XII. Multivariate and Unsteady Conduction

A. governing partial differential equation and assumptions

- see example 9 of XI.C

Assumptions:

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same mathematical equation is called "diffusivity equation" in PGE 323, because

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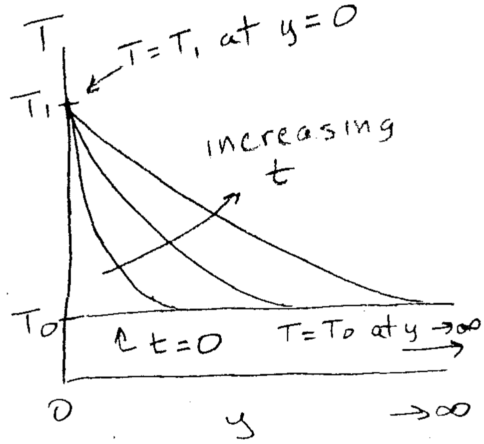
all obey the same partial differential equation

B. Tabulated 1D solutions

1. Semi-infinite slab (BSL Sect. 4.1, Ex. 12 [redacted].1-1)

a. initial and boundary conditions



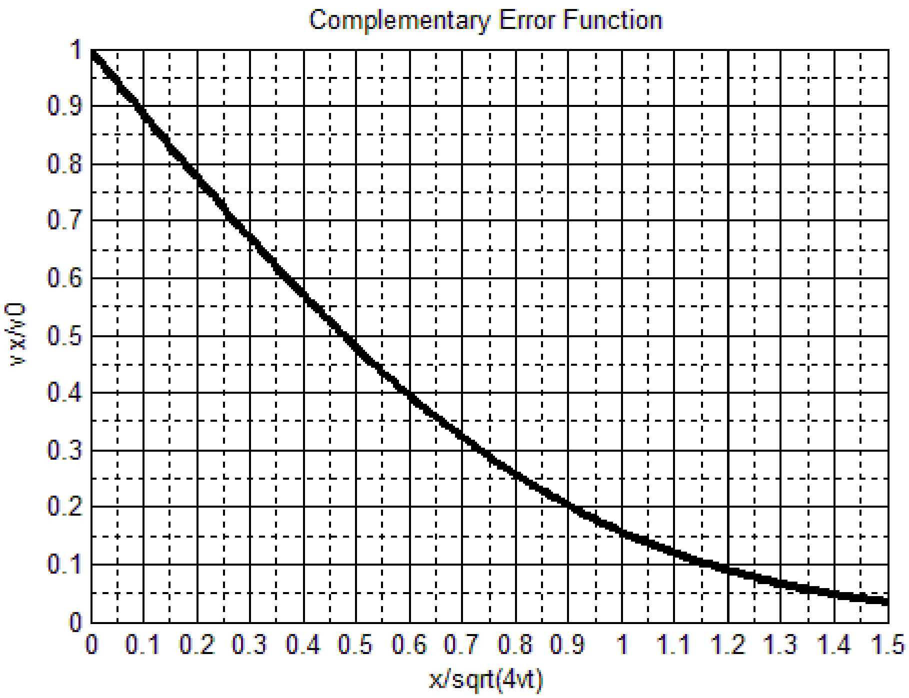
$T = T_0$ at $t = 0$, for $y \geq 0$

$T = T_1$ at $y = 0$ for $t > 0$

$T = T_0$ at $y \to \infty$ for $t > 0$

b. solution given by Eq. 12 [redacted].1-8 and Fig. 4.1-2

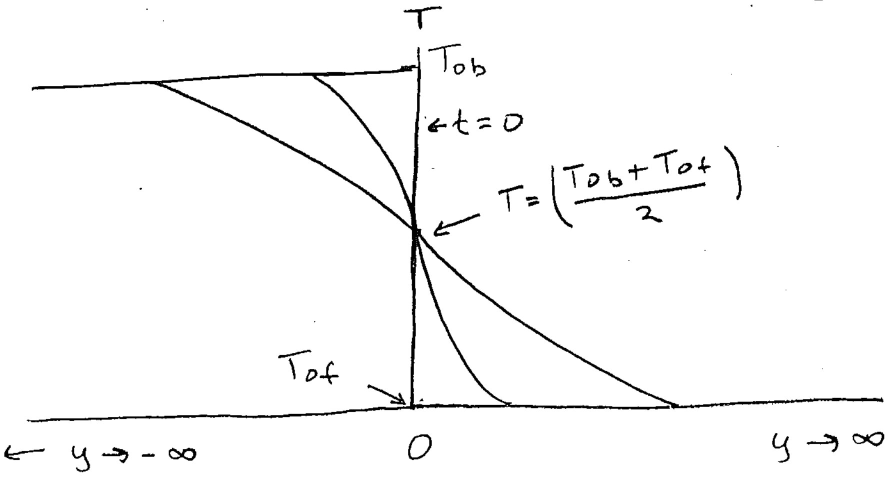
c. heat flux at y = 0



Velocity distribution in dimensionless form for flow in the neighborhood of a wall suddenly set in motion. (The function is a complementary error f unction.)

