VI. Radiation
   A. Stefan-Boltzmann law
      1. value of σ
      2. emissivity *e*
      3. absorptivity *a*
   B. Simple examples
      1. Two spherical bodies, far apart
      2. Two broad flat plates facing each other
      3. Spherical body surrounded by distant surface

VII. Heat-transfer coefficients
   A. review
   B. general definition of h
   C. flow in tubes
      1. definition of h
      2. energy balance on tube
      3. correlations for h_{ln}
      4. using the correlations for h_{ln}
      5. combination with conduction through cylindrical walls
      6. worked examples

VI. Radiation (text, section 3.7)

A. Stefan-Boltzmann law (eq. 3.163)

1. σ = Stefan-Boltzmann constant =

2. Real objects emit some fraction of the value given above. That fraction is *e*, the emmisivity.

"black body": "white body":

3. Real objects do not absorb all radiation that strikes them. The fraction that is absorbed is the absorptivity, *a*.

"black body": "white body":

B. Simple examples. In general, to determine the heat transfer in a given situation, one must account for *e* and *a*, distance between the objects, and orientation. (For instance, the radiation heat transfer from the sun is less in the winter because the angle of the sun is farther from 90° in the given hemisphere. But in some simple cases easy estimates are possible.

1. two spherical bodies, far apart (assume black bodies), one much larger and hotter
   draw a sphere around the hotter body with r = distance to other body

   total heat emitted by hotter body

   fraction striking smaller body (use circular
   cross section for smaller body)

   net heat transfer from hotter body to colder body

2. two broad, flat solids facing each other over relatively narrow distance: all radiation from one surface strikes the other; fraction (1-*e*) [should this be *a*?] is not absorbed, but radiated back. Net heat transfer: eq. 3.177.

3. spherical body radiates into space surrounded by cooler surface far away
   All energy radiated by the body strikes a surface at the cooler temperature
   See example section 3.15 of *FTI*

[redacted]

[redacted]

~~For further details, see BSL ch. 16. *PGE 322K does not cover radiation.*~~

VII ~~XV~~ **Heat-transfer coefficients**

A. Review

B. General definition of h

C. Flow in tubes (BSL Sects. 14.1, 2)

1. Definitions

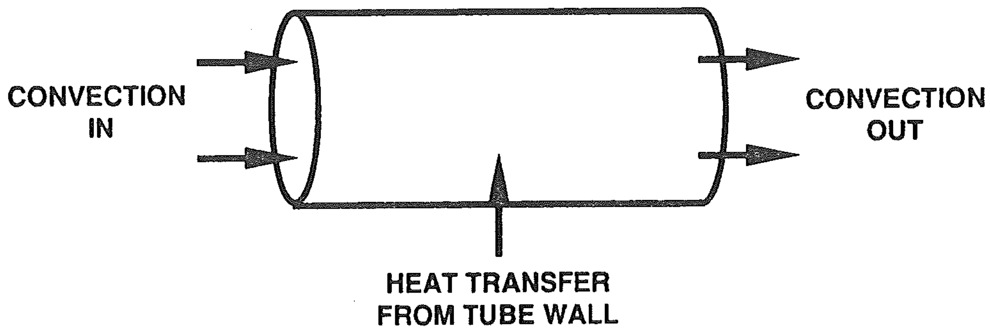
Note:

-
-
-

2. Energy balance for flow in tube

a. macroscopic balance

(Can use macroscopic energy balance because don't care about details of how T varies with position within the tube. T_{b1} is average temperature of fluid at inlet; T_{b2} is average temperature of fluid at outlet.)



Define system as fluid within pipe. Assume fluid is incompressible. Therefore average velocity v is same at inlet and outlet.

Terms in energy balance:

- convection of thermal energy in: $(\pi R^2)\, v\, \rho\, \hat{C}_p\, T_{b1}$
  Define mass rate of flow $w \equiv (\pi R^2)\, \rho\, v$;
  then convection in = $w\, \hat{C}_p\, T_{b1}$
- convection of thermal energy out: $w\, \hat{C}_p\, T_{b2}$
- heat transfer from walls: Q
- No accumulation at steady state - T does not change with *time* at any fixed location in the pipe (though T does vary with *position* along the pipe).
- No heat generation within fluid.

