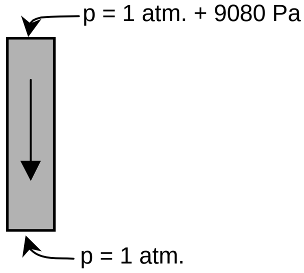
ta3220 Final Examination (retake)
Spring 2013 - 27 June 2013

Write your solutions *on your answer sheet*, not here. In all cases ***show your work***.
To avoid any possible confusion,
state the equation numbers and figure numbers of equations and figures you use.
Beware of unnecessary information in the problem statement.

1. An engineer is measuring downward flow rate of water (ρ = 1000 kg/m^3, μ = 0.001 Pa s) through sand packed in a cylindrical tube 61 cm long and 5 cm in diameter and held vertically. At the (upper) entrance to the tube, absolute pressure is 1.3 psi (9080 Pa) above atmospheric pressure. At the (lower) outlet of the tube, pressure is atmospheric pressure. (Atmospheric pressure is 1.01 x 10^5 Pa.)
   a. What is the total potential difference $\Delta\mathscr{P}$ driving this flow?
   b. The sand grains in this packing have rather large diameter - 2 mm. The porosity is 0.35. What is the flow rate Q (in m^3/s) through the tube? If you are unable to complete part (a), just assume a value for potential difference $\Delta\mathscr{P}$ (and make it very clear what you assume).

   (20 points)



2. A long cylindrical wire of radius R is heated by electrical resistance at a uniform rate throughout the wire S_e (in units W/(m^2 K)). After a time the temperature profile in the wire T(r) comes to steady state. At the outer surface of the wire, temperature is not fixed, as in the section 9.2 of BSL that we studied in class, but heat is lost by radiation according to the following equation:

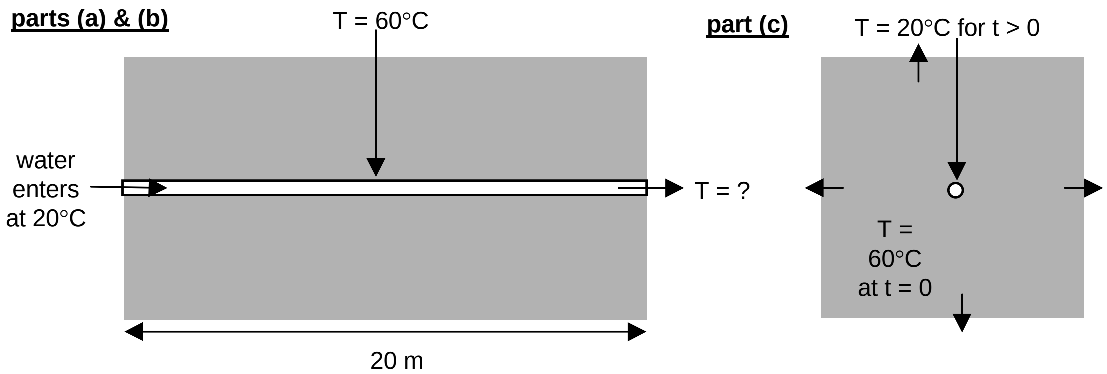
   at r = R, $\quad q_r = A\,(T^4 - T_0^4)$

   where A is a constant and T_0 is the fixed temperature of the surroundings. All you need know about radiation for this problem is the given equation.
   a. You may notice this is similar to the problem in BSL section 9.2. The relevant pages from section 9.2 are attached at the back of this exam. What is the *last* equation you can use from BSL section 9.2, without alteration, in solving this problem? Write that equation number on your answer sheet.
   b. Complete the solution for steady-state T(r) within the wire. In the process, you may find that you come to a point where you need to solve a complicated algebraic equation. If you can't figure out how to solve this equation, state very clearly what steps you *would* take to finish the problem from there. Be specific.