2. Two semi-infinite slabs brought together

- slabs have identical properties, different initial temperatures



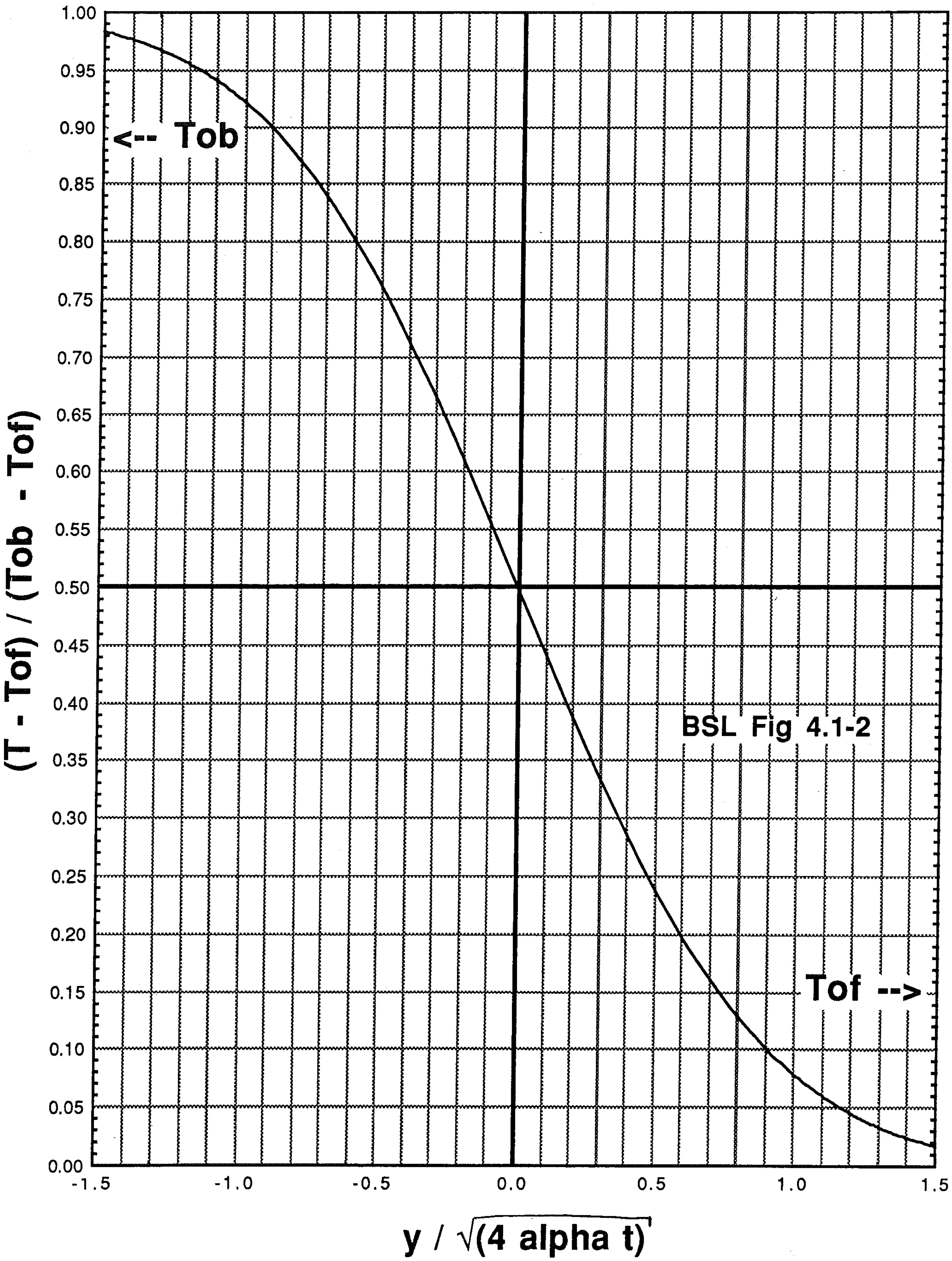
a. boundary and initial conditions

$T = T_{of}$	at $t = 0$,	for $y > 0$
$T = T_{of}$	at $y \rightarrow \infty$,	for $t > 0$
$T = T_{ob}$	at $t = 0$,	for $y < 0$
$T = T_{ob}$	at $y \rightarrow -\infty$,	for $t > 0$
$T(y \rightarrow 0^+) = T(y \rightarrow 0^-)$		for $t > 0$
$q_y(y \rightarrow 0^+) = q_y(y \rightarrow 0^-)$		for $t > 0$

b. solution

(note $\mathrm{erf}(-x) = -\mathrm{erf}(x)$; thus equation applies to $y > 0$ and $y < 0$)
