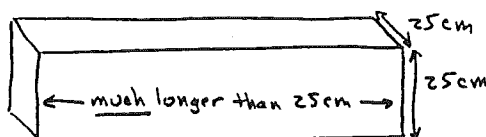


1. a) Dr. Kluge, the famous TV police doctor, has a murder case on his hands. The naked body was found at midnight completely surrounded by ice-cold water (0°C); before the murder, its temperature was, of course, about 37°C . When found, the body's internal temperature was 5°C at its warmest point. Assuming the body has the same thermal properties as water, and can be treated as a long rectangular solid roughly 25 cm deep \times 25 cm wide, what is the shortest possible time the body could have been in the water, consistent with these assumptions? Justify your answer.

$\rho = 1000\text{ kg/m}^3$ Properties of water: $\hat{c}_p = 4190\text{ J/(kg }^{\circ}\text{K)}$ $k = 0.67\text{ W/(m }^{\circ}\text{K)}$



- b) When the murderer confesses, Dr. Kluge is stunned to discover that the murder took place 48 hours before the body was found. Given the fact that the body's internal temperature was 5°C after 48 hr in the cold water, what is the lowest possible value of the heat-transfer coefficient h between the water and the surface of the body? Assume the body shape and thermal properties are those given in part (a).

1. a) There are two modes of heat transport here: convection to the surface + unsteady conduction within. Given the parameters governing unsteady conduction, the shortest time is that for infinitely fast convection heat transfer: i.e., assuming surface $T = 0^{\circ}\text{C}$ starting at $t = 0$.

T at the center = 5°C ; $\frac{T_1 - T}{T_1 - T_0} = \frac{5}{37} = 0.135$. Since one dimension is much greater than the others, we can ignore it. We have then a 2D product-method problem: $\left(\frac{T_1 - T}{T_1 - T_0}\right)_{z=0} = 0.135$; $\left(\frac{T_1 - T}{T_1 - T_0}\right)_{z=0} = 0.368$.
For $y/b = 0$, this corresponds to roughly $\frac{x^2}{b^2} \approx 0.50$. $\alpha = \frac{0.67}{(1000)(4190)}$
 $\rightarrow \alpha = 1.60 \cdot 10^{-7}\text{ m}^2/\text{s}$. $t = (0.50)(0.125)^2 / (1.60 \cdot 10^{-7}) = 4.9 \cdot 10^4\text{ sec}$,
or about 13 hr 30 min.

- b) The body cooled off a lot more slowly than estimated ignoring convection. Therefore convection must have dominated the process. The lowest possible value of h assumes that internal conduction offered no resistance at all; i.e., that T was uniform in the body.

Energy balance on body: $\rho c_p V \frac{dT}{dt} = -h A (T - T_w)$
with $V = \text{volume} = (0.25)^2 L$; $A = (0.25)L$ (ignoring ends);

$T_w = 0^{\circ}\text{C}$.

$\rightarrow \frac{dT - T_w}{(T - T_w)} = \frac{-h L}{(1000)(4190)(0.25)^2 L} dt = -3.82 \cdot 10^{-6} h dt$

$\ln(T - T_w) = (-3.82 \cdot 10^{-6} h) t + C$

$\rightarrow \frac{T - T_w}{(T - T_w)_{t=0}} = \exp[-3.82 \cdot 10^{-6} h t] \rightarrow \frac{5}{37} = \exp[-3.82 \cdot 10^{-6} h (49(60)^2)]$

$\therefore \frac{5}{37} = -0.66 h \rightarrow h = 3.03\text{ W/m}^2\text{K}$