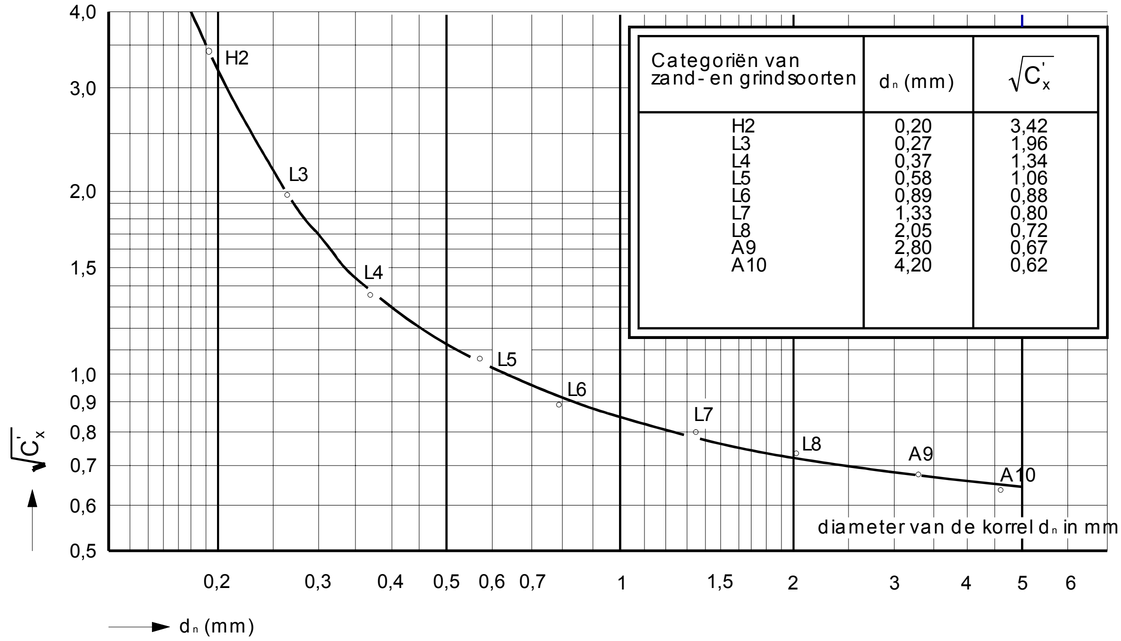

Figure 4.2. Modified particle Froude number $Fr_{vt}^{-1} = \frac{\sqrt{gd}}{v_t}$, here marked $\sqrt{C_x'}$, determined experimentally for various sorts of sand and gravel by Durand et al.

The plot on Fig. 4.2 can be used to estimate the $\frac{\sqrt{gd}}{v_t}$ value for sand or gravel particles.

According to van den Berg (1998) the relationship between Fr_{vt}^{-1} and d can be obtained also from the "best fit curve" correlation

$$Fr_{vt}^{-1} = \frac{\sqrt{gd}}{v_t} = 0.69 - 0.01d^{1.5} + \frac{0.161}{d^{1.5}} \quad (4.5).$$

An effect of broad particle size distribution is taken into account by determining an average value of the particle Froude number from values of Froude number for soil fractions p_i of different sizes d_i

$$Fr_{vt}^{-1} = \frac{\sqrt{gd}}{v_t} = \frac{\sum_i Fr_{vt,i}^{-1} \cdot p_i}{100} = \frac{\sum_i \frac{\sqrt{gd_i}}{v_{t,i}} \cdot p_i}{100} \quad (4.6).$$

Over the years the Durand type of correlation has been tested by using different experimental data bases and a considerable number of values has been proposed for K and n by various investigators (see survey in Kazanskij, 1978).

C. Discussion on the applicability of the Durand correlation:

An *advantage* of the Durand method is that it provides a simple relationship that covers a wide range of slurry flow conditions and correlates all basic parameters influencing the behaviour of slurry flow in a pipeline.

A major *disadvantage* is the inaccuracy of the determination of Φ for the extreme values of the coefficient Ψ (see Fig. 4.1). The Φ - Ψ curve is very steep at low Ψ. A small difference in Ψ may create a big difference in Φ (so predicted I_m may differ by from ten per cent to several hundred per cent). At high Ψ values the coefficient Φ decreases very slowly with increasing Ψ. The regions of the extreme Ψ values represent a transition from heterogeneous flow to the extreme slurry flow patterns: fully-stratified for the lowest values of Ψ and fully-suspended (pseudo-homogeneous) for the highest values of Ψ. The insensitivity of the correlation at its extremes reveals the fact that the model does not reflect different slurry flow patterns.

Doubts about the applicability of the correlation to a wide range of slurry flow conditions have been confirmed by a number of tests during the years. A large discrepancy between Durand's prediction and experimental data has been experienced specifically for a coarse slurry flow exhibiting considerable stratification. The correlation might, however, provide a satisfactory prediction for medium and medium to coarse sand mixtures at flows falling within the approximate range $4 < \Psi < 15$.

DEPOSITION-LIMIT VELOCITY (CRITICAL VELOCITY)

The deposition-limit velocity V_{dl} is given by an empirical correlation based on visual observations of the initial formation of a stationary bed in pipelines for different mixture flow conditions.

Durand experiments showed that the Froude number $Fr^2 = \frac{V_e^2}{gR_h}$ remained constant for pipeline flow when a stationary bed was formed and gradually became thicker under decreasing V_m. The Froude number for flow above the stationary bed was based on the velocity above a stationary bed, V_e, and on the hydraulic radius, R_h, of discharging area above the stationary bed. The constant value for Fr^2 was experienced in flows of constant solids density, particle diameter and delivered concentration. For flow conditions at the beginning of the stationary bed ($V_e = V_m = V_{dl}$ and $D = 4R_h$) this condition was written as

$$Fr^2 = \frac{V_{dl}^2}{gD} = \text{const.} \qquad (4.7)$$

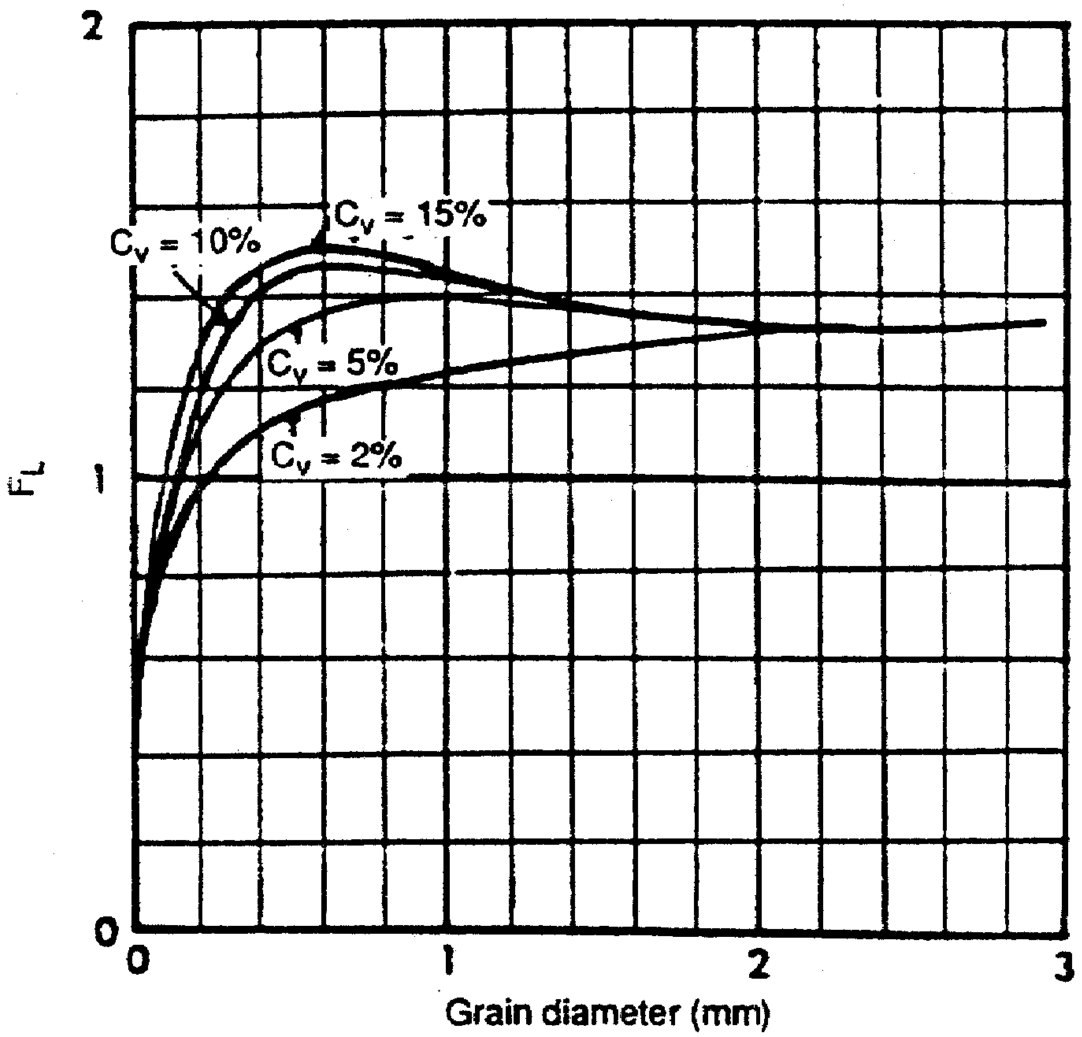

Figure 4.3. Modified Froude number F_L for a determination of the deposition-limit velocity according to the Durand et al. model.

An effect of various particle diameters and delivered concentrations C_{vd} on the value of the V_{dl} was expressed in the empirical relationship $F_L = f(d, C_{vd})$ presented as a graph (Fig. 4.3).

The correlation for deposition-limit velocity by Durand et al. is

$$V_{dl} = F_L \sqrt{2g(S_s - 1)D} \qquad (4.8)$$

V_{dl}	deposition-limit velocity (critical velocity)	[m/s]
F_L	empirical coefficient; graph $F_L = f(d, C_{vd})$	[-]
g	gravitational acceleration	[m/s^2]
S_s	relative density of solids	[-]
D	pipe diameter	[m].

4.1.2 Führböter model

FRICTIONAL HEAD LOSS

A. Experimental observations:

Experimental data were collected for slurry flow conditions in a 300 mm laboratory pipeline for sand and gravel of particle size range between 0.15 mm and 1.8 mm. Based on the experimental results, the following issues were proposed for the hydraulic gradient I_m in the heterogeneous slurry flow characterised by constant particle size d and pipeline size D.

B. Construction of the Führböter correlation and its ultimate version:

The correlation was found

$$I_m - I_f = S_k \frac{C_{vi}}{V_m} \qquad (4.9)$$

in which S_k was the empirical coefficient dependent on solids properties.
Practical calculations are done for C_{vd} instead of C_{vi}, thus the slip effect was incorporated to obtain C_{vd} in the above equation. The constant value of the slip ratio C_{vd}/C_{vi} = 0.65 was considered to hold for all mixture flow conditions. The transport factor S_{kt} was obtained by $S_{kt} = S_k.C_{vi}/C_{vd}$.

The values of the factor S_{kt} were empirically determined:

$S_{kt} = 2.59\ d_m - 0.037$	for $0.2 < d_m < 1.1$ mm
S_{kt} from graph on Fig. 4.4	for $1.1 < d_m < 3.0$ mm
S_{kt} is approximately 3.3	for $d_m > 3.0$ mm.

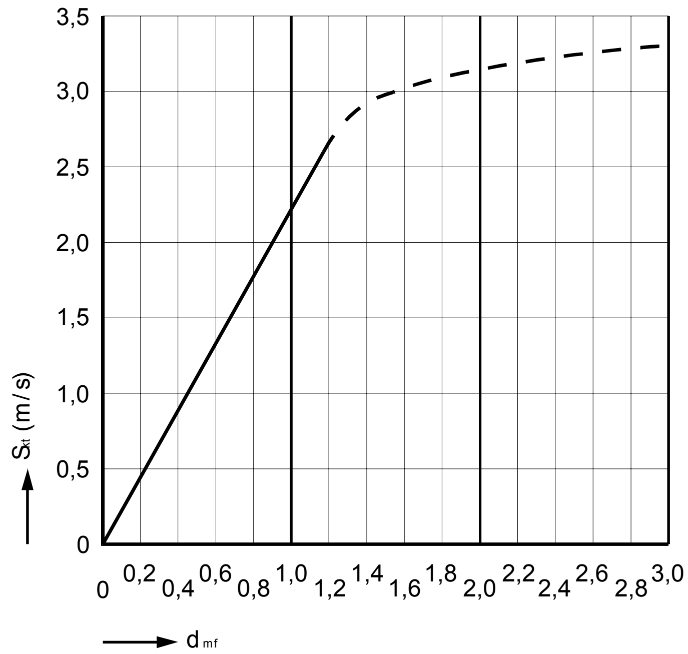

Figure 4.4. Transport factor S_{kt} for the Führböter correlation.

The Führböter correlation is

$$I_m - I_f = S_{kt} \frac{C_{vd}}{V_m} \qquad (4.10)$$

I_m	hydraulic gradient for mixture flow	[-]
I_f	hydraulic gradient for liquid flow	[-]
S_{kt}	transport factor	[m/s]
C_{vd}	delivered volumetric solids concentration	[-]
V_m	mean mixture velocity in a pipeline	[m/s]

C. Discussion on the applicability of the Führböter correlation:

Advantage:
The model is very easy to use and to calibrate by own data (only one coefficient has to be determined).

Disadvantage:
The transport factor S_{kt} must cover all effects of particle settling process (the settling velocity is not handled by the model) and effects of various soil and liquid densities (these are not explicitly handled by the model) on energy dissipation in a mixture flow. Thus the S_{kt} factor value, determined experimentally for certain flow conditions, can hardly be considered applicable to any different conditions.
Furthermore, the assumption of a constant slip ratio value is unacceptable for mixture flows of different particle sizes, mean mixture velocities and solids concentrations.
The model offers no possibility to introduce an effect of a broad PSD on flow resistance.

4.1.3 Jufin - Lopatin model

This model was constructed as a proposal for the Soviet technical norm in 1966. The authors did not submit a new model but selected the best combination of correlations for the frictional head loss and the critical velocity from four models submitted by different Soviet research institutes.

FRICTIONAL HEAD LOSS

A. Experimental observations:

Four submitted models were tested by a large experimental database collected by a number of researchers. The database contained data from both laboratory and field measurements (including data from dredging installations). The data covered a wide range of pipeline sizes (103 – 800 mm) and particle sizes (sand and gravel, 0.25 - 11 mm).

B. Construction of the Jufin - Lopatin correlation and its ultimate version:

The correlation was based on the empirical experience suggesting that the hydraulic gradient I_m at the minimum velocity V_{min} was independent of the mixture flow properties and it was three times higher than the hydraulic gradient of water flow at the same velocity in a pipeline. Thus $I_m = 3I_f$ at V_{min}. This was experienced also in the American dredging industry (see Turner, 1996).

The frictional-head-loss correlation by Jufin & Lopatin (in the revised version by Kobernik, 1968) is

$$I_m = I_f\left(1+2\left[\frac{V_{min}}{V_m}\right]^3\right) \quad (4.11).$$

The minimum velocity V_{min} is given by the empirical correlation

$$V_{min} = 5.3\left(C_{vd}.\psi^*.D\right)^{\frac{1}{6}} \quad (4.12)$$

in which the parameter $\psi^* = f(d)$ is determined either using a table by Jufin & Lopatin (see Tab. 4.1) or calculated as modified Froude number of a solid particle, $\psi^* = Fr_{vt}^{1.5}$.

I_m	hydraulic gradient for mixture flow	[-]
I_f	hydraulic gradient for liquid flow	[-]
V_m	mean mixture velocity in a pipeline	[m/s]
V_{min}	minimum velocity	[m/s]
C_{vd}	delivered volumetric solids concentration	[-]
ψ^*	particle settling parameter	[-]
D	pipeline diameter	[m]

Table 4.1. Particle settling parameter for the Jufin-Lopation model.

size fraction of solids, d [mm]	particle settling parameter, ψ* Jufin & Lopatin (1966)	particle settling parameter, ψ* Jufin (1971)
0.05 - 0.10	0.0204	0.02
0.10 - 0.25	0.093	0.2
0.25 - 0.50	0.404	0.4
0.50 - 1.00	0.755	0.8
1.0 - 2.0	1.155	1.2
2.0 - 3.0	1.50	1.5
3.0 - 5.0	1.77	1.8
5 - 10	1.94	1.9
10 - 20	1.97	2.0
20 - 40	1.80	2.0
40 - 60	1.68	2.0
> 60	1.68	2.0

An effect of broad particle size distribution is taken into account by determining an average value of the modified particle Froude number from values of modified Froude number for soil fraction p_i of different size d_i

$$\psi^* = Fr_{vt}^{1.5} = \frac{\sum_i Fr_{vt,i}^{1.5} \cdot p_i}{100} = \frac{\sum_i \psi^*(d_i) \cdot p_i}{100} \qquad (4.13).$$

C. Discussion on the applicability of the Jufin-Lopatin correlation:

Advantage:
The model was based on experiments carried out on large pipelines and thus it is considered suitable for pipeline-flow predictions in dredging.

DEPOSITION-LIMIT VELOCITY (CRITICAL VELOCITY)

Jufin and Lopatin proposed the following correlation for the deposition-limit velocity

$$V_{dl} = 8.3 D^{\frac{1}{3}} \left(C_{vd} \cdot \psi^* \right)^{\frac{1}{6}} \qquad (4.14).$$

4.1.4 Wilson - GIW model

The semi-empirical Wilson - Georgia Iron Works model for heterogeneous flow in slurry pipelines is based on considering the heterogeneous flow as a transition between two extreme flows governed by different mechanisms for support of a solid particle in the stream of the carrying liquid: the *fully-stratified flow* (all particles are transported as a contact load) and the *fully-suspended flow* (all particles are transported as a suspended load).

FRICTIONAL HEAD LOSS

A. Experimental observations:

Circuit tests in the experimental laboratory of the GIW Inc. provided a data base for a verification of the friction-loss correlation. The circuits are of the pipeline size 200 mm, 440 mm respectively. Data for medium to coarse sands in mixtures of delivered concentrations up to 0.16 were used.

B. Construction of the correlation and its ultimate version:

The model for partially-stratified (heterogeneous) flow operates with a parameter V_{50} expressing the mean slurry velocity at which one half of the transported solid particles contribute to a suspended load and one half to a contact load. An equation for this velocity expresses the influence of suspension mechanisms from the carrier turbulent diffusion and the hydrodynamic lift acting on particles larger than the sub-layer thickness in the near-wall region.

The energy dissipation due to the presence of solid particles in a carrier flow is predominantly due to mechanical friction between contact-load particles and a pipeline wall. Basically, a resisting force of the contact bed against the carrier flow is related to the submerged weight of the bed via the coefficient of mechanical friction.
Thus at velocity $V_m = V_{50}$ the pressure loss due to presence of solids (ΔP_m - ΔP_f) is due to the submerged weight of the moving bed containing one half of the total solid fraction $[0.5C_{vd}(\rho_s - \rho_f)g]$ times the friction coefficient (μ_s). Rewritten for head losses this basic balance is

$$I_m - I_f = 0.5\mu_s C_{vd}(S_s-1) \qquad (4.15)$$

for the condition $V_m = V_{50}$.
Experimental data, plotted in log-log coordinates, showed a linear relationship between the ratio $\frac{I_m - I_f}{C_{vd}(S_s - 1)}$ (called the relative solids effect) and the mean mixture velocity V_m. The relationship was found the same for flows of different concentrations in pipes of different sizes (Fig. 4.5). A slope of the line in the plot was considered sensitive only on the particle size distribution of transported solids.