solution is plotted on next page

XII.B.2.b Unsteady Conduction Between Two Semi-Infinite Slabs in Contact



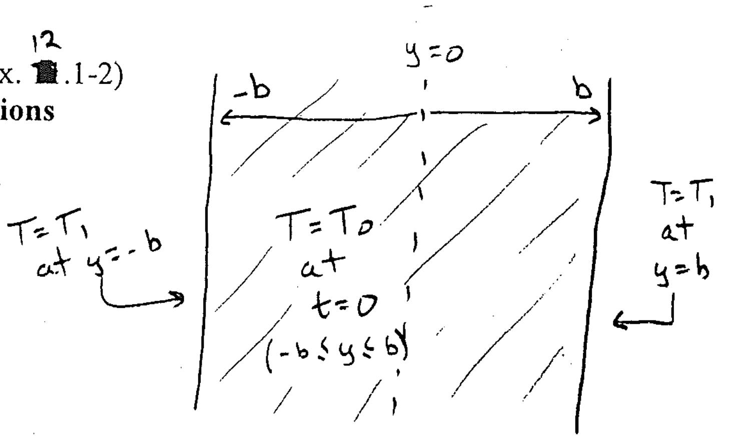
3. Slab of finite thickness (BSL Ex. 12.1-2)

a. boundary and initial conditions

$T = T_0$ at $t = 0$, for $-b \leq y \leq b$

$T = T_1$ at $y = b$ for $t > 0$

$T = T_1$ at $y = -b$ for $t > 0$



b. solution: BSL Figure. 12.1-1

(note curves for small $\alpha t/B^2$ do *not* go below horizontal axis)

c. an aside: how to estimate heat flux at surface from this chart?

