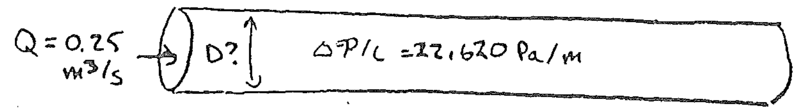
Friction-factor problems

Examples of trial-and-error method

To put out the oilfield fires in Kuwait, firefighters used hoses that could deliver up to 4000 gal/min (0.25 m^3/s) of seawater. The hoses must extend long distances, so suppose $\Delta P/L$ is limited to 1 psi/ft (22,620 Pa/m). The density and viscosity of water are roughly 1,000 kg/m^3 and 0.001 Pa s, respectively. What hose diameter D is required? Assume the roughness factor k/D is 0.004. Note the figure given at the end of the exam.
(25 points)



In this problem, we don't know Re since we don't know D. There are at least 2 ways to solve this problem.

1) Trial + error. $f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\Delta P}{\frac{1}{2}\rho\langle v\rangle^2}\right)$. Since Q is fixed, not $\langle v\rangle$, $\langle v\rangle = Q/[(\pi/4)D^2]$

$$\rightarrow f = \frac{1}{4}\frac{D}{L}\frac{\Delta P\left(\frac{\pi}{4}D^2\right)^2}{\frac{1}{2}\rho Q^2} = \frac{1}{4}\frac{\Delta P}{L}\frac{(\pi/4)^2 D^5}{\frac{1}{2}\rho Q^2} = \frac{1}{4}(22{,}620)\frac{(\pi/4)^2 D^5}{(\frac{1}{2})(1000)(0.25)^2} = 111.6\, D^5 \quad \boxed{\mathrm{I}}$$

$$Re = \frac{D\langle v\rangle\rho}{\mu} = \frac{DQ\rho}{\mu\frac{\pi}{4}D^2} = \frac{Q\rho 4}{\mu\pi D} = \frac{(0.25)(1000)(4)}{(0.001)\pi D} = 3.18\cdot 10^5/D \quad \boxed{\mathrm{II}}$$

Guess $D = 0.1$ m $\rightarrow Re = 3.2\cdot 10^6$; from chart, $f = 0.0072 = 111.6\, D^5 \rightarrow D = 0.145$ m

Guess $D = 0.145$ m $\rightarrow Re = 2.19\cdot 10^6$; from chart, $f = 0.0072$ again. $\rightarrow D = 0.145$ m

done. $D = 0.145$ m (about $5\frac{3}{4}$ in.)

Stonewall has been assigned to design a large pipeline to transport 0.2 m^3/s oil products with a pressure gradient of 10 psi/mile (42.8 Pa/m). The pipeline is horizontal. The oil products have density 850 kg/m^3 and viscosity 10 cp (0.01 Pa s). Stonewall figures that even without knowing the pipe diameter, a reasonable estimate of roughness is k/D = 0.01. What pipe diameter should he recommend?

Without knowing Re, one can't proceed directly. With D unknown, we don't know v or D. $v = Q/(\pi D^2/4) = 0.2/(\pi D^2/4) = 0.255/D^2$.

$Re = Dv\rho/\mu = D[0.255/D^2]\,850/(0.01) = 21645/D$

$f = \frac{1}{4}\frac{D}{L}\frac{\Delta P}{\frac{1}{2}\rho v^2} = (1/4)\,D\,(42.8)/[\frac{1}{2}(850)(0.255/D^2)^2] = 0.387\, D^5$ (eq. 6.1-4)

From here, one can proceed in several ways.

1) Trial + error. Guess $D = 1$ m, $Re = 21{,}645$. From $f(Re)$, $f \approx 0.0105 = 0.387\, D^5$

$\rightarrow D = 0.486 \rightarrow Re = 44{,}500 \rightarrow f(Re) \approx 0.0099 = 0.387\, D^5 \rightarrow D = 0.480$

$\rightarrow Re = 45100 \rightarrow f \approx 0.0099$ again. Done. $D = 0.48$ m.

An example of trial and error in falling-sphere problem

Safety experts warn gun owners that firing into the air can be dangerous, because the bullet can fall to earth with enough velocity to hurt someone. Suppose a bullet were a sphere of lead (density 11,300 kg/m^3) weighing 1/2 ounce (0.0156 kg). What would be the terminal velocity of such a sphere falling from a great height through air (with viscosity $\mu = 1.8 \times 10^{-5}$ Pa s and density $\rho = 1.3$ kg/m^3)?
(20 points)

We don't know v_t or Re. ∴ some sort of special or trial-and-error solution is needed. ∃ several possible ways to proceed.

i) trial + error: $Re = DV\rho/\mu$; What is D? $\frac{4}{3}\pi R^3 \rho = 0.0156$.