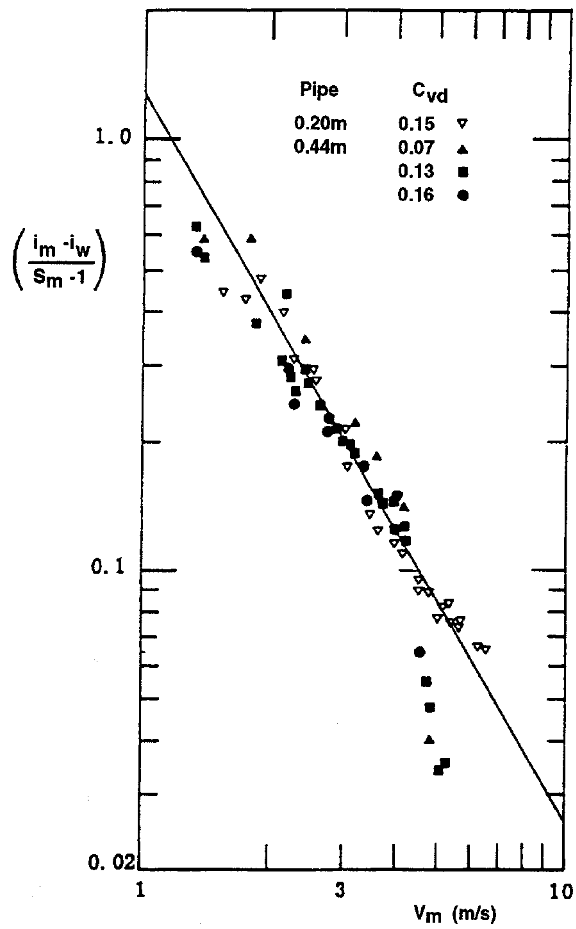

Figure 4.5. Relationship between relative solids effect and mean slurry velocity for masonry sand mixture (d_{50} = 0.42 mm), after Clift et al. (1982).

This led to the following form of the Wilson - GIW correlation

$$\frac{I_m - I_f}{C_{vd}\left(S_s - 1\right)} = 0.5\mu_s\left(\frac{V_m}{V_{50}}\right)^{-M} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-M} \qquad (4.16)$$

I_m	hydraulic gradient for mixture flow	[-]
I_f	hydraulic gradient for liquid flow	[-]
C_{vd}	delivered volumetric solids concentration	[-]
S_s	relative density of solids	[-]
V_m	mean mixture velocity in a pipeline	[m/s]
V_{50}	value of V_m at which one half of solids is suspended in a carrier flow	[m/s]
μ_s	coefficient of mechanical friction between solids and the pipeline wall (μ_s = 0.44)	[-]
M	empirical exponent sensitive on PSD	[-]

V_{50} should be obtained experimentally or estimated roughly by the approximation

$$V_{50} \approx 3.93 (d_{50})^{0.35} \left(\frac{S_s - 1}{1.65} \right)^{0.45} \qquad (4.17).$$

in which the particle diameter d_{50} is in mm and the resulting V_{50} in m/s. The exponent M is given by the approximation

$$M \approx \left[\ln \left(\frac{d_{85}}{d_{50}} \right) \right]^{-1} \qquad (4.18).$$

M should not exceed 1.7, the value for narrow-graded solids, nor fall below 0.25.

C. Discussion on the correlation:

The Wilson - GIW model gives a scale-up relationship for friction loss in slurry pipelines of different sizes transporting solids of different sizes at different concentrations. It is based on the assumption that there is a power-law relationship between the relative solids effect and the mean slurry velocity that is valid in all slurry flow conditions. The exponent M of this relationship is assumed to be dependent on the particle size distribution only.

4.1.5 MTI Holland model

DEPOSITION-LIMIT VELOCITY (CRITICAL VELOCITY)

In MTI Holland the correlation has been developed (see e.g. van den Berg, 1998) for the threshold velocity between the "fully suspended heterogeneous flow" regime and the regime of "flow with the first particles settling to the bottom" of a pipeline. This velocity was considered as the lowest acceptable velocity for a economic and safe operation of a dredging pipeline and was therefore also called *the critical velocity*

$$V_{crit} = 1.7\left(5 - \frac{1}{\sqrt{d_{mf}}}\right)\sqrt{D}\left(\frac{C_{vd}}{C_{vd} + 0.1}\right)^{\frac{1}{6}}\sqrt{\frac{S_s - 1}{1.65}} \qquad (4.19)$$

In Eq. 4.19 the particle diameter d_{mf} is in millimetres and the pipe diameter D in meters.

The correlation has an advantage of being based on data including those from various dredging pipelines. MTI recommends the correlation for grains of sand and gravel size and pipelines larger than 200 mm.

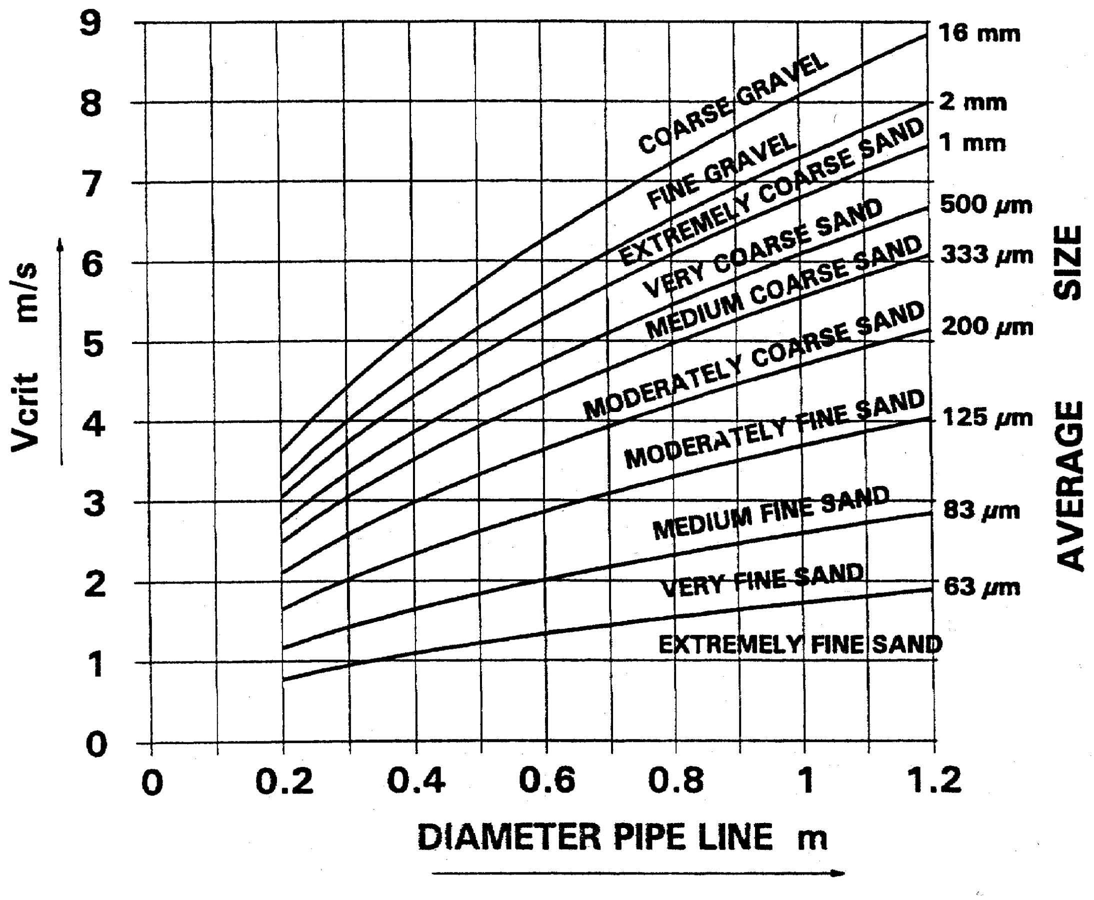

Figure 4.6. Critical velocity according to the MTI model (Eq. 4.19). The nomograph does not include the effect of deliverde concentration C_{vd}.

4.2 PHYSICAL MODELING

4.2.1 Prediction of fully-stratified flow of mixture using NOMOGRAPHS and/or approximations based on outputs of a physical two-layer model

FRICTIONAL HEAD LOSS

The frictional head loss in the **fully-stratified flow** is can be predicted successfully by using a two-layer model. It can be computed in its original shape (a set of mass and force balance equations) by iteration. To avoid these computations the nomograph was constructed as an interpolation of typical outputs from the original two - layer model.

A nomograph on Fig. 4.7 provides the values of I_m in the fully stratified flow for various combinations of input d, D, S_s, C_{vd} and V_m. The nomograph is composed of a locus curve, determining the boundary of the stationary deposit zone, and of a set of fit-function curves relating

- the relative excess pressure gradient $\frac{I_m - I_f}{I_{pg}} = \frac{I_m - I_f}{2\mu_s(S_s - S_f)C_{vb}}$ with

- the relative velocity $V_r = \frac{V_m}{V_{sm}}$ for different

- relative concentrations $C_r = \frac{C_{vd}}{C_{vb}}$.

I_m	hydraulic gradient for mixture flow	[-]
I_f	hydraulic gradient for liquid flow	[-]
I_{pg}	hydraulic gradient for plug flow	[-]
μ_s	coefficient of mechanical friction between solids and the pipeline wall	[-]
S_s	relative density of solids	[-]
S_f	relative density of carrying liquid	[-]
C_{vd}	delivered volumetric solids concentration	[-]
C_{vb}	loose-poured bed concentration, typically C_{vb} = 0.60	[-]
V_m	mean mixture velocity in a pipeline	[m/s]
V_{sm}	maximum value of V_{dl} for various solids concentrations in flowing mixture	[m/s]

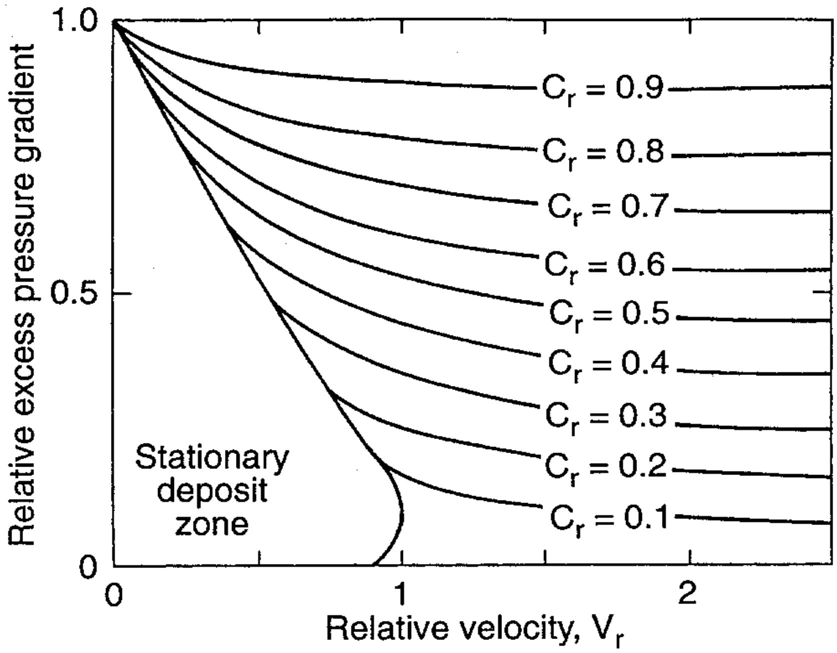

Figure 4.7. Curves of relative excess pressure gradient, from Wilson et al. (1992).