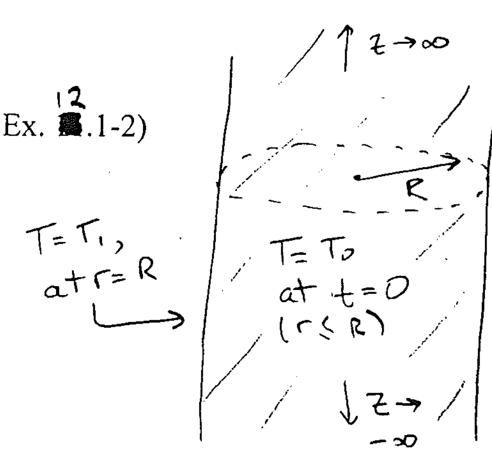
4. Conduction within cylinder ($0 \leq r \leq R$) (BSL Ex. 12.1-2)

a. boundary and initial conditions

$T = T_0$ at $t = 0$, for $0 \leq r \leq R$

$T = T_1$ at $r = R$ for $t > 0$

T finiteat $r = 0$ for $t > 0$

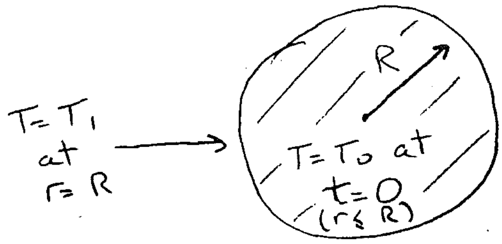


b. solution: BSL Figure. 12.1-2

(note curves for small $\alpha t/R^2$ do *not* go below horizontal axis)

5. Conduction within sphere ($0 \le r \le R$) (BSL Ex. 12.1-2)

a. boundary and initial conditions



$T = T_o$ at $t = 0$, for $0 \le r \le R$

$T = T_1$ at $r = R$ for $t > 0$

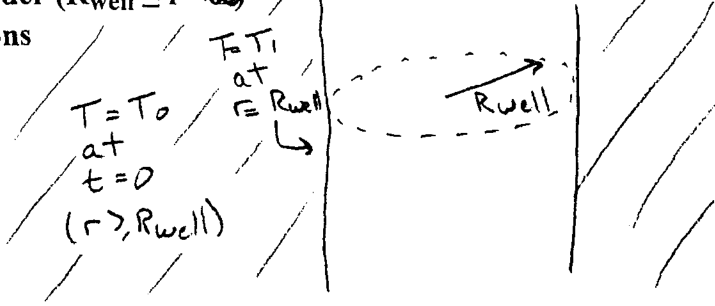
T finiteat $r = 0$ for $t > 0$

b. solution: BSL Figure. 12.1-3

(note curves for small $\alpha t/R^2$ do *not* go below horizontal axis)

6. Conduction *outwards* from cylinder ($R_{well} \le r < \infty$)

a. boundary and initial conditions



$T = T_o$ at $t = 0$, for $r \ge R_{well}$

$T = T_1$ at $r = R_{well}$ for $t > 0$

$T = T_o$ at $r \to \infty$ for $t > 0$

b. solution: Carslaw and Jaegar, *Conduction of Heat in Solids*, Figs. 41 and 42 (see attached)

7. Comments on 1D solutions

The trend shown here, in which dimensionless temperature $[(T-T_o)/(T_1-T_o)]$ is a function of dimensionless position and dimensionless time, which is proportional to (t / L^2), where L is the characteristic length of the solid, applies to *any* homogeneous solid of any shape, with uniform initial temperature T_o and a sudden change in surface temperature to T_1.