<u>Energy balance</u>:

$$w\, \hat{C}_p\, T_{b1} - w\, \hat{C}_p\, T_{b2} + Q = 0; \quad \text{or} \quad Q = w\, \hat{C}_p\, (T_{b2} - T_{b1}) \quad - \quad \text{"Eq. II"}$$

Combine with definition of h_{ln} ("Eq. I") from above and rearrange terms:

$$\frac{h_{ln}\, D}{k} \equiv Nu_{ln} = \frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{ln}} \left(Re\ Pr\, \frac{D}{4L} \right) \quad - \quad \text{"Eq. "III"}$$

Note: *IF* T_0 is uniform along tube ($T_{01} = T_{02} \equiv T_0$), then

$$\frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{ln}} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}} \right)$$

- In PGE 383, for simplicity, we usually assume $T_{01} = T_{02} \equiv T_0$.
- Eq. "III" is derived from energy balance, and therefore represents conservation equation in solution of heat-transfer problems in tubes.

3. Correlations for h_{ln}

As with friction factors, correlations for h_{ln} relate dimensionless groups:

Correlations for Nu_{ln} are transport "laws" - one combines these correlations with the conservation eq., "Eq. III," to complete the solution.

The primary correlation is

Note:

- similarities to f(Re) chart

- differences with f(Re) chart

  ... *because correlation is only for smooth pipes*

- solution in laminar region is from analytical solution to pde derived in BSL Sect. ~~9~~ 10.8; in turbulent region, based on experimental results

- third label for the vertical axis of Fig. 14.3-2 *combines transport law and conservation eq:* thus can solve problem directly

- although $[(h_{ln} D/k) Re^{-1}]$ declines with increasing Re across much of chart, h_{ln} increases monotonically with increasing Re. *Increasing convection and turbulence increases h_{ln}.*

4. using the correlations for Nu_{ln}

Re < 2100

a) Is wall temperature roughly uniform?
- no - must use differential equation for T_b with h_{loc}
  - any correlation below (Eq. 14.3-17, Fig. 14.3-2 or Fig. 14.2-1 "const. wall T (tube)" curve) is OK for h_{loc}
  - *NOT COVERED IN PGE 322K*
- yes - continue - find h_{ln}

b) Re Pr D/L >> 10? (Or $[(T_{02}-T_{b2})/(T_{01}-T_{b1})] > 0.2$?)
(in other words, is T relatively far from equilibrium at outlet?)
- yes: then can solve in any of 3 ways:
  - use Fig 14.3-2
  - Eq. 14.3-17 and Eq. "III;"
  - Fig. 14.2-1 "const. wall T (tube)" curve and Eq. "III"
- no: then can use only Fig. 14.2-1 "const. wall T (tube)" & Eq "III"

Re > 2100

Use Fig. 14.3-2

If Re > 20,000 can also use Eq. 14.3-16 and Eq. "III"

Non-cylindrical Tubes: Use "hydraulic radius" approximation
Same approach as in friction-factor problems
Substitute $4R_h$ X 4 (S/Z) for D
Calculate <v> for given tube shape
Calculate h or T as for circular tube
- *VALID ONLY FOR TURBULENT FLOW*

Note: for *any* Re, if $T_{01}=T_{02}\equiv T_0$, then
$(T_{b2}-T_{b1})/(T_0-T_b)_{ln} = \ln[(T_0-T_{b1})/(T_0-T_{b2})]$

5. Additional notes

a. Don't sweat the other correlations for h_{ln} in BSL Section 14.2, 14.3

average bulk temperature and μ_0 is the viscosity at the arithmetic average wall temperature.[1] Then we may write

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Re}, \mathrm{Pr}, L/D, \mu_b/\mu_0) \tag{14.3-15}$$

This type of correlation seems to have first been presented by Sieder and Tate.[2] If, in addition, the density varies significantly, then some free convection may occur. This effect can be accounted for in correlations by including the Grashof number along with the other dimensionless groups. This point is pursued further in §14.6.

Let us now pause to reflect on the significance of the above discussion for constructing heat transfer correlations. The heat transfer coefficient h depends on *eight* physical quantities (D, $\langle v \rangle$, ρ, μ_0, μ_b, $\hat{C}_p$, k, L). However, Eq. 14.3-15 tells us that this dependence can be expressed more concisely by giving Nu as a function of only *four* dimensionless groups (Re, Pr, L/D, μ_b/μ_0). Thus, instead of taking data on h for 5 values of each of the eight individual physical quantities (5^8 tests), we can measure h for 5 values of the dimensionless groups (5^4 tests)—a rather dramatic saving of time and effort.

