ta 3220 Final Exam April 2012

$V = \frac{Q}{A} = \frac{0.002}{\pi(0.025)^2} = 1.019\ m/s$

1. $Re = \frac{Dv\rho}{\mu} = \frac{(0.05)(1.019)1000}{0.001} = 5.10 \cdot 10^4$ turbulent flow

$\frac{\varepsilon}{D} = \frac{0.0005}{0.05} = 0.01$ (roughness factor)

from chart (Fig 5.10, p. 224) $f \triangleq 0.037$

$0.037 = \frac{1}{4}\frac{D}{L}\frac{\Delta\mathcal{P}}{1/2\,\rho v^2} \rightarrow \Delta\mathcal{P} = 0.037\frac{100}{0.05}\frac{1}{2}(1000)(1.019)^2$

$= 3.84 \cdot 10^4\ Pa$

$3.84 \cdot 10^4 = (P_0 - 1.01 \cdot 10^5) + 4(1000)(9.8)$ ← outlet lower; gravity helps flow

$P_0 = 1.002 \cdot 10^5\ Pa$

(minimum p is a little less than atm. pressure)

2. For slit, $D_h = 2 \times gap = 0.04\ m$, $Re = \frac{D_h v\rho}{\mu} = \frac{(0.04)(0.5)(1000)}{0.001} = 20{,}000$

turbulent eq., + use of hydraulic radius are OK.

$Nu = (0.026)(20{,}000)^{0.8}(6.16)^{1/3} = (0.026)\,Re^{0.8}\,Pr^{1/3}$ (Eq. 3.120)

$= 131.5$ $\left(Pr = \frac{C_p\mu}{k} = \frac{4190\,(0.001)}{0.68} = 6.16\right)$

$131.5 = \frac{hD_h}{k} = \frac{h\,(0.04)}{0.68} \Rightarrow h = 2236\ W/m^2K$

3. This is simply a special case of Eq. 9.6-31 of BSL.

$\frac{1}{u} = \frac{1}{h} + \frac{1}{h} = \frac{2}{h}$

$u = h/2$ The rest of the information is irrelevant.

see end of solution for a better derivation

4. Some possibilities:

Put Ketchup in a layer of known thickness on a flat plate and tip the plate. If it is PL fluid, it would start to flow (perhaps slowly) at any angle If it is BP, there are small angles where it doesn't flow at all.

Put ketchup in tube and tip tube at different angles. Same idea as above.

If have series of balls of various diameters of slightly greater density, see if they all sink, or small balls stay suspended.

In any case, need to be able to look in limit as $\tau \to 0$ and see if flow stops.

5. A macroscopic momentum balance on fluid in pipe:

pressure force x X-section area = drag on cylindrical surface

$\pi R^2 \Delta \mathcal{P} = 2\pi R L \tau$

$\tau = \Delta \mathcal{P} \, R/2 = 2 \cdot 10^4 (0.05)/2 = 500$ Pa.

Nothing in this analysis depends on nature of fluid or whether flow is turbulent.

6. We can jump to Eq. 9.2-6 as long as we recognize that S_e is not a constant: $S_e = S_{eo} + Br$

$\frac{d}{dr}(r q_r) = (S_{eo} + Br) r = S_{eo} r + Br^2$ Integrate.

$r q_r = S_{eo} \frac{r^2}{2} + B \frac{r^3}{3} + C_1$ Divide by r

$q_r = S_{eo} \frac{r}{2} + B \frac{r^2}{3} + \frac{C_1}{r}$

BC: q_r blows up at $r=0$ unless $C_1 = 0$

$q_r = S_{eo} \frac{r}{2} + B \frac{r^2}{3} = -k \frac{dT}{dr}$

$\frac{dT}{dr} = -S_{eo} \frac{r}{2k} - B \frac{r^2}{3k}$

$T = -\frac{S_{eo} r^2}{4k} - B \frac{r^3}{9k} + C_2$

BC: at $r=R$, $T = T_0 \rightarrow T_0 = -\frac{S_{eo} R^2}{4k} - B \frac{R^3}{9k} + C_2$

$C_2 = T_0 + \frac{S_{eo} R^2}{4k} + B \frac{R^3}{9k}$

$T - T_0 = \frac{S_{eo}}{4k}(R^2 - r^2) + \frac{B}{9k}(R^3 - r^3)$

(other equivalent ways of grouping terms also OK)

7. a) Heat lost by cube = heat gained by water ← There was some confusion. Temperatures were 7.5, 8.2 °C, not 7500, 8200 °C)

$= V\rho C_p (8.2)$

$= (0.03)(1000)(4190)\, 8.2 = 125{,}700 \; 8.2 = 1.031 \cdot 10^6$ J

b) Heat lost by cube $= (V\rho C_p)_{cube} (100 - 8.2)$

$1.031 \cdot 10^6 = (0.2)^3 \rho C_p (91.8) \rightarrow \rho C_p = 1.40 \cdot 10^6$

c) After 10 minutes, water has gained heat

$V\rho C_p (7.5) = (125{,}700)(7.5) = 9.428 \cdot 10^5$

heat lost by cube: $(0.2)^3 (1.4 \cdot 10^6)(100 - T) = 9.428 \cdot 10^5$

$T = 16°C$

(Could also work this by ratio of ΔT's)

8. a) If internal conduction is the only process, can use Fig. 3.16 (p. 132) of FT

$$\frac{T_1 - \langle T \rangle}{T_1 - T_0} = \frac{0-8}{0-100} = 0.08 \rightarrow Fo \doteq 0.05 \text{ (chart)}$$

$$Fo = \frac{\alpha t}{D^2} = \frac{\alpha(600)}{(0.2)^2} = 0.05 \rightarrow \alpha = 3.33 \cdot 10^{-6}$$

These are modes in series.

b) We assumed no resistance from convective heat transfer. In reality, convective heat transfer puts up some resistance, and internal conduction puts up <u>less</u> resistance. α is <u>larger</u> than estimated using assumption that it is the only resistance.

Better derivation for problem 3

This problem is similar to BSL Eq. 9.6-31.

At steady state, heat into fluid from one side = heat out other side (but opposite in sign). Let $T_f = T$ fluid.

$$q = h(T_1 - T_f) \rightarrow T_1 - T_f = q/h$$

$$-q = h(T_0 - T_f) \qquad T_0 - T_f = -q/h$$

$$(T_1 - T_0) = (T_1 - T_f) + (T_f - T_0) = q/h + q/h = q(1/h + 1/h)$$

$$q = \frac{T_1 - T_0}{(2/h)} \equiv U(T_1 - T_0)$$

$$U = h/2 \text{ or } \frac{1}{1/h + 1/h}$$