The nomograph curves can be approximated by the simple expression

$$\frac{I_m - I_f}{C_{vd}(S_s - 1)} = \left(\frac{V_m}{0.55 V_{sm}}\right)^{-0.25} \tag{4.20}$$

in which V_{sm} is the maximum value of deposition-limit velocity for different solids concentrations in slurry flow of certain S_s, d and D. The V_{sm} is determined from the "demi-McDonald" nomographic chart (see further below) or by its approximation

$$V_{sm} = \frac{8.8\left[\frac{\mu_s (S_s - S_f)}{0.66}\right]^{0.55} D^{0.7} d_{50}^{1.75}}{d_{50}^{2} + 0.11 D^{0.7}} \tag{4.21}$$

D	pipeline diameter	[m]
d_{50}	mass-median particle diameter	[mm]

in which d_{50} is in millimetres and D in metres.

DEPOSITION-LIMIT VELOCITY

The computation of the force balance at incipient motion of the bed in fully-stratified flow gives a locus curve - the curve relating the deposition-limit velocity, V_{dl}, with the position of an interface between the layers in the pipeline cross section. The locus curve has a maximum (see Fig. 4.7) which determines the maximum velocity at the limit of stationary deposition V_{sm}.

Maximum velocity at the limit of stationary deposition, V_{sm}:

Wilson (1979) processed V_{sm} values obtained as the model outputs for a variety of values of input parameters (d_{50}, D, S_s) to the nomographic chart (Fig. 4.8), sometimes called the demi-McDonald.

The demi-McDonald curve has a *turbulent branch* (for small particles of diameter less than approximately 0.5 mm) and a *fully-stratified branch* (for particles larger than approximately 0.5 mm). The threshold particle size delimits two branches at the peak of the nomographic curve.

The *fully-stratified branch* of the demi-McDonald curve was constructed from outputs of the model for fully-stratified flow. According to this part of the demi-McDonald curve, V_{sm} decreases with increasing particle size in a pipeline of a certain D. Thus a lower V_m is required to initiate motion in a coarser bed. An explanation of this phenomenon requires knowledge of physical principles mixture motion in a stratified flow expressed using a force balance in a two-layer model. This will be discussed later in paragraph 4.2.2. Basically, the faster flowing of the coarser bed, when compared to the finer beds, is caused by a fact that the top of the coarser bed is rougher (the roughness is related to the size of particles occupying the bed top). Via the rougher interface the higher driving force is transmitted to the bed from the faster flowing carrier above the bed.
V_{sm} in *turbulent branch* is affected by a variable thickness of bed at an incipient motion under different flow conditions. The bed thickness diminishes owing to a turbulent suspension process that picks up the particles from the bed surface and suspend them in flow above the bed. Thus the mixtures containing fine particles create thinner bed than mixture of coarse particles. Lower mean flow velocity is required to put the thinner bed into a motion. Therefore the maximum deposition-limit velocity drops with a size of particles transported in a pipeline.

The entire demi-McDonald nomograph can be approximated by the fit function (Eq. 4.21)

$$V_{sm} = \frac{8.8\left[\frac{\mu_s (S_s - S_f)}{0.66}\right]^{0.55} D^{0.7} d_{50}^{1.75}}{d_{50}^{2} + 0.11 D^{0.7}}.$$

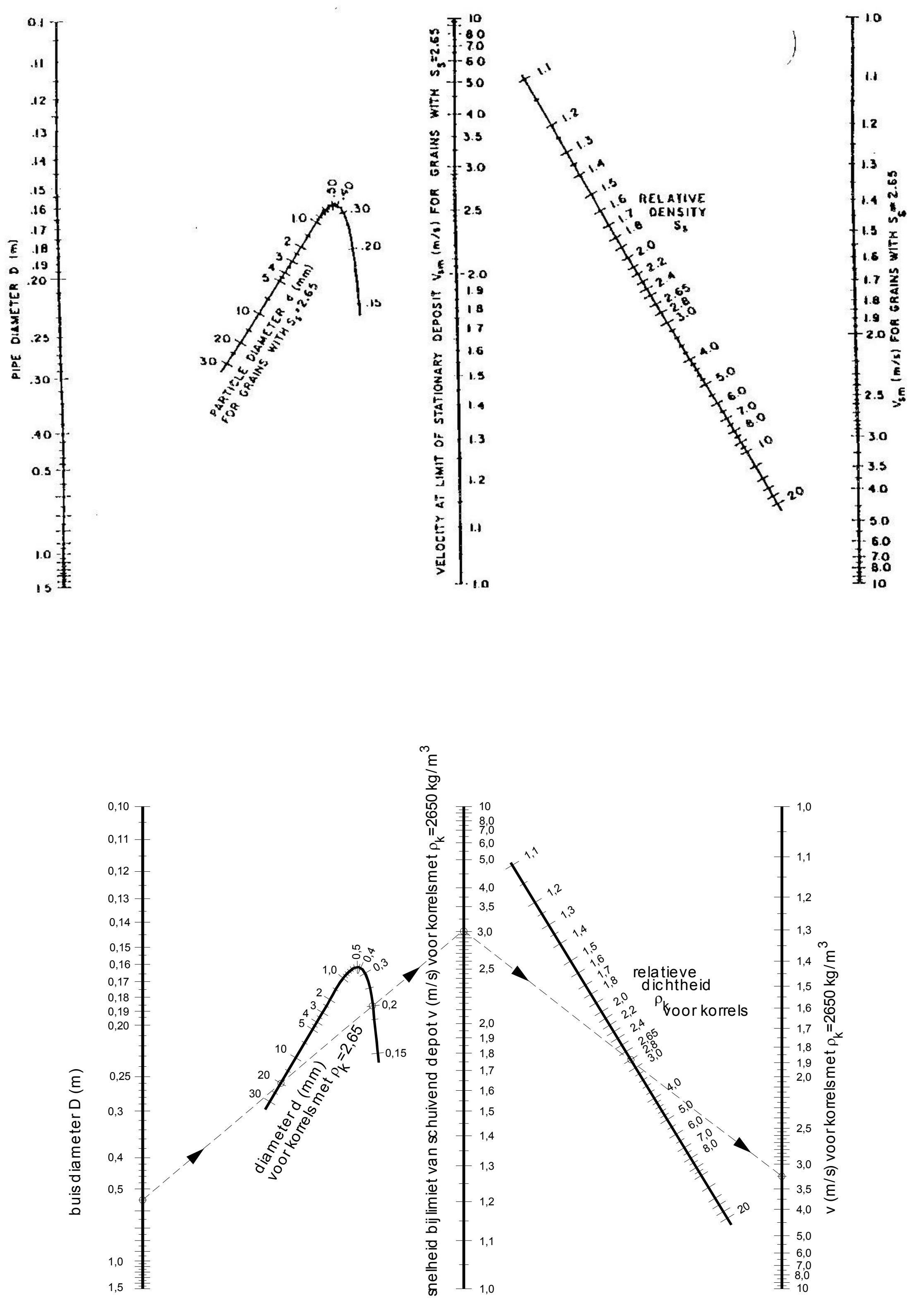

Figure 4.8. Nomographic chart for maximum velocity at limit of stationary deposition V_{sm} after Wilson (1979).

The *incorporation of a shear layer* to the pattern of a two-layer model has recently led to a modification of the demi-McDonald nomograph (Wilson, 1992). If the sharp interface between a bed and the carrier flow above the bed is replaced by the shear layer the driving force transmitted from the upper-layer flow to the bed is no longer dependent on the roughness of the top of a bed. Therefore the particle size does not directly influence the maximum deposition-velocity V_{sm}. Wilson (1992) proposed that the V_{sm} for fully-stratified flow with the shear layer (marked $V_{sm, max}$) should be determined by an approximation

$$\frac{V_{sm,max}}{\sqrt{2gD(S_s-1)}} = \left(\frac{0.018}{\lambda_f}\right)^{0.13} \qquad (4.22).$$

$V_{sm,max}$	value of V_{sm} for fully-stratified flow with the shear layer	[m/s]
g	gravitational acceleration	[m/s^2]
S_s	relative density of solids	[-]
D	pipeline diameter	[m]
λ_f	Darcy-Weisbach friction coefficient for liquid flow (from Moody diagram)	[-].

The $V_{sm,max}$ by the approximation is considered the V_{sm} value if it is lower than the V_{sm} value from the nomograph (i.e. also from the Eq. 4.21).

Deposition-limit velocity V_{dl} - the effect of solids concentration the V_{sm} value:

V_{sm} gives the maximum value on the locus curve delimiting the stationary deposit zone (see Fig. 4.7). The position on the curve is given by a solids concentration. A locus curve is a product of a two-layer model, thus a physical explanation of the curve is given by the principles of force balance in a two-layer flow pattern. A shape of a locus curve is dependent on the several parameters from which the size of a particle and a pipeline are the most important ones.

Nomographs were developed to determine the critical velocity V_{dl} from V_{sm} without a necessity to compute an original two-layer model. The nomograph curves were also approximated by fit functions. A process of V_{dl} determination goes in following steps:

1. V_{sm} using demi-McDonald nomograph (Fig. 4.8) or its approximating fit function (Eq. 4.21)

2. C_{rm}, the concentration at which V_{sm} occurs, using the nomograph Fig. 4.9 or its approximating fit function (Eq. 4.23)

3. V_{dl}/V_{sm}, the relative deposit velocity, using the nomograph Fig. 4.10 or its approximating fit functions (Eqs. 4.24 & 4.25).

ad 2. The relative solids concentration C_{rm}

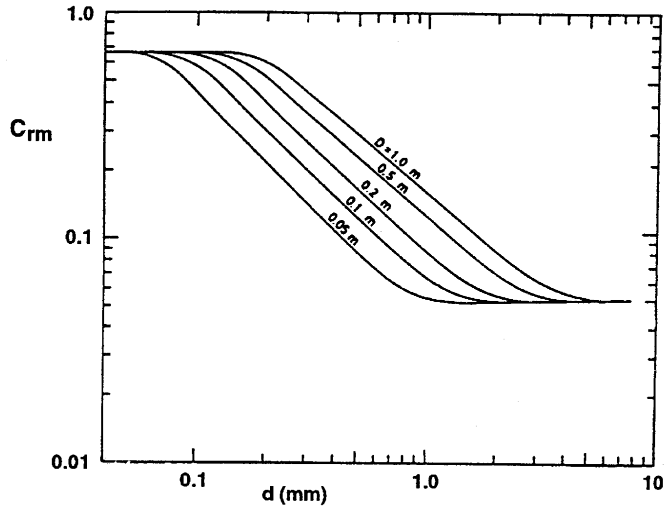

Figure 4.9. Computer output for relative solids concentration C_{rm} at maximum deposit velocity, from Wilson (1986).