A good global view of heat transfer in circular tubes with nearly constant wall temperature can be obtained from the Sieder and Tate[2] correlation shown in Fig. 14.3-2. This is of the form of Eq. 14.3-15. It has been found empirically[2,3] that transition to turbulence usually begins at about Re = 2100, even when the viscosity varies appreciably in the radial direction.

For *highly turbulent flow*, the curves for $L/D > 10$ converge to a single curve. For $\mathrm{Re}_b > 20{,}000$ this curve is described by the equation

$$\mathrm{Nu}_{\mathrm{ln}} = 0.026\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu_b}{\mu_0}\right)^{0.14} \tag{14.3-16}$$

This equation reproduces available experimental data within about ±20% in the ranges $10^4 < \mathrm{Re}_b < 10^5$ and $0.6 < \mathrm{Pr} < 100$.

For *laminar flow*, the descending lines at the left are given by the equation

$$\mathrm{Nu}_{\mathrm{ln}} = 1.86\left(\mathrm{RePr}\,\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_0}\right)^{0.14} \tag{14.3-17}$$

---

[1] One can arrive at the viscosity ratio by inserting into the equations of change a temperature-dependent viscosity, described, for example, by a Taylor expansion about the wall temperature:

$$\mu = \mu_0 + \left.\frac{\partial \mu}{\partial T}\right|_{T=T_0}(T - T_0) + \cdots \tag{14.3-15a}$$

When the series is truncated and the differential quotient is approximated by a difference quotient, we get

$$\mu \cong \mu_0 + \left(\frac{\mu_b - \mu_0}{T_b - T_0}\right)(T - T_0) \tag{14.3-15b}$$

or, with some rearrangement,

$$\frac{\mu}{\mu_0} \cong 1 + \left(\frac{\mu_b}{\mu_0} - 1\right)\left(\frac{T - T_0}{T_b - T_0}\right) \tag{14.3-15c}$$

Thus, the viscosity ratio appears in the equation of motion and hence in the dimensionless correlation.

[2] E. N. Sieder and G. E. Tate, *Ind. Eng. Chem.*, **28**, 1429–1435 (1936).

[3] A. P. Colburn, *Trans. AIChE*, **29**, 174–210 (1933). **Alan Philip Colburn** (1904–1955), provost at the University of Delaware (1950–1955), made important contributions to the fields of heat and mass transfer, including the "Chilton–Colburn relations."

b. h_{ln} corresponds to h_o of BSL Sect. 10.6
- i.e., heat-transfer coefficient for Newton's law on inner surface of tube

- Can combine $h_{ln} = h_o$ derived with methods of BSL ch. 14 with formula for conduction through composite cylindrical layers from Section 10.6 to get overall heat-transfer coefficient U_o for heat transfer through tubes

- (in this course, still have no way to calculate h_3 on the outer tube surface; for this problem, see other sections of BSL ch. 14 or other texts and handbooks. Principles are still same: greater convection and increasing turbulence increase h_3 as they do $h_{ln} \equiv h_o$)

c. for h for packed beds, see BSL Sect. 14.5 - not covered in PGE 322K

d. Eq. III describes heat transfer between the inner of surface of a tube (assumed to be at fixed T_0) and fluid within the tube.
What if instead have uniform T_f *of fluid outside* the tube?

Eqs. 10.6-29 through 31 apply, with

e. Eqs. 10.6-29 to 31 give U_o; then how to calculate fluid T along tube?
Conservation equation is modified form of Eq. III:

6. Example problems

a. BSL ex. 14.3-1

Examples of Heat Transfer in Tubes

1) Heat transfer to pore walls.

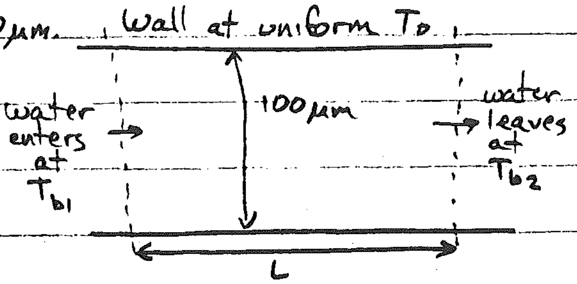
Assume pore is circular tube, $D = 100\,\mu m$.

