Outline of ta3220 - opening lectures

I. laminar flow of Newtonian and non-Newtonian fluids
  A. Fluid properties: Newtonian fluids
    1. thought experiment defining viscosity
    2. magnitudes of viscosity; trends with T, p
    3. Newton's law in *FTI*
    4. meaning of shear stress in tensor form
    5. how one can measure viscosity without infinite plates
  B. shell-balance problems for Newtonian fluids in laminar flow
    1. procedure (cf. also Sect. 5.6 of *FTI*)
    2. examples in *FTI*
      a. shear flow between parallel plates - no pressure or gravity driving force
      b. flow in a slit - no pressure driving force
      c. flow in a tube - no gravity driving force
      d. film condensation (falling film)
    3. examples from *BSL*
      a. falling film
      b. flow in a tube (both pressure and gravity)
      c. flow in an annulus (both pressure and gravity)
      d. flow in a slit (both pressure and gravity) - only final equation given
    4. limits to laminar flow in terms of Re
  C. shell-balance problems for non-Newtonian fluids
    1. properties of non-Newtonian fluids
      a. Bingham plastic (beware of how *FTI* writes this equation!)
      b. power-law fluid
      c. effective viscosity for a non-Newtonian fluid
    2. flow derivations
      a. flow of Bingham plastic in tube
      b. power-law fluid in a tube
      c. definition of effective viscosity for tube flow
      d. flow in a slit
        i. Newtonian fluid reconsidered
        ii. Bingham plastic
        iii. power-law fluid
      e. non-Newtonian fluids in an annulus
    3. *cf.* derivations in *FTI*
      a. Bingham plastic in slit (only gravity, no pressure driving force)
      b. power-law fluid in slit (no pressure driving force)
    4. conditions for suspending solid particles in Bingham plastic

I. Laminar flow of Newtonian and non-Newtonian fluids

A. Fluid properties: Newtonian fluid

1. Thought experiment

F/A =

$\Phi''_{p_y,x} = \tau_{yx} =$

Units:

SI unit of viscosity: Pa s

2. Magnitudes of viscosity

3. Newton's law in FTI (see Eq. 2.4)

Trends of viscosity with t, p

for pure, single-component liquids:

for gases:

for crude oils with dissolved gas

4. Meaning of shear stress in tensor form

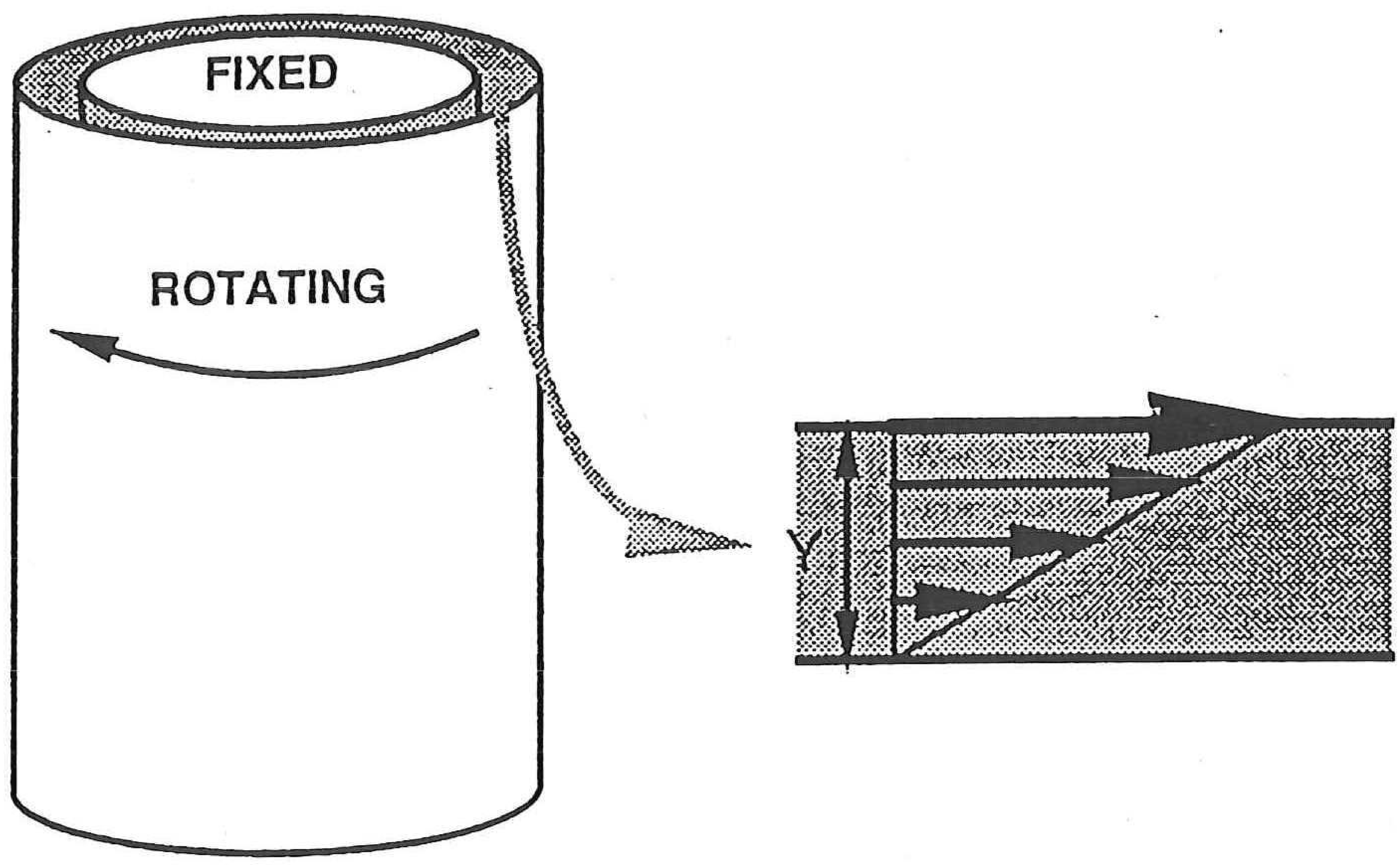
$$\Phi''_m = \tau_{yx} =$$

5. How one can measure viscosity without infinite plates

# <u>INFINITE</u> PARALLEL PLATES ARE IMPOSSIBLE, OF COURSE.

THEN HOW CAN ONE MEASURE VISCOSITY?

USING CONCENTRIC CYLINDERS. IF GAP WIDTH$\rightarrow$0, GAP APPROXIMATES PLANAR GEOMETRY.




IF GAP WIDTH $\nrightarrow$0, NEED CORRECTION FACTORS

## . . . THE IDEA BEHIND "FANN" VISCOMETER

## I. B. II. outline of shell momentum balance approach (BSL ch. 2)

**Procedