Power-law fluid in a slit

Since the geometry and momentum balance are the same as for a Newtonian fluid, we can jump directly to "Eq II" of the ~~homework~~ solution for the Newtonian fluid in a slit

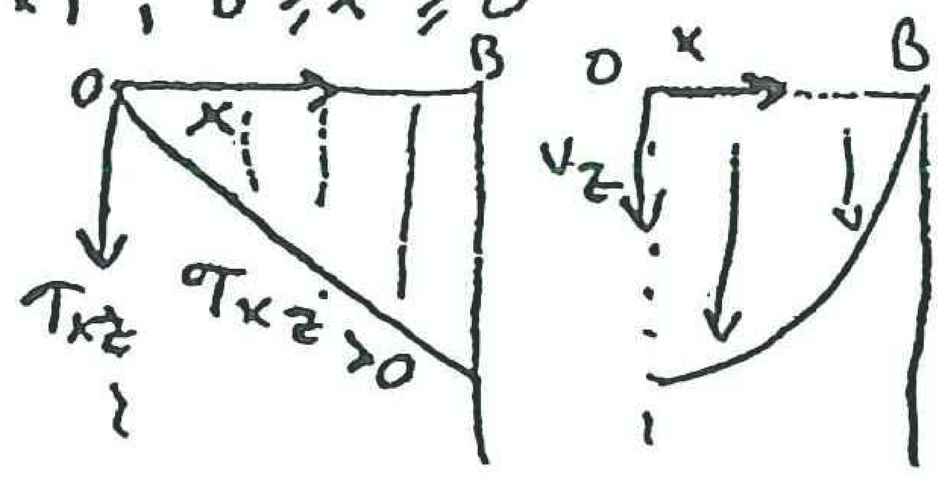
$$\tau_{xz} = [(\mathcal{P}_0 - \mathcal{P}_L)/L]\, x + C_1$$

Here we'll use BC (1) $v_z = 0$ at $x = B$ and (2) $\tau_{xz} = 0$ at $x = 0$, and we'll solve for $0 \le x \le B$ only. BC2 $\rightarrow$

$$\tau_{xz} = 0 = [(\mathcal{P}_0 - \mathcal{P}_L)/L](0) + C_1 \rightarrow C_1 = 0$$

$$\therefore \quad \tau_{xz} = [(\mathcal{P}_0 - \mathcal{P}_L)/L]\, x = -m\,|dv_z/dx|^{n-1}(dv_z/dx), \quad B \ge x \ge 0$$

As with flow through a tube, a sketch helps:



$\tau_{xz} > 0$, and we expect $dv_z/dx < 0$, since $v_z = \max > 0$ at $x = 0$ and $v_z = 0$ at $x = B$. $\therefore$

$dv_z/dx = -|dv_z/dx|$ and

$$[(\mathcal{P}_0 - \mathcal{P}_L)/L]\, x = m\,|dv_z/dx|^n$$

$$\rightarrow |dv_z/dx|^n = [(\mathcal{P}_0 - \mathcal{P}_L)/(mL)]\, x$$

$$|dv_z/dx| = [(\mathcal{P}_0 - \mathcal{P}_L)/(mL)]^{1/n}\, x^{1/n}$$

Finally, since $|dv_z/dx| = -dv_z/dx$

$$dv_z/dx = -[(\mathcal{P}_0 - \mathcal{P}_L)/(mL)]^{1/n}\, x^{1/n}$$

Integrate: $v_z = -[(\mathcal{P}_0 - \mathcal{P}_L)/(mL)]^{1/n}\, x^{[1+(1/n)]}/(1+(1/n)) + C_2$.

BC: $v_z = 0$ at $x = B \rightarrow \quad C_2 = [(\mathcal{P}_0 - \mathcal{P}_L)/(mL)]^{1/n}\, B^{[1+(1/n)]}/[1+(1/n)]$

$$\rightarrow \quad v_z = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{mL}\right)^{1/n} \frac{B^{[1+(1/n)]}}{1+(1/n)} \left[1 - \left(\frac{x}{B}\right)^{[1+(1/n)]}\right]$$