Assume vel. = 10 ft/d = $3.5 \cdot 10^{-5}$ m/s

Assume $\rho = 1000$ kg/m³

$\mu = 0.001$ Pa·s $\quad \hat{C}_p = 4190$ J/kg

$k = 0.67$ W/mK.



What is L for $\frac{T_{02} - T_{b2}}{T_{01} - T_{b1}} = 0.01$? (99% of way to equilibrium)

Solution: $Re = \frac{D v \rho}{\mu} = \frac{(10^{-4})(3.5 \cdot 10^{-5})(1000)}{(0.001)} = 0.0035 \ll 2100$

$\frac{T_{02} - T_{b2}}{T_{01} - T_{b1}} < 0.2$; ∴ cannot use Fig. 14.3-2 or Eq. 14.3-17.

Use Fig 14.2-1 "const. wall T (tube)" and Eq. "III".

Eq. III: $Nu_{ln} = \frac{h_{ln} D}{k} = \frac{T_{b2} - T_{b1}}{(T_0 - T_b)_{ln}} \, Re \, Pr \, \frac{D}{4L}$

Since T_0 is uniform, $\frac{T_{b2} - T_{b1}}{(T_0 - T_b)_{ln}} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) = \ln(100)$

$Re = 3.5 \cdot 10^{-5}$; $Pr = \frac{\hat{C}_p \mu}{k} = (4190)(0.001)/(0.67) = 6.25$

Eq. III → $Nu_{ln} = \ln(100)(0.0035)(6.25)\frac{10^{-4}}{4L} = 2.52 \cdot 10^{-6}/L$

Fig 14.2-1 without knowing L, we need some sort of trial + error here. Note if $\frac{L}{D\,Re\,Pr} > 0.3$, $Nu_{ln} \approx 3.657$. First assume $\frac{L}{D\,Re\,Pr} > 0.3$, then check at end.

If $Nu_{ln} = 3.657 = 2.52 \cdot 10^{-6}/L$, $L = 6.9 \cdot 10^{-7}$ m

Check: $\frac{L}{D\,Re\,Pr} = \frac{6.9 \cdot 10^{-7}}{(10^{-4})(3.5 \cdot 10^{-3})(6.25)} = 0.31 > 0.3$. Assumption verfied.

Note that time for equilibration is $\frac{L}{\langle v \rangle} = \frac{6.9 \cdot 10^{-7}}{3.5 \cdot 10^{-5}} = 0.02$ s.

Nearly istantaneous equilibration. Why? Small D.

2) Heat transfer in 4-in pipe.

All parameters same as in (1), except

$D = 0.1$ m $\qquad \langle v \rangle = 0.5$ m/s.

$$Re = \frac{Dv\rho}{\mu} = \frac{(0.1)(0.5)(1000)}{(0.001)} = 50{,}000$$

For this high Re, can use Eq. 14.3-16 and Eq. "III."

Eq. 14.3-16: $Nu_{ln} = 0.026\,(Re)^{0.8}\,(Pr)^{1/3}\left(\frac{\mu_b}{\mu_0}\right)^{0.14}$ ignore last term

$$Nu_{ln} = (0.026)(50{,}000)^{0.8}(6.25)^{1/3} = 275$$

Eq. III: $Nu_{ln} = \frac{T_{b2}-T_{b1}}{(T_0-T_b)_{ln}}\,Re\,Pr\left(\frac{D}{4L}\right) = \ln(100)(5\cdot10^4)(6.25)\,\frac{0.1}{4L}$

$$= \frac{36{,}000}{L} = 275$$

$L = 131$ m.

Time for equilibration is $\frac{L}{\langle v \rangle} = \frac{131}{0.5} = 260$ s (4 min, 20 sec)

Why so slow? Large D. Turbulence actually speeds up the equilibration. If there were no turbulence (somehow suppressed in spite of high Re), then Fig 14.2-1 would apply again, instead of Eq. 14.3-16. $Nu_{ln} = 3.66$ instead of 275, and $L \cong 1000$ m!