   (20 points)

3. When the Boulder dam was constructed in the U.S., massive slabs of concrete were poured into molds and then allowed to harden. The hardening of concrete involves a chemical reaction that releases heat. There was some concern that the heat released from this reaction might cause the concrete to crack. Therefore pipes were laid in the concrete to carry cooling water through the slabs and thereby carry the heat out of the concrete. We consider the overall heat-transfer process in two steps.
   a. Consider a pipe with 4 in. (0.1 m) inner diameter. Suppose water enters the pipe at a temperature of 20°C. The slab of concrete is maintained constant and uniform at 60°C by the chemical reaction. The slab is 20 m thick, so the pipe is 20 m long. (The slab is much larger in the other two directions.) Suppose the velocity of the water is 2 m/s. What is the temperature of the water leaving the pipe? For the purposes of this part, assume that the inner surface of the pipe is maintained at 60°C.
   b. What is the total rate of heat transfer Q out of the slab from this one pipe?
   c. Let's look at this problem differently. Suppose the chemical reaction is allowed to be completed and it heats the concrete uniformly to 60°C, and then the water flows through the pipes to cool off the slab. Suppose cold water maintains the pipe surface at 20°C; the solid concrete is initially at 60°C, and heat is conducted into the pipe. What is the total rate of heat transfer Q into the pipe 30 days after the water flow begins?
   d. Call the values of Q you got in parts (b) and (c) Q_b and Q_c. Based on the two values of Q you computed, which is more important to the heat-transfer process 30 days into the process: heat transfer within the pipes, or conduction in the concrete? Briefly justify your answer. If you were unable to reach an answer in parts (b) and (c), tell how you would answer the question if you had values for Q_b and Q_c.

(40 pts)



<u>properties of water</u>

ρ = 1000 kg/m^3 μ = 0.001 Pa s k = 0.680 W/(m K) C_p = 4190 J/(kg K)

<u>properties of concrete</u>

ρ = 2000 kg/m^3 k = 0.6 W/(m K) C_p = 2200 J/(kg K)

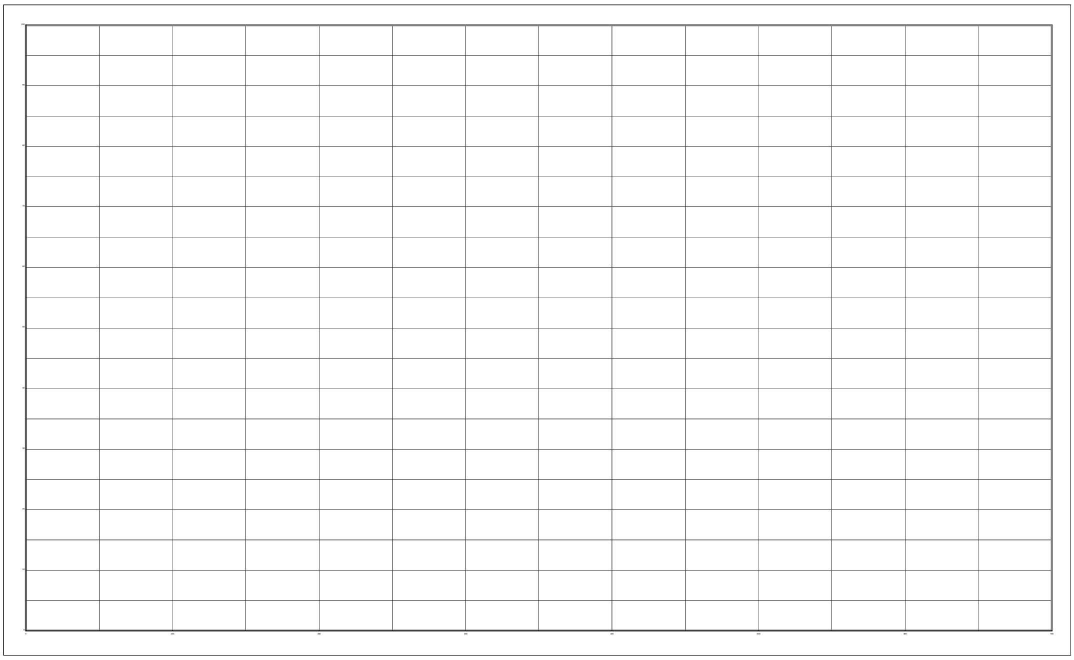
4. In a past year, a student made the following measurements for a brand of Honey Mustard. The data are taken using a Fann viscometer. The "revolution/min." reading is proportional to shear rate, ($-dv_X/dy$). Don't worry about converting it into units of s^{-1}:

