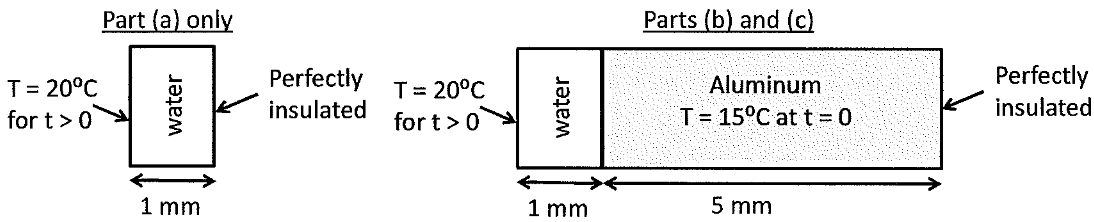
ta3220 Final Examination
Spring 2013 - 11 April 2013

Write your solutions *on your answer sheet*, not here. In all cases ***show your work***.
To avoid any possible confusion,
state the equation numbers and figure numbers of equations and figures you use.
Beware of unnecessary information in the problem statement.

1. A laboratory apparatus is supposed to operate at 20°C, but at the start of the day it is often at 15°C. There is a layer of water with dissolved polymer, 1 mm thick, between the heater and a layer of aluminum, which is 5 mm thick. The polymer does not change the properties of water, but it does prevent any convection in the water layer. The entire apparatus is at 15°C at t = 0, but starting at t = 0 the heater is immediately heated to 20°C and maintained at that temperature.
   Consider this problem in three parts:
   a. Ignore the aluminum layer, and suppose all we have to do is warm the water. (See figure on left, below.) Assume the opposite surface is perfectly insulated. How long would it take the opposite side of the water layer to heat to 19.5°C?
   b. For the rest of the problem, we assume that the water layer is always at steady state. That is, *heating* the water (as in part (a)) offers no resistance, but heat transfer *across* the water to the aluminum layer may be important, and we consider the time to heat the metal layer. (See figure on right, below.) Assume all the resistance to heat transfer is in heat transfer across the water layer. How long does it take to heat up the aluminum layer to 19.5°C?
   c. Assume that all the resistance is in unsteady conduction *within* the aluminum layer. How long does it take the opposite side of the aluminum layer to heat up to 19.5°C?
   d. Based on your results to parts (a), (b) and (c), which is the better approximation? (If you couldn't get answers to part (a), (b) or (c), state how you would decide which is the better answer.) Is the true time to warm the apparatus greater or less than that you estimate here?

(26 pts)



<u>properties of water</u>

$\rho = 1000$ kg/m^3 $\mu = 0.001$ Pa s $k = 0.680$ W/(m K) $C_p = 4190$ J/(kg K)

<u>properties of aluminum</u>

$\rho = 2700$ kg/m^3 $k = 230$ W/(m K) $C_p = 938$ J/(kg K)

2. An aluminum sphere of diameter 1.5 m is initially at 100°C. At time t = 0 its surface temperature is reduced to 0°C, and then, after 10 minutes, the surface temperature is immediately raised back to 100°C. What is the temperature at the center of the sphere at t = 30 minutes, i.e. 20 minutes after the second change? Use the properties of aluminum from problem 1.

(12 pts)

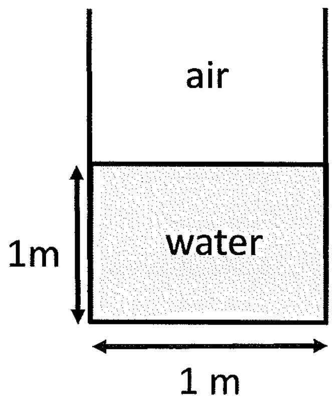
3. A sphere of radius R is initially at temperature T_o at time t = 0. Starting at t = 0, heat is released inside the sphere at a uniform and constant rate S (in units W/m^3). The sphere is surrounded (some distance away) by a larger spherical surface maintained at temperature T_0. As the sphere heats up, the most important heat-transfer process is radiation from its surface, which occurs at a rate $Q = [\sigma\, e\, (T^4 - T_o^4)]$, where σ and e are constants, and where Q > 0 means heat is lost to the surroundings. The sphere can be assumed to be uniform in temperature at all times.
   a. What is the final steady-state temperature of the sphere?
   b. Derive a differential equation for T, the temperature of the sphere, as a function of time for the time before the sphere reaches steady state. **You do not need to solve this equation.**

   NOTE: To solve this problem, you don't need to know any more about radiation than is given in the problem statement.

(15 pts)

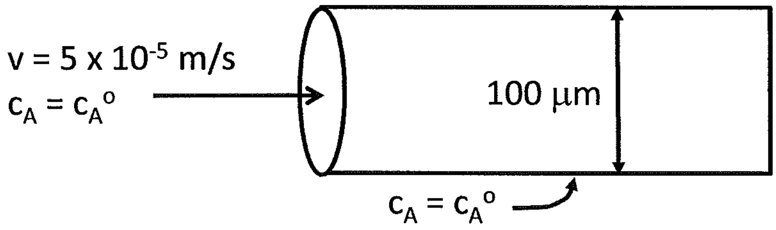
4. Water flows through an open rectangular channel, 1 m wide, and open at the top; the channel is at an angle 3° from the horizontal along the direction of flow. The water is 1 m deep in the channel. The properties of water are given in problem 1. The walls are all roughened on a scale of about 5 cm. What is the total flow rate through the channel in m^3/s?

(12 pts)



5. Water with concentration c_A^o of solute A flows into a cylindrical pore with diameter 100 μm with velocity 5 x 10^{-5} m/s. Solute A reacts immediately with the mineral on the pore wall, so that $c_A = 0$ at the pore wall. How far does the water flow before the average concentration c_A in the water is reduced to ($0.5\ c_A^o$)? The properties of water are given in problem 1. The diffusion coefficient of the solute in water $\mathscr{D}$ is 1 x 10^{-9} m^2/s.

(15 pts)



6. An engineer is conducting an experiment with Newtonian fluid flowing through a cylindrical tube in highly turbulent flow (*very* large Re). The tube is not infinitely smooth.
   a. Suppose the engineer doubles the flow rate. How much does the potential gradient ($\Delta\mathscr{P}/L$) increase?
   b. This engineer doesn't realize that flow is turbulent. Instead, he thinks he is working with a power-law fluid in laminar flow. What power-law exponent n would he infer for this fluid to explain the result in part (a)? (If you are unable to answer part (a), then guess an answer and use it to solve part (b).)

(12 pts)

7. A pipe is 1 km long. Along its length, it rises 100 m vertically. Water (properties in problem 1) flows through the pipe.
   a. What is the magnitude of the potential gradient ($\Delta\mathscr{P}/L$) driving flow through the pipe?
   b. Does flow go up or down the pipe (i.e., to the right or to the left)?

(8 pts)