Fit function:

$$C_{rm} = 0.16 D^{0.40} d^{-0.84} \left(\frac{S_s - S_f}{1.65} \right)^{-0.17} \tag{4.23}$$

in which d [mm] and D [m].

ad 3. The relative deposit velocity $\frac{V_{dl}}{V_{sm}}$

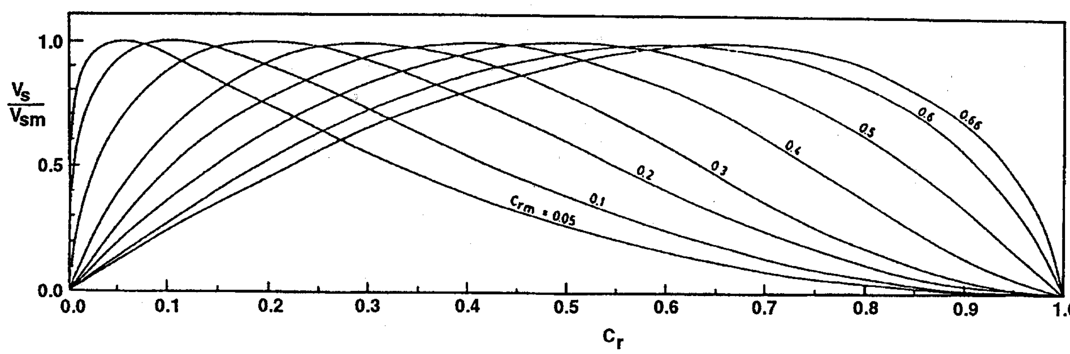

Figure 4.10. Plot of relative deposit velocity V_{dl}/V_{sm} versus $C_r = C_{vd}/C_{vb}$ for various values of C_{rm}, from Wilson (1986).

Fit functions:

for $C_{rm} \leq 0.33$

$$\frac{V_{dl}}{V_{sm}} = 6.75 C_r^{\frac{\ln(0.333)}{\ln C_{rm}}} \left[1 - C_r^{\frac{\ln(0.333)}{\ln C_{rm}}} \right]^2 \quad (4.24)$$

and for $C_{rm} > 0.33$

$$\frac{V_{dl}}{V_{sm}} = 6.75 (1 - C_r)^{2\frac{\ln(0.666)}{\ln(1-C_{rm})}} \left[1 - (1 - C_r)^{\frac{\ln(0.666)}{\ln(1-C_{rm})}} \right] \quad (4.25).$$

4.2.2 TWO-LAYER MODEL: principles and mathematical formulation

When solids such as sand or gravel are transported in a slurry pipeline some degree of slurry flow stratification usually occurs. This is the effect of the tendency of solid particles in the carrying liquid to settle. Stratified slurry flow forms a particle-rich zone and a particle-lean zone in the pipeline cross section. According to the shape of its concentration profile, the slurry flow may be considered fully-stratified or partially-stratified.

4.2.2.1 Principles of the model

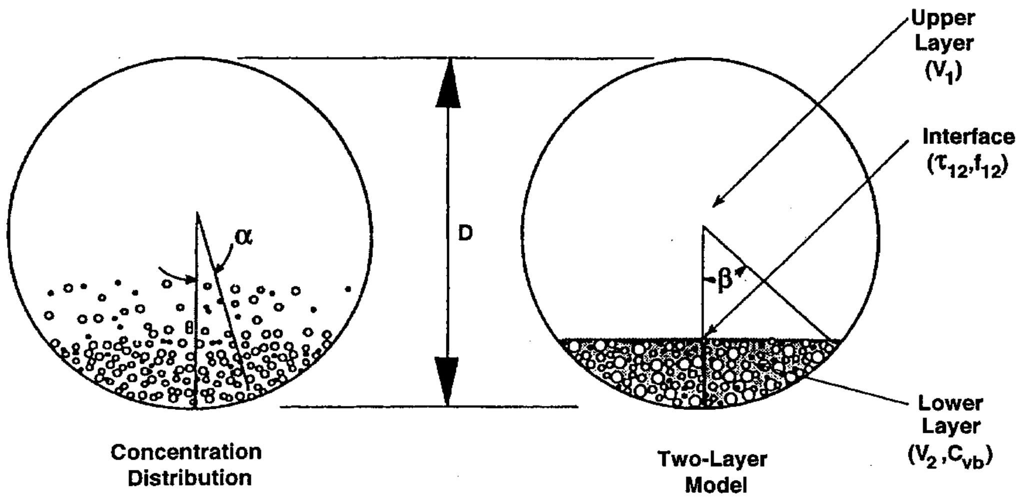

Figure 4.11. Definition sketch for two-layer model of stratified flow, after Wilson et al. (1992).

The two-layer model takes into account the slurry flow stratification and transforms a real concentration profile in a pipeline cross section into a simplified two-layer pattern. When the slurry flow is fully stratified solid particles transported in the carrying liquid are all accumulated in a granular bed sliding at the bottom of the pipeline (Fig. 4.11). **All particles in this lower layer are in mutual contact**. The volumetric concentration of solids in the lower layer of the fully-stratified flow approaches the concentration value of a loose-poured bed. The stream of the carrying liquid above the granular bed is particle-free. The position of an interface between two layers is determined by the angle β.

In a partially-stratified flow a considerable fraction of the total transported solids mass is suspended in the carrier stream. **Suspended particles are assumed not to be in contact with other particles and the flow boundaries**. Velocity distribution - as well as the concentration distribution - is idealised as being uniform within both the upper and the lower layers (Fig. 4.12). The distribution of the suspended particles within an idealised two-layer pattern has been subjected to investigation. Early versions of the model anticipated a suspension only in the upper layer. For an idealised flow pattern the recent modification of the model assumes a uniform distribution of suspended particles along the entire pipeline cross section.

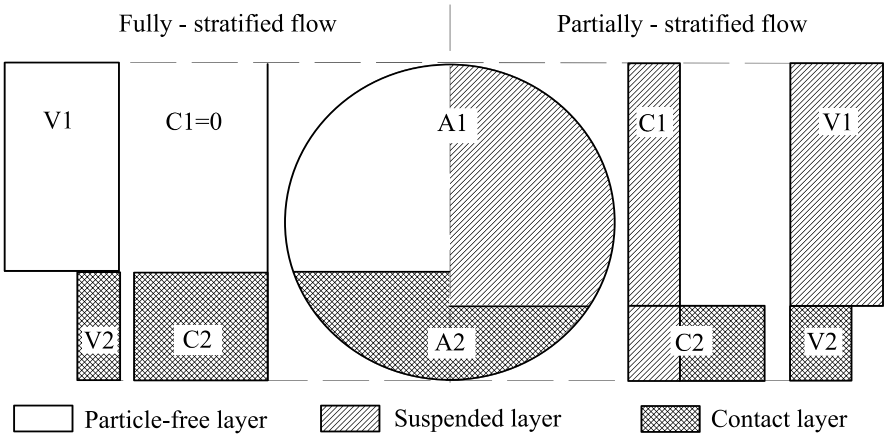

Figure 4.12. Schematic cross section for two-layer model.

The model is based on the assumption that **there are two physical mechanisms for solid particle support in a pipeline - interparticle contact and particle suspension in a carrying liquid**. Thus solids are transported as both suspended and contact loads. According to Bagnold (1956), the suspended particles transfer their submerged weight directly to the carrier, while the submerged weight of the non-suspended particles is transferred via interparticle contacts to the pipeline wall.

According to the model **the behaviour of the flow is governed by the principle of force balance between driving and resisting forces in the flow in two layers**. The driving force in the flow in a pressurised horizontal pipeline is produced by the pressure gradient over a pipeline length section. The resisting force is represented by shear stress exerted by flowing matter at a flow boundary. The same formulation of the force balance between the driving and resisting forces, combined with a friction coefficient equation, gives the Darcy-Weisbach equation (Eq. 1.20) for friction losses in a water pipeline. The Darcy-Weisbach equation is obtained from a two-layer model for the limiting case in which the particle-free upper layer occupies the whole pipeline cross section.

4.2.2.2 Mathematical formulation of the model

The model is composed of a set of equations expressing the conservation of mass and momentum in a mixture flow in both layers in the pipe section. A set of conservation equations is computed by iteration. The layer occupying a pipe length L is considered to be a control volume. Flow in the control volume is steady and uniform. The quantities describing the properties of the layer are given by values averaged in the control volume. This can be seen in Fig. 4.12 where V_1 and V_2 denote the mean velocity of mixture in the upper (lower respectively) layer. The same is valid for mean volumetric concentrations C_1 and C_2 in the layers. Slip between solid phase and liquid phase is considered negligible within both the suspension flow and the flow of contact particles. The model-equation parameters defining the geometry of the schematic cross section for a two-layer model are described in Fig. 4.13.

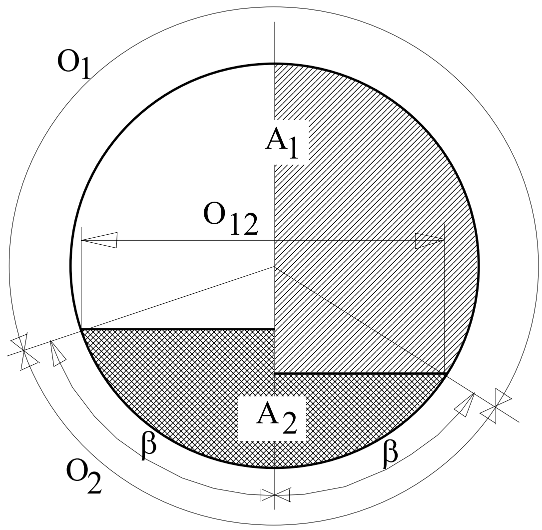

Figure 4.13. Geometry of schematic cross-section for two-layer model.