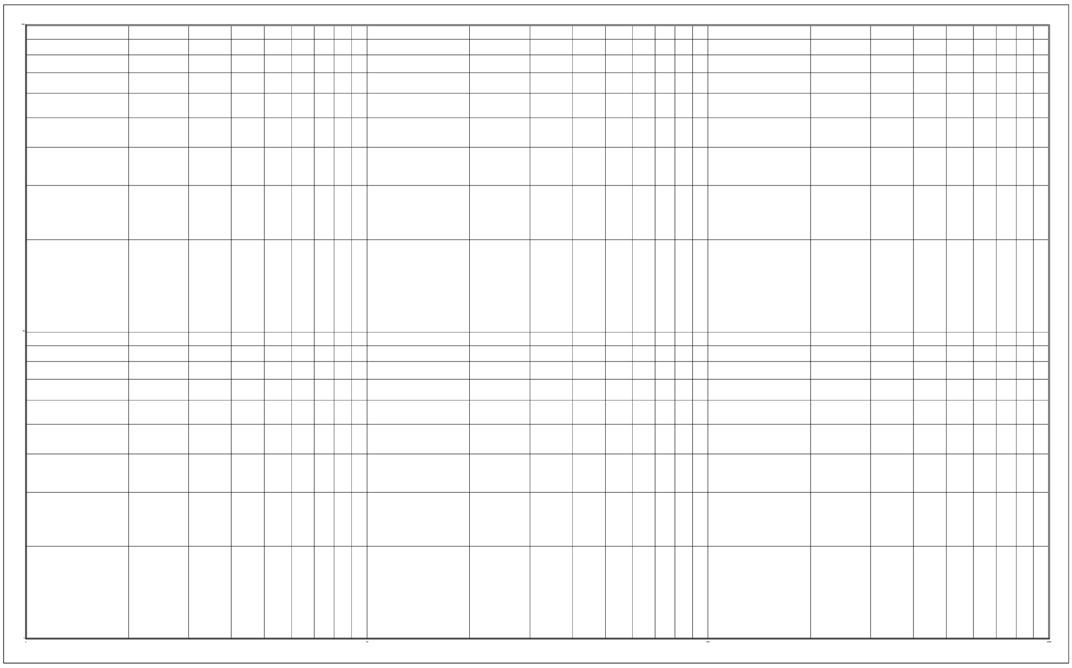
rev/min	shear stress, Pa
10	123
2	70

a) Assuming this is a Bingham plastic, determine the value of τ_o.
b) Assuming it is a power-law fluid, determine the power-law exponent, n.
c) Describe one or more further measurement(s) you would make to unambiguously determine whether this is a Bingham plastic or power-law fluid. Be absolutely clear: describe the measurement quantitatively, and then write "if it were a Bingham plastic, then __ [describe result]; if it were a power-law fluid, then __ [describe result]."

There are figures attached on the next page that you can use if you like; you do not need to use these figures unless you want to. If you do use those figures, be sure to attach them to your answer sheet and explain how you use them.
(20 points)

Figures for use in problem 4





pages from BSL Section 9.2, for problem 2:

C. Examples in BSL, 1. Heating in electrical wire

dependence of either the thermal or electrical conductivity need be considered. The surface of the wire is maintained at temperature T_0. We now show how one can determine the radial temperature distribution within the heated wire.

