ta 3220 Final Exam Spring 2009 (3 April)

Solution

1. a) $\frac{\Delta\mathcal{P}}{L} = \frac{(1\text{ atm} + 10000 - 1\text{ atm}) + [0.5\sin(30°)]\,1000(9.8)}{0.5}$ flows down, so gravity adds to $\Delta\mathcal{P}$

$= \frac{10000 + 2450}{0.5} = 24900$ Pa/m

b) Can use hydraulic radius approx. as long as flow is turbulent. Don't know v, so don't know Re yet. Use trial + error.

$\frac{Dv\rho}{\mu} = ?$ First, need hydraulic diameter; for slit, it is

2 × gap width = 0.003 m

"Very smooth" → $\varepsilon/D = 0$

$\frac{Dv\rho}{\mu} = \frac{0.003(v)1000}{0.001} = 3000\,v$

Eq. 5.30, generalized to case with gravity

Guess $v = 1$ m/s Re = 3000 $4f \approx 0.042$ $v = \left(\frac{D_h}{L}\frac{2\Delta\mathcal{P}}{\rho}\frac{1}{4f}\right)^{1/2}$

$v = \left(0.003(2)(24900)\frac{1}{1000}\frac{1}{0.042}\right)^{1/2} = 1.89 = \left(\frac{0.1494}{4f}\right)^{1/2}$

v = 1.89	Re = 5660	4f ≅ 0.035	v = 2.07
2.07	6198	0.033	2.13
2.13	6383	can't read any difference in 4f. Done	

V = 2.13 m/s

2 a) Bingham plastic flows if shear stress at wall > τ_0.

For slit, example in FT text has only gravity. In our case, $\tau_{xz} = \frac{\Delta\mathcal{P}}{L}x$, at wall $\tau_{xz} = (24900)\frac{0.0015}{2} = 18.7$

If $\tau_0 > 18.7$ Pa, Bingham plastic flows.

b) For Power-law fluid, $Q \sim \left(\frac{\Delta\mathcal{P}}{L}\right)^{1/n}$. If $\frac{\Delta\mathcal{P}}{L}$ increases by a factor of 1.5, and n = 3, Q increases by a factor $(1.5)^{1/3} = 1.144$. $Q_2 = 1.144\,Q_1$.

3. a) FT Eq. 3.120 applies for Re > 10000. Here, $Re = \frac{(0.003)(1)(1000)}{0.001} = 3000$

Eq. 3.116 is for laminar flow, also not relevant. Fig 14.3-2 of BSL applies, though.

For Re = 3000, $\frac{h_{ln} D}{K} Re^{-1} Pr^{-1/3} = 0.0029$ $\qquad \frac{L}{D} = \frac{0.5}{0.003} = 167$

$$Pr = \frac{C_p \mu}{K} = \frac{4190(0.001)}{0.68} = 6.16$$

$$D = D_h = 0.003$$

$$\frac{h_{ln}(0.003)}{0.68}(3000)^{-1}(6.16)^{-1/3} = 0.003$$

$$h_{ln} = 3789 \ \frac{W}{m^2 K}$$

b) Could use either $\frac{hD}{K} = Nu$ above in "Eq. III" in lecture notes, or other option in Fig 14.3-2

$$0.003 = \frac{T_{b2} - T_{b1}}{(T_0 - T_b)_{ln}} \cdot \left(\frac{D}{4L}\right) Pr^{2/3} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) \frac{D_h}{4L} (Pr)^{2/3}$$

$$0.003 = \ln\left(\frac{0 - 25}{0 - T}\right) \frac{0.003}{4(0.5)} (6.16)^{2/3}$$ [No need to convert T units because it is in dimensionless ratio]

$\ln\left(\frac{25}{T}\right) = 0.595$ $\qquad \frac{25}{T} = 1.81$; $T = 13.8°C$

4 a) This is just the situation in section 9.6 of BSL

e.g., eq. 9.6-15 : $q_0 = \frac{25 - 0}{\left(\frac{1}{h_0} + \frac{0.001}{0.25}\right)}$

Better eq. 9.6-16, $U_0 = \left[\frac{1}{h_0} + \frac{0.001}{0.25}\right]^{-1} = \left[\frac{1}{3789} + 0.004\right]^{-1} = 234 \ \frac{W}{m^2 K}$

b) The two resistances are $\frac{1}{3789} = 0.000267$ and $\frac{0.001}{0.25} = 0.004$.

The plastic has a huge effect. Note that it reduces heat transfer coefficient by a factor of $\frac{234}{3789} = 0.063$

It is by far the biggest resistance.

5. The geometry (cylindrical) and terms in the energy balance (conduction, generation, no convection or accumulation) are the same as in BSL sect. 9.2. Everything is same through Eq. 9.2-7, but 9.2-9 does not apply: r=0 is not part of an annulus. Annulus extends from $r = R_0$ to $r = R_1$

$$q_r = \frac{Sr}{2} + \frac{C_1}{r}$$

BC: at $r = R_1$, $q_r = 0$ (perfectly insulated)

$$q_r = 0 = \frac{SR_1}{2} + \frac{C_1}{R_1} \rightarrow C_1 = -\frac{SR_1^2}{2}$$

$$\therefore q_r = \frac{Sr}{2} - \frac{SR_1^2}{2r} = -k\frac{dT}{dr}$$

integrate $-\frac{1}{k}\frac{Sr^2}{4} + \frac{1}{k}\frac{SR_1^2}{2}\ln r = T + C_2$

BC: at $r = R_0$, $T = T_0$

$$-\frac{1}{k}\frac{SR_0^2}{4} + \frac{1}{k}\frac{SR_1^2}{2}\ln R_0 = T_0 + C_2$$

$$-\frac{1}{k}\frac{SR_0^2}{4} + \frac{1}{k}\frac{SR_1^2}{2}\ln R_0 - T_0 = C_2$$

$$T = \frac{1}{k}\frac{S}{4}(R_0^2 - r^2) + \frac{1}{k}\frac{SR_1^2}{2}\ln\left(\frac{r}{R_1}\right)$$

6. Perfect insulation on the top, bottom, left + right means this is unsteady conduction in a slab. Perfect insulation on the right means this is like a slab 40 cm long, cooled on both sides. Using the chart on p. 133 (Wakke plot),

$\left(F_o = \frac{\alpha t}{D^2};\ D = 0.4;\ \alpha = \frac{k}{\rho c_p} = 1.66 \cdot 10^{-4};\ F_o = \frac{(1.66 \cdot 10^{-4})t}{(0.4)^2} \right)$

We want $\frac{T_1 - T_c}{T_1 - T_0} = \frac{0-5}{0-25} = 0.2$

For plate, this is perhaps $F_o \cong 0.185 = \frac{1.66 \cdot 10^{-4}\, t}{(0.4)^2}$

$\rightarrow t \cong 178$ sec.

One could also use Eq 12.1-3. In that case

$\frac{T - T_0}{T_1 - T_0} = \frac{5-25}{0-25} = 0.8$, $\frac{\alpha t}{b^2} \approx 0.77$, $b = 0.2\ (= D/2)$

$\rightarrow t \cong 185$ sec.