$\frac{4}{3}\pi R^3 (11,300) = 0.0156 \rightarrow R = 0.00691$ m; $D = 0.0138$ m

$$Re = \frac{(0.0138) v (1.3)}{(1.8 \cdot 10^{-5})} = 997\ V$$

$$f = \frac{4}{3} gD \frac{1}{V^2}\left(\frac{\rho_s - \rho}{\rho}\right) = \frac{4}{3}(9.9)(0.0138)\frac{1}{V^2}\left(\frac{11,300-1.3}{1.3}\right) = 1567/v^2$$

guess $v = 1$ m/s. $Re = 997$. $f \approx 0.45 = 1567/v^2 \rightarrow V = 59.0$ m/s
↑ from Fig 6.3-1

guess $v = 59.0$. $Re = 5.9 \cdot 10^4$. from Fig 6.3-1, $f \doteq 0.49$ $v = \sqrt{\frac{1567}{0.49}} = 56.6$

guess $v = 56.5$ $Re = 5.64 \cdot 10^4$. " $f = 0.49$ No change from last iteration; ∴ done. $v = 56$ m/s

Two engineers are pumping a Newtonian liquid through a sandpack of permeability 10 Darcies. For this sandpack, porosity is 30%. The liquid viscosity is 0.001 Pa-s and its density is 1000 kg/m^3. The two engineers are arguing about the onset of non-Darcy flow.

a) Bubba believes in the bundle-of-uniform-tubes model. At what pressure gradient ($\Delta p/L$) does he predict that turbulent flow begins?
b) Rocky uses the correlations for packed beds. At what pressure gradient ($\Delta p/L$) does he predict that the laminar-flow model begins to break down?

a) Bubba thinks turbulence begins when $Re \equiv \frac{Dv\rho}{\mu} \geq 2100$, with D & v the values for a "bundle-of-tubes." For this model, $D = 2R = 2\sqrt{8K/\phi}$

$= 2(8(10\cdot10^{-12})/0.3)^{1/2} = 32.7\ \mu m$ or $3.27\cdot10^{-5}$ m.

If $Re = 2100 = \frac{Dv\rho}{\mu} = (3.27\cdot10^{-5})\,v\,(1000)/(0.001)$, then $v = 64.3$ m/s.

From BSL eq. 2.3-18, $\langle v \rangle = \frac{\Delta P R^2}{8\mu L} \rightarrow 64.3 = \frac{\Delta P}{L}\left(\frac{3.27\cdot10^{-5}}{2}\right)^2 (8(0.001))^{-1}$

$\rightarrow \frac{\Delta P}{L} = 1.93\cdot10^{9}$ Pa/m (about 85,000 psi/ft!)

(Obviously he's not seriously worried about turbulence.)

b) Rocky expects deviation from Darcy flow when the Blake-Kozeny equation for packed tubes begins to break down. According to BSL Fig. 6.4-1, this occurs for $Re \hat{=} 10 = \frac{D_p G_o}{\mu}\frac{1}{1-\phi}$. (If he were conservative, he might take a somewhat lower value, if less conservative, a higher value.) For this correlation, $G_o = \rho v_o$ with $v_o \equiv$ Darcy velocity. For Blake-Kozeny flow, (Eq. 6.4-9)

$$v_o = \frac{\Delta P}{L}\frac{D_p^2}{150\mu}\frac{\phi^3}{(1-\phi)^2}. \quad \boxed{IV} \quad \text{Therefore}$$

$$\frac{D_p\rho v_o}{(1-\phi)\mu} = 10 = \frac{D_p\rho}{\mu}\left[\frac{\Delta P}{L}\frac{D_p^2}{150\mu}\frac{\phi^3}{(1-\phi)^2}\right]\frac{1}{1-\phi} = \frac{D_p^3\rho}{\mu^2(150)}\frac{\phi^3}{(1-\phi)^3}$$

$$\frac{\Delta P}{L} = 10\,\frac{(0.001)^2(150)(0.7)^3}{D_p^3(1000)(0.3)^3} = 1.90\cdot10^{-5}\left(\frac{1}{D_p}\right)^3$$

What is D_p? From Eq. $\boxed{IV}$, $K = \frac{D_p^2}{150}\frac{\phi^3}{(1-\phi)^2} = 10\cdot10^{-12} = \frac{D_p^2}{150}\frac{(0.3)^3}{(0.7)^2} \Rightarrow D_p = 1.65\cdot10^{-4}$ m

$\frac{\Delta P}{L} = 1.90\cdot10^{-5}/(1.65\cdot10^{-4})^3 = 4.23\cdot10^{6}$ Pa/m; about 190 psi/ft.