For the energy balance we select as the system a cylindrical shell of thickness Δr and length L. (See Fig. 9.2–1.) The various contributions to the energy balance are

rate of thermal energy in across cylindrical surface at r	$(2\pi rL)(q_r\|_r)$	(9.2–2)
rate of thermal energy out across cylindrical surface at $r + \Delta r$	$(2\pi(r + \Delta r)L)(q_r\|_{r+\Delta r})$	(9.2–3)
rate of production of thermal energy by electrical dissipation	$(2\pi r\,\Delta r L)S_e$	(9.2–4)

The notation q_r means "flux of energy in the r-direction," and $|_r$ means "evaluated at r." Note that we take "in" and "out" to be in the positive r-direction.

We now substitute these three expressions into Eq. 9.1–1. Division by $2\pi L\,\Delta r$ and taking the limit as Δr goes to zero gives

$$\left\{\lim_{\Delta r\to 0}\frac{(rq_r)|_{r+\Delta r} - (rq_r)|_r}{\Delta r}\right\} = S_e r \tag{9.2–5}$$

The expression within braces is just the first derivative of rq_r with respect to r, so that Eq. 9.2–5 becomes

$$\frac{d}{dr}(rq_r) = S_e r \tag{9.2–6}$$

This is a first-order ordinary differential equation for the energy flux, which may be integrated to give

$$q_r = \frac{S_e r}{2} + \frac{C_1}{r} \tag{9.2–7}$$

The integration constant C_1 must be zero because of the boundary condition

B.C. 1: at $r = 0$ q_r is not infinite (9.2–8)

Hence the final expression for the energy flux distribution is

$$\boxed{q_r = \frac{S_e r}{2}} \tag{9.2–9}$$

This states that the heat flux increases linearly with r.

We now substitute Fourier's law (see Eq. 8.1–2) in the form $q_r = -k(dT/dr)$ into Eq. 9.2–9 to obtain

$$-k\frac{dT}{dr} = \frac{S_e r}{2} \qquad (9.2\text{–}10)$$

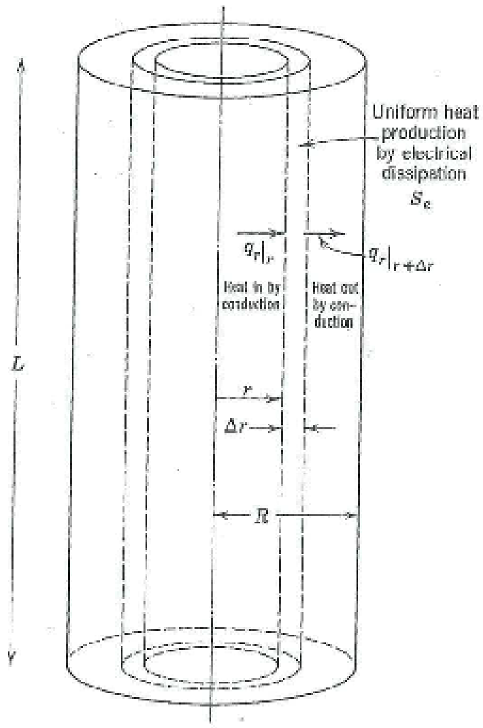


Fig. 9.2–1. Cylindrical shell over which energy balance is made in order to get temperature distribution in an electrically heated wire.

When k is assumed to be constant, this first-order differential equation may be integrated to give

$$T = -\frac{S_e r^2}{4k} + C_2 \qquad (9.2\text{–}11)$$

The integration constant C_2 is determined from

B.C. 2: at $r = R$ $T = T_0$ (9.2–12)

Hence C_2 is found to be $T_0 + (S_e R^2/4k)$ and Eq. 9.2–11 becomes

$$\boxed{T - T_0 = \frac{S_e R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]} \qquad (9.2\text{–}13)$$