Adapted from Carslaw & Jaeger, Conduction of Heat in Solids, 2nd edition, 1959, Oxford

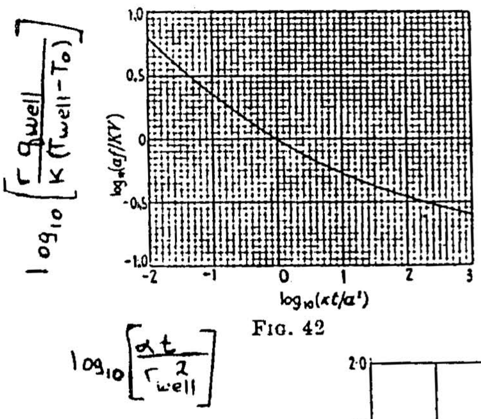


FIG. 42

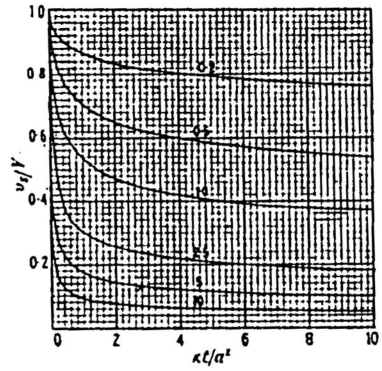


FIG. 43

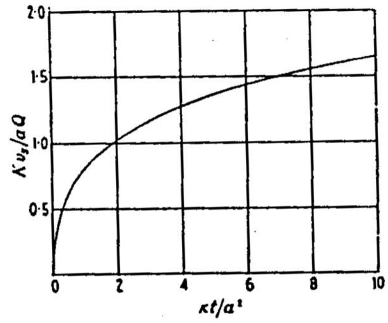


FIG. 44

FIG. 44

FIG. 42. The flux f at the surface of the region bounded internally by a circular cylinder of radius a, with zero initial temperature and constant surface temperature V.
FIG. 43. Surface temperature v_s of the region bounded internally by a circular cylinder of radius a, with constant initial temperature V and radiation at its surface into a medium at zero. The numbers on the curves are values of ah.
FIG. 44. Surface temperature v_s of the region bounded internally by a circular cylinder of radius a, with zero initial temperature and constant flux Q at the surface.

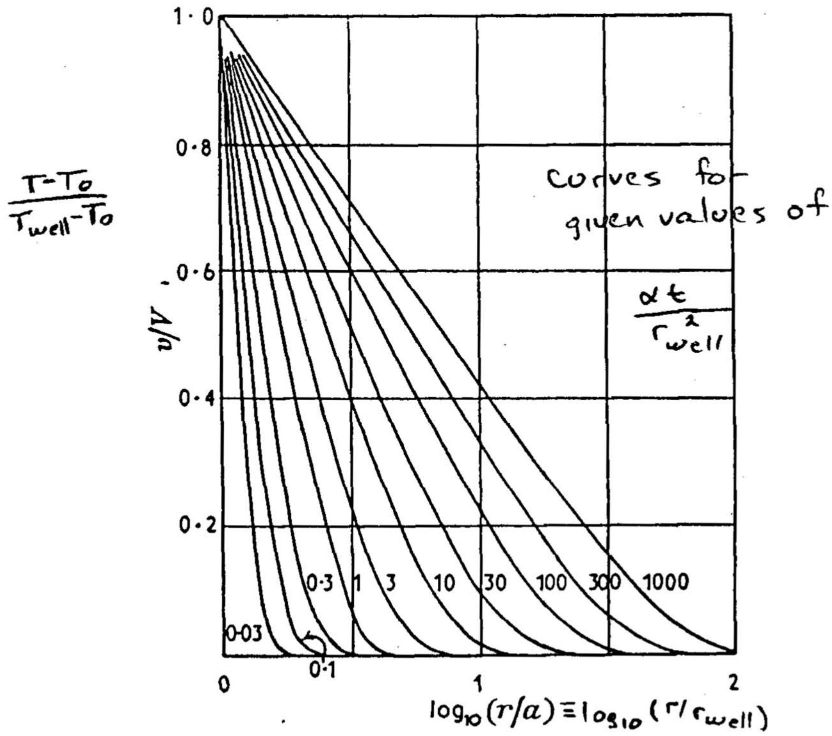


FIG. 41. Temperatures in the region bounded internally by the cylinder $r = a$, with zero initial temperature and constant surface temperature V. The numbers on the curves are the values of $\kappa t/a^2$.