The following equations (Eq. 4.26, 4.29, 4.30, 4.31, 4.32, 4.34, 4.38, 4.39, 4.40, 4.42, 4.43, 4.44, 4.45) form the two-layer model:

Mass balance for flow in two layers

The application of the mass conservation law to a two-layer pattern gives the balances

for slurry flow rate: $Q_m = Q_{m1} + Q_{m2}$

$$V_m A = V_1 A_1 + V_2 A_2 \quad (4.26),$$

for solids flow rate: $Q_s = Q_{s1} + Q_{s2}$

$$A_s V_s = C_{vi} A V_s = C_1 A_1 V_1 + C_2 A_2 V_2 \quad (4.27)$$

and for liquid flow rate:

$$Q_f = Q_{f1} + Q_{f2}$$

$$A_f V_f = (1-C_{vi}) A V_f = (1-C_1) A_1 V_1 + (1-C_2) A_2 V_2 \quad (4.28).$$

The solids volume balance is written as

$$C_{vi}A = C_1A_1 + C_2A_2 \qquad (4.29).$$

Since $C_{vd} = Q_s / Q_m$ the Eq. 4.27 for solids flow rate can be written as

$$C_{vd}AV_m = C_1A_1V_1 + C_2A_2V_2 \qquad (4.30).$$

Momentum balance for flow in two layers

A law of conservation of momentum is formulated as force balance between driving and resisting forces acting on the flow boundaries of each layer in a horizontal pipeline of the length L:

DRIVING FORCES = RESISTING FORCES.

The force balance for the *upper layer* is written as

$$\Delta PA_1 = \tau_1 O_1 L + \tau_{12} O_{12} L \qquad (4.31)$$

and for the *lower layer* as

$$\Delta PA_2 + \tau_{12} O_{12} L = (\tau_{2f} + \tau_{2s}) O_2 L \qquad (4.32).$$

A summation of these two equations gives a force balance in the whole pipeline section

$$\Delta PA = \tau_1 O_1 L + (\tau_{2f} + \tau_{2s}) O_2 L \qquad (4.33).$$

Resistance forces against flow are due to viscous or mechanical friction at flow boundaries.

4.2.2.3 Friction mechanisms by two-layer model

Mechanical friction between solids and pipeline wall

Solid particles in contact with each other and with the pipeline wall transmit their submerged weight to the pipeline wall. This is the source of the resisting force exerted by the contact load solids against the flow driving forces. The force is due to the solids stress acting at the pipeline wall. In a horizontal pipeline the stress σ_s between the solids grains and the pipeline wall acts in a radial direction in the pipeline cross section, so that it is normal to the pipeline wall. The normal stress σ_s produces the (Coulombic) intergranular shear stress at the pipeline wall $\tau_s = \mu_s \sigma_s$. In this relationship μ_s is the coefficient of mechanical friction between solid particles and the pipeline-wall material. The total resistance force exerted by the sliding granular bed is

$$\mu_s F_N = \tau_{2s} O_2.$$

The total normal force, F_N, exerted by the normal intergranular stress against the pipeline wall is obtained by integrating the normal stress over the pipeline perimeter O_2. The result of integrating is

$$F_N = g(\rho_s - \rho_f)C_{vb}\frac{D^2}{2}(\sin\beta - \beta\cos\beta) \qquad (4.34).$$

The force F_N differs from F_W, which is the submerged weight of the granular bed. The force F_W, which represents the gravitational effect on a granular body, is integrated from the intergranular stress component σ_W. Only this component can act to support the bed weight. At each local pipeline-wall position, given by angle α, the stress $\sigma_W = \sigma_s\cos\alpha$. By integrating over the perimeter O_2 of the interface between a bed and a pipeline wall

$$F_W = g(\rho_s - \rho_f)C_{vb}\frac{D^2}{4}(\beta - \sin\beta\cos\beta) \qquad (4.35)$$

where $\frac{D^2}{4}(\beta - \sin\beta\cos\beta) = A_2$ and therefore

$$F_W = g(\rho_s - \rho_f)C_{vb}A_2 \qquad (4.36).$$

For a dense-phase flow (called also the plug flow) $F_N = 2F_W$. Force balance at an initial motion of a plug flow is written as

$$\frac{\Delta P}{L}A = \mu_s 2F_W \quad \text{that is} \quad \frac{\Delta P}{L}A = 2\mu_s g(\rho_s - \rho_f)C_{vb}A \ .$$

Hydraulic gradient required to initiate a plug sliding is then

$$I_{pg} = \frac{\Delta P}{L}\frac{1}{\rho_f g} = 2\mu_s(S_s - 1)C_{vb} \qquad (4.37).$$

The shear stress, τ_{2s}, due to mechanical friction between granular bed forming a contact layer and pipeline wall is velocity-independent. It is determined from σ_s, the normal intergranular stress at the pipeline wall. A resisting force due to mechanical friction between a contact layer and a pipeline wall is perpendicular to the normal intergranular force F_N exerted against the pipeline wall and it is related with F_N by $\mu_s F_N$.

Viscous friction at flow boundaries

Viscous friction between the flowing carrying liquid and the flow boundary is a velocity-dependent process described by the boundary shear stress (τ_1, τ_{12}, τ_{2f}). Shear stress is related to the velocity gradient between the flowing carrier and the flow boundary by a friction coefficient expressing flow conditions at the boundary (see Chapter 2). The conditions are given by the flow regime and the boundary roughness. The Darcy-Weisbach friction coefficient (Eq. 1.18) is related to the Reynolds number of the flow, Re, and/or the boundary roughness, k. The friction coefficient for water flow in a pipeline is obtained from the Moody diagram or its computational version (Churchill, 1977). The Reynolds number characterising the flow in the layer is calculated from the hydraulic diameter, D_h, of the layer (D_{h1} = $4A_1/O_1$, $D_{h2} = 4A_2/O_2$) by $Re = \frac{VD_h\rho_f}{\mu_f}$.

Friction coefficients λ_1, for flow in the upper layer over the pipeline wall of perimeter, O_1, is

$$\lambda_1 = fn\left(\frac{V_1 D_{h1}\rho_f}{\mu_f}, k\right) \qquad (4.38)$$

(λ_1 determined using the Moody diagram) and *λ_2, for flow in the lower layer* over the pipeline wall of perimeter, O_2,

$$\lambda_2 = fn\left(\frac{V_2 D_{h2}\rho_f}{\mu_f}, k\right) \qquad (4.39)$$

(λ_2 determined using the Moody diagram).

The *coefficient λ_{12} for flow in the upper layer over the interface between two layers* (the perimeter O_{12}) differs according to the conditions at the interface. When the interface is represented by a clearly identifiable flat surface of a contact bed it can be considered to have a roughness proportional to the diameter of the particles occupying the bed surface. The interfacial friction law is given by a formula for turbulent liquid flow over a fully-rough boundary, e.g.

$$\sqrt{\frac{8}{\lambda_{12}}} = \frac{4\log\left(\frac{D}{d_{12}}\right) + 3.36}{\sqrt{0.5 + X}} \qquad (4.40)$$

in which $X = 5 + 1.86\log\left(\frac{d_{12}}{D}\right)$ for $d_{12}/D > 0.002$ and $X = 0$ otherwise. In the equation, d_{12} is the diameter of particle at the interface. This is determined by assuming that all particles larger than the particle of the d_{12} size are below the interface.

The condition of a flat and sharp interface is fulfilled more likely in pipeline flow containing very coarse particles. In flow containing finer particles the top of a contact bed is usually sheared off and a sharp interface is replaced by a transition zone, called shear layer, with concentration and velocity gradient. Thus an interface becomes virtual rather than real. For the virtual interface the particle-size roughness is no longer a parameter determining interfacial friction. The Wilson analysis of a flow at high shear stress above a stationary granular bed revealed that the thickness of the shear layer is a crucial parameter determining the interfacial friction. This is related to the hydraulic gradient in the total flow so that the interfacial friction coefficient can be determined as

$$\lambda_{12} = 0.87\left(\frac{I_m}{S_s - 1}\right)^{0.78} \qquad (4.41).$$

The *boundary liquid-like shear stresses* are written
for the pipe wall in the upper layer

$$\tau_1 = \frac{\lambda_1}{8}\rho_f V_1^2 \quad (4.42),$$

for the pipe wall in the lower layer

$$\tau_{2f} = \frac{\lambda_2}{8}\rho_f V_2^2 \quad (4.43),$$

and for the interface between the upper layer and the lower layer

$$\tau_{12} = \frac{\lambda_{12}}{8}\rho_f (V_1 - V_2)^2 \quad (4.44).$$

4.2.2.4 Model coefficients

The two-layer model for fully-stratified flow has the following coefficients:
- the coefficient of mechanical friction, μ_s, between a granular bed and a pipe wall,
- the viscous friction coefficient, λ_{12}, for liquid flow at the interface between two layers,
- the viscous friction coefficient, λ_1, for liquid flow at the boundary between a liquid flow in an upper layer and a pipe wall,
- the viscous friction coefficient, λ_2, for liquid flow at the boundary between a granular bed and a pipe wall,
- the volumetric spatial concentration, C_{vb}, in the contact layer.

These have to be either prepared as inputs to the model (μ_s, C_{vb}) or determined in the model (λ_1, λ_2, λ_{12}).

4.2.2.5 Model computation: inputs and outputs

To *determine the frictional head loss* for a certain value of the mean mixture velocity, V_m, in a pipeline the following input parameters are required.

Input parameters:	Liquid:	S_f, ν_f
	Solids:	d, S_s, μ_s
	Slurry flow:	V_m, C_{vd}, C_{vb}
	Pipe:	L, k, D

The model provides the following output parameters, characterising the friction, slip and simplified concentration and velocity distribution in a fully-stratified flow.

Output parameters:
ΔP (I_m respectively)
Y_{12}/D
C_{vi}
V_1, V_2.

To *determine deposition-limit velocity*, V_{dl}, various values of V_m are used as inputs to the model and the force balance is sought for V_m value at which $V_2 = 0$.

4.2.2.6 Model adaptation to partially-stratified flow

The two-layer model can be used as a predictive tool for the partially-stratified (heterogeneous) flow also. The only condition is that the mixture flow is sufficiently stratified, i.e. it contains a granular bed that is of any significance to the mixture flow behavior. It seems that this condition is fulfilled for flows of medium sand (if travelled at velocities near the deposition-limit value) and coarser. In such flows only one part of solids occupies a granular bed and the rest is suspended in the carrying liquid. The particle suspension is predominantly due to the dispersive effect of liquid turbulent eddies, at certain flow conditions, however, the transported particles might be also suspended due to the shearing of a top of a granular bed. In the sheared layer the particles have sporadic rather than permanent contact so that at each moment a portion of solid particles within the sheared layer might be considered suspended. The shear layer is a transition region between the granular bed where all particles are in permanent mutual contact and the upperst layer in which particles are either not present or they are present but have no mutual contact with each other and with a pipe wall – they are suspended in a carrier stream.