15.9

(35 pts) Water flows with velocity 1 m/s through a 3-m length of tubing of 0.01-m inner diameter. Water properties are given below. The tube wall is 0.01 m thick, with thermal conductivity k = 100 W/(m s). Surrounding the pipe is a fluid at temperature 0°C, with properties given below. Water enters the pipe at 100°C and leaves at 50°C. What is the heat-transfer coefficient at the outer surface of the pipe?

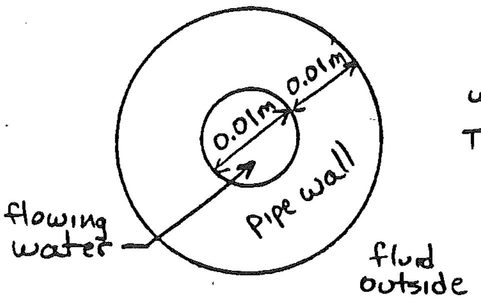
Approximate properties of water

ρ = 1000 kg/m^3 $\hat{C}_p$ = 4180 J/(kg K) k ~ 0.65 W/(m K) μ = 0.001 Pa s

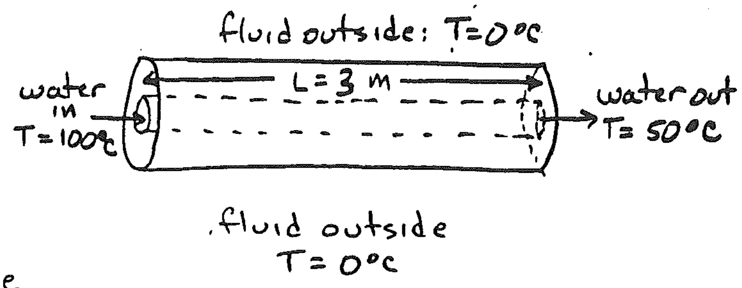
Approximate properties of fluid outside pipe

ρ = 700 kg/m^3 $\hat{C}_p$ = 3000 J/(kg K) k ~ 0.5 W/(m K) μ = 0.01 Pa s

end view



axial view



Eq. 10.6-31 gives the overall heat-transfer coefficient for the system. First we need to compute h_{ln} between the fluid inside the pipe and the pipe:

What is Re? $Re = Dv\rho/\mu = (0.01)(1)(1000)/(0.001) = 10{,}000$. For this Re, only Fig 14.3-2 applies. From this fig,

$$\frac{h_{ln} D}{K} (Re)^{-1} (Pr)^{-1/3} \cong 0.0039. \quad (I)$$

What is Pr? $Pr = \hat{C}_p \mu / K = (4180)(0.001)/(0.65) = 6.43$

(I) → $h_{ln} = (0.0039)(10{,}000)(6.43)^{1/3}(0.65)/(0.01) = 4{,}714 \ W/(m^2 K)$

This is "h_o" in Eq 10.6-31:

$$U_o = \frac{1}{0.005}\left[\frac{1}{(0.005)4714} + \ln\left(\frac{0.015}{0.005}\right)/100 + \frac{1}{(0.015)h_3}\right]^{-1} = \frac{1}{0.005}\left[0.0424 + 0.0109 + \frac{66.7}{h_3}\right]^{-1}$$

Now we have $U_o(h_3)$; we still need to relate U_o to the fluid-temperature change in the tube, using the equation given in class:

$$\frac{U_o D}{K} = \frac{T_{b2} - T_{b1}}{(T_f - T_b)_{ln}}\left(Re\, Pr \frac{D}{4L}\right) = \ln\left(\frac{T_f - T_{b1}}{T_f - T_{b2}}\right)\left(Re\, Pr \frac{D}{4L}\right)$$

$$\frac{U_o D}{K} = \ln\left(\frac{0-100}{0-50}\right)\left((10{,}000)(6.43)\frac{0.01}{4(3)}\right) = 0.693(53.58) = 37.14$$

Now $$\frac{U_o D}{K} = 37.14 = \frac{1}{0.005}\left[(0.0424) + (0.0109) + \frac{66.7}{h_3}\right]^{-1} \frac{0.01}{0.65}$$

$$0.0828 = \left[(0.0424) + (0.0109) + \frac{66.7}{h_3}\right] \rightarrow h_3 = 2260 \ (W/m^2K)$$