A reliable application of the two-layer model to partially-stratified flow is still subject to investigation. It requires experimental experience based on the measurements of the concentration (and velocity) profiles in mixture flow under various conditions. To date only few such experiments have been carried out. Most recently the experiments were carried out in the Laboratory of Dredging Technology and Bulk Transport of Delft University of Technology. This gives us an opportunity to discuss this subject on a basis of certain experimental experience lacking when earlier model assumptions had been made.

- the threshold between fully-stratified and partially-stratified flow

An analysis of the interaction between settling particles and turbulent carrier flow gives a condition for the initiation of particle suspension: the length scale of liquid turbulence (represented by the mixing length, discussed in Chapter 2) has to be larger than the particle size. Only particles smaller than a certain portion of the mixing length could be supported by the eddies, otherwise the turbulent dispersive mechanism is not effective in suspending transported particles. The turbulent length scale is considered to be dependent on the local position within a pipeline flow and thus the average mixing length depended on the pipeline diameter. This gives to rise to rather complex relationships between the mean mixture velocity at the beginning of turbulent suspension and particle/pipeline size. However, a rough estimation of a threshold between the fully-stratified flow and the partially-stratified flow can be satisfied by a simple d/D ratio value. The flow would be fully stratified for $d/D > 0.018$ according to Wilson, our data from a 150 mm pipeline suggest rather higher value of the d/D ratio, 0.03 approximately.

- the additional model coefficient: the stratification ratio

For fully-stratified flow, the two-layer model considers the upper layer as particle-free and the lower layer as occupied by particles, all of which are in continuous contact. In a partially-stratified flow the solids are transported in a carrying liquid both as a contact load and as a suspended load. The amount of solids occupying a slurry pipeline is given by the volumetric spatial concentration C_{vi} that is the sum of a solids fraction in suspension, C_s, and a solids fraction in contact, C_c. A suitable method must be used to predict the amounts of suspended solids C_s or of solids in contact C_c. Assuming a two-layer pattern according to Fig. 4.14, the C_c determines the concentration of solids in contact within the lower layer, C_{2c}, by recalculating of C_c from the cross-sectional area of the entire pipeline, A, to the cross-sectional area of the lower layer, A_2, using $C_{2c} = C_c.A/A_2$.

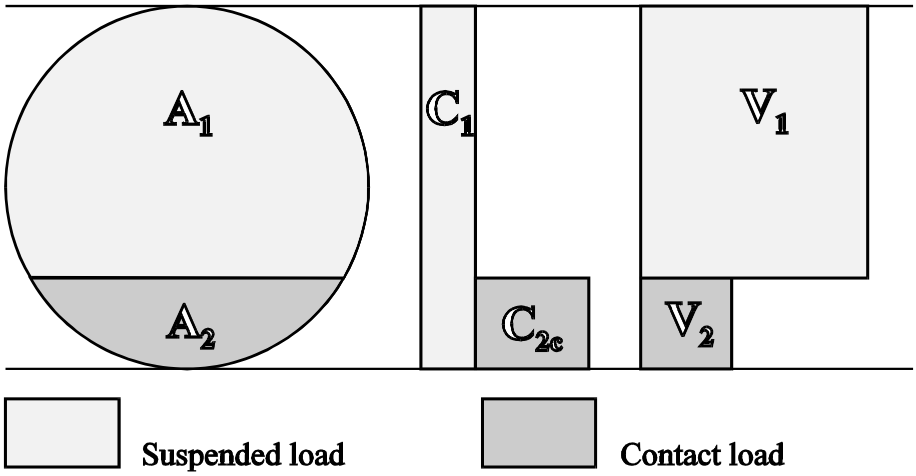

Figure 4.14. Two-layer pattern for the model for partially-stratified flow.

An additional coefficient has to be introduced to the two-layer model to predict the partially-stratified flow. This coefficient is called the stratification ratio C_c/C_{vi}. Although subject to further investigation the stratification-ratio correlation for overall mixture flow conditions can be written as

$$\frac{C_c}{C_{vi}} = \exp\left(-X\frac{V_m}{v_t}\right) \quad (4.45).$$

A value of the empirical coefficient X was found equal to 0.018 according to Gillies et al. (1990) data and 0.024 from tests in our laboratory.

- the buoyancy effect on the bed submerged weight

Modification of the two-layer flow pattern for the partially-stratified flow (see Fig. 4.14) required modification of the method used to determine the normal intergranular force against the pipeline wall F_N. The buoyancy effect associated with the presence of suspended coarse particles (d > 0.074 mm) and fine particles (d < 0.074 mm) in the lower layer is included to the equation for the normal solids stress at the pipeline wall, σ_s. In the lower layer the suspended coarse particles, the fine particles smaller than 0.074 mm and the liquid form a mixture of density ρ_{2f} determined as

$\rho_{2f}(1\text{-}C_{2c}) = \rho_s C_1 + \rho_{fines}(1\text{-}C_{2c}\text{-}C_1)$, so that

$$\rho_{2f} = \frac{\rho_{fines}(1 - C_2) + \rho_s C_1}{1 - C_2 + C_1} \quad (4.46).$$

in which ρ_{fines} is density of a mixture composed of the liquid and fine particles smaller than 0.074 mm and $C_2 = C_1 + C_{2c}$. The normal force F_N is integrated from Eq. 4.46 as

$$F_N = g(\rho_s - \rho_f)C_{vb}\frac{D^2}{2}(\sin\beta - \beta\cos\beta) \quad (4.47).$$

4.3 SUMMARY: GENERAL TRENDS FOR FRICTIONAL HEAD LOSS AND DEPOSITION-LIMIT VELOCITY UNDER VARIOUS FLOW CONDITIONS

4.3.1 Pressure drop due to friction

The total frictional pressure drop in mixture flow is composed of the frictional pressure drop in a carrying liquid and an additional frictional pressure drop, called the solids effect, due to a presence of solid particles in a mixture. The solids effect extends the frictional loss of a carrier alone if mixture flows at velocity round the deposition-limit value. However, at higher velocities the water friction creates a major part of a total frictional loss in a mixture flow. This is particularly valid for low concentrated flows.

Pressure drop in flow of carrying water

Frictional loss in flow of carrying water (I_f) is particularly *sensitive to flow velocity and pipeline diameter*. Higher throughputs (flow rates) in a pipeline of a certain diameter are always paid in higher pressure losses due to friction. Less energy is dissipated due to friction in flow through a larger pipeline than through a smaller pipeline at the same velocity. Additionally, the *roughness of a pipeline wall* affects the losses. The rougher wall the higher frictional losses.

Solids effect in a mixture flow

The solids effect (I_m-I_f) on the total frictional losses is predominantly due to mechanical friction between transported particles and a pipeline wall. Thus a thickness of a granular bed is a major indicator of the solids effect for flow under certain conditions.

The solids effect is *sensitive to flow velocity*, particularly for flows with a considerable change in a degree of flow stratification within a operational range of mixture velocities. A granular bed diminishes in flow with an increasing mixture velocity and so diminishes the solids effect.

A coarse material tends to form a thicker bed than a fine material in a flow of certain velocity. Thus the solids effect (I_m-I_f) increases with the *size of particles* in mixture flow. For very coarse particles, however, all solids are transported as a bed load and a settling tendency of particles is not of importance for flow friction. The solids effect of fully-stratified flow is virtually independent of a particle size. The solid effect is further sensitive to the *particle size distribution*. A broad graded material might cause lower friction losses than a narrow graded material of the same mass-median size (d_{50}) and the same concentration in mixture flow. This is so if a broad graded material contains a considerable portion of fine particles that can be easily suspended in carrying liquid. A granular bed is thinner when compared to flow with a narrow graded material and thus the solids effect is smaller.

The solids effect (I_m-I_f) grows with the *concentration of solids* in a pipeline. The relationship can be estimated as linear, at least according to various experiments for flows of C_{vd} between 0.05 and 0.25 approximately.

Scaling of mixture flow parameters obtained in one pipeline to pipelines of different sizes (diameters) is a source of uncertainty. A lack of data of appropriate range and quality from large pipelines prevents to evaluate the pipeline-size effect on flow mechanisms. There is no agreement among the predictive models with regard to the influence of a *pipeline size* on the solids effect (I_m-I_f). The Wilson model and the Führböter model do not predict any difference in the solids effect if flow of certain parameters is scaled to pipelines of different diameters. The Durand model predicts an increase in the solids effect with the pipeline diameter. Jufin and Lopatin predict the opposite trend - the solids effect should be smaller in a larger pipeline.

4.3.2 Deposition-limit velocity

A physical description of a force balance between two layers gives an appropriate basis for an explanation of the trends in the velocity of initial sliding of a granular bed in a stratified mixture flow.

Deposition-limit velocity tends to increase with a *particle size* for flow of fine and fine to medium sands. This can be explained by a fact that larger particles form a thicker bed and higher velocity is required to put the thicker bed to motion. For coarser materials (coarser than approximately 0.4-0.5 mm), however, the critical velocity does not grow further. This is because a bed at its initial sliding has approximately the same thickness for different particle sizes in a flow. Coarser particles are not suspended and tend to increase a thickness of a bed but at the same time a top of a bed is sheared off so that the effective thickness of a stationary bed does not change. For very coarse particles the deposition-limit velocity even drops for still coarser particles since the bed composed of these particles is subjected to increasing driving force from the flow above the bed. This force acts at the top of a bed as a viscous shear stress related to the roughness of the bed surface. The roughness is given by a size of particles occupying a bed surface. The coarser particle, the rougher bed surface and the higher shear stress acting to the bed.

The value of deposition-limit velocity is higher in a larger pipe than in a smaller one for flow under identical other conditions. In a smaller pipe a higher pressure drop is built up over a pipe section than in a larger pipe (see the relationship between the hydraulic gradient and the *pipeline diameter* in the Darcy-Weisbach equation). The pressure drop is a source of a major driving force acting to put a granular bed to motion in a pipe.
For *concentration*s usually handled in dredging pipelines ($C_{vd} > 0.10$) the value of the deposition-limit velocity tends to drop in more concentrated flows. This is basically because more concentrated suspension flowing above the bed exerts higher driving force to the bed than the flow of low concentrated suspension or particle-free carrying liquid. This has been detected in both the laboratory pipe and a field dredging pipeline (Matousek, 1997). In a dredging practice the effect of V_{dl} reduction in concentrated mixtures is not taken into account for a flow prediction. The concentration of solids is difficult to control during a dredging operation and it may vary within a rather wide range.

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CASE STUDY 4

Mixture flow in a horizontal pipeline

An aqueous mixture of fine sand or medium gravel (see previous Case studies) is transported from a dredge to a deposit site through a dredging pipeline that is 1.5 kilometer long and has an internal diameter of 900 millimeter.

Propose a suitable transport velocity for mixture in a pipeline and determine the energy lost due to friction, the specific energy consumption and the production for mixture transport at the chosen velocity. The absolute roughness of a pipeline wall is 20 microns.

Remark: Consider a horizontal pipeline and no boosters. Consider $1.1V_{dl}$ (deposition-limit velocity) a suitable transport velocity of mixture in the pipeline. For a simplification consider a narrow graded soil characterised by the median diameter only.

Inputs:

$\mathbf{d_{50} = 0.120\ mm}$ of $\mathbf{d_{50} = 6.0\ mm}$
$\boldsymbol{\rho_s = 2650\ kg/m^3}$
$\boldsymbol{\rho_f = 1000\ kg/m^3}$
$\boldsymbol{\nu_f = 0.000001\ m^2/s}$
$\mathbf{C_{vd} = 0.27}$
$\mathbf{L = 1500\ m}$
$\mathbf{D = 900\ mm}$
$\mathbf{k = 0.00002\ m}$

Solution:

a. The deposition-limit velocity

Fine sand (d = 0.120 mm)

Durand: $V_{dl} = F_L\sqrt{2g(S_s - 1)D}$
$F_L = 1.05$ (see a nomograph in Fig. 4.3)
$V_{dl} = 1.05\sqrt{2x9.81(2.65-1)0.9} = 5.67$ m/s.

Wilson: $V_{sm} = 1.72$ m/s (zie Eq. 4.21 for $\mu_s = 0.4$)
$C_{rm} = 0.66$ (see a nomograph in Fig. 4.9)
$C_r = C_{vd}/C_{vb} = 0.27/0.60 = 0.45$
$V_{dl} = 0.86^{*}\ V_{sm} = 1.48$ m/s.

MTI: $V_{dl} = 3.23$ m/s (see a nomograph in Fig. 4.6 or Eq. 4.19).

Medium gravel (d = 6.0 mm)

Durand: $V_{dl} = F_L\sqrt{2g(S_s - 1)D}$

F_L = 1.35 (see a nomograph in Fig. 4.3)

$V_{dl} = 1.35\sqrt{2x9.81(2.65-1)0.9}$ = 7.29 m/s.

Wilson: V_{sm} = 5.21 m/s (zie a nomograph in Fig. 4.8 or Eq. 4.21 for μ_s = 0.4)

($V_{sm,max}$ = 5.83 m/s according to Eq. 4.22)

C_{rm} = 0.05 (see a nomograph in Fig. 4.9)

$C_r = C_{vd}/C_{vb}$ = 0.27/0.60 = 0.45

V_{dl} = 0.33* V_{sm} = 1.72 m/s.

MTI: V_{dl} = 7.03 m/s (see a nomograph in Fig. 4.6 or Eq. 4.19).

The MTI results are taken as predicted values of V_{dl}.

The suitable transport velocity for sand-water mixture: ***V_m*** = 1.1V_{dl} ***= 3.60 m/s***.
The suitable transport velocity for gravel-water mixture: ***V_m*** = 1.1V_{dl} ***= 7.70 m/s***.

b. Energy loss due to friction

Fine sand (d = 0.120 mm)

Water flow:

Re = 3.6*0.9/0.000001 = 3.24 x 10^6

k/D = 0.00002/0.9 = 2.2 x 10^{-5} (D/k = 45000)

λ_f = 0.0107 (see Moody diagram, Fig. 1.6)

Friction head loss from the Darcy-Weisbach equation (Eq. 1.20)

$$I_f = \frac{\lambda_f}{D}\frac{V_m^2}{2g} = \frac{0.0107}{0.900}\frac{3.6^2}{19.62} = 0.00785 \quad [-].$$

Mixture flow:

Durand model: (Eq. 4.4, for the v_t value see Case study I)

$$\frac{I_m - I_f}{I_f C_{vd}} = 180\left(\frac{V_m^2}{gD}\frac{\sqrt{gd}}{v_t}\right)^{-1.5} = 180\left(\frac{3.6^2}{9.81x0.9}\frac{\sqrt{9.81x0.00012}}{0.00947}\right)^{-1.5} = 14.68$$

I_m = 0.00785 + 14.68 x 0.27 x 0.00785 = 0.0390 [-].

Wilson model for heterogeneous flow: (Eqs. 4.16 and Eq. 4.17)

$$V_{50} \approx 3.93(d_{50})^{0.35}\left(\frac{S_s - 1}{1.65}\right)^{0.45} = 3.93(0.12)^{0.35}1 = 1.87 \text{ m/s.}$$

$$\frac{I_m - I_f}{C_{vd}(S_s - 1)} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-M} = 0.22\left(\frac{3.60}{1.87}\right)^{-1.7} = 0.07225$$

I_m = 0.00785 + 0.07225 x 0.27 x 1.65 = ***0.0400 [-]***.

Energy head, H [meter water column, mwc], lost over a pipeline length L = 1500 metre:

$\boldsymbol{H} = I_m \text{ x } L = 0.0400 \text{ x } 1500 = \boldsymbol{60.0\ mwc}$.

Medium gravel (d = 6.0 mm)

Water flow:

Re = 7.7*0.9/0.000001 = 6.93 x 10^6

k/D = 0.00002/0.9 = 2.2 x 10^{-5} (D/k = 45000)

λ_f = 0.010 (see Moody diagram, Fig. 1.6)

$$I_f = \frac{0.010}{0.900}\frac{7.7^2}{19.62} = 0.03358 \ [-] \ \text{(see Eq. 1.20, i.e. Darcy-Weisbach eq.)}$$

Mixture flow:

Durand model: (Eq. 4.4, for the v_t value see Case study I)

$$\frac{I_m - I_f}{I_f C_{vd}} = 180\left(\frac{V_m^2}{gD}\frac{\sqrt{gd}}{v_t}\right)^{-1.5} = 180\left(\frac{7.7^2}{9.81\text{x}0.9}\frac{\sqrt{9.81\text{x}0.006}}{0.27374}\right)^{-1.5} = 12.40$$

I_m = 0.03358 + 12.40 x 0.27 x 0.03358 = 0.1460 [-].

Wilson model for fully-stratified flow: (Eq. 4.20)

V_{sm} = 5.21 m/s (zie a nomograph in Fig. 4.8 or Eq. 4.21 for μ_s = 0.4).

$$\frac{I_m - I_f}{C_{vd}(S_s - 1)} = \left(\frac{V_m}{0.55V_{sm}}\right)^{-0.25} = \left(\frac{7.70}{0.55\text{x}5.21}\right)^{-0.25} = 0.7810$$

I_m = 0.03358 + 0.7810 x 0.27 x 1.65 = 0.3815 [-].

The ratio d/D = 6/900 = 0.0067 < 0.018, i.e. the flow is not considered fully stratified.

Wilson model for heterogeneous flow: (Eq. 4.16 and Eq. 4.17).

$$V_{50} \approx 3.93(d_{50})^{0.35}\left(\frac{S_s - 1}{1.65}\right)^{0.45} = 3.93(6.0)^{0.35}1 = 7.36 \text{ m/s}$$

$$\frac{I_m - I_f}{C_{vd}(S_s - 1)} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-M} = 0.22\left(\frac{7.70}{7.36}\right)^{-1.7} = 0.20374$$

$\boldsymbol{I_m}$ = 0.03358 + 0.20374 x 0.27 x 1.65 = ***0.1243 [-]***.

Energy head, H [meter water column, mwc], lost over a pipeline length L = 1500 metre:

$\boldsymbol{H} = I_m \text{ x } L = 0.1243 \text{ x } 1500 = \boldsymbol{186.5\ mwc}$.

c. Specific energy consumption (Eq. 3.6)

Fine sand (d = 0.120 mm)

$$SEC = 2.7\frac{I_m}{S_s.C_{vd}} = 2.7\frac{0.0400}{2.65x0.27} = \boldsymbol{0.151\ [kWh/(tonne.km)]}.$$

Medium gravel (d = 6.0 mm)

$$SEC = 2.7\frac{I_m}{S_s.C_{vd}} = 2.7\frac{0.1243}{2.65x0.27} = \boldsymbol{0.469\ [kWh/(tonne.km)]}\ .$$

d. Production

Fine sand (d = 0.120 mm)

Production of solids: (Eq. 3.3)

$$Q_s = \frac{\pi}{4}D^2V_mC_{vd}3600 = \frac{\pi}{4}0.9^2 3.6x0.27x3600 = 2226.1\ [m^3/hour].$$

Production of in situ soil: (for porosity n = 0.4) (Eq. 3.4)

$$Q_{si} = \frac{\pi}{4}D^2V_mC_{vdsi}3600 = \frac{Q_s}{1-n} = \boldsymbol{3710.2\ [m^3/hour]}.$$

Medium gravel (d = 6.0 mm)

Production of solids: (Eq. 3.3)

$$Q_s = \frac{\pi}{4}D^2V_mC_{vd}3600 = \frac{\pi}{4}0.9^2 7.7x0.27x3600 = 4761.4\ [m^3/hour].$$

Production of in situ soil: (for porosity n = 0.4) (Eq. 3.4)

$$Q_{si} = \frac{\pi}{4}D^2V_mC_{vdsi}3600 = \frac{Q_s}{1-n} = \boldsymbol{7935.6\ [m^3/hour]}.$$

Summary of the results:

Fine sand (d = 0.12 mm):	
suitable transport velocity:	***V_m = 3.60 m/s***
frictional head loss:	***I_m = 0.0400 [-]***
head lost over the pipeline 1500 m long:	***H = 60.0 mwc***
specific energy consumption:	***SEC = 0.151 kWh/(tonne.km)***
production of in situ soil:	***Q_{si} = 3710.2 m^3/hour***
Medium gravel (d = 6.00 mm):	
suitable transport velocity:	***V_m = 7.70 m/s***
frictional head loss:	***I_m = 0.1243 [-]***
head lost over the pipeline 1500 m long:	***H = 186.5 mwc***
specific energy consumption:	***SEC = 0.469 kWh/(tonne.km)***
production of in situ soil:	***Q_{si} = 7935.6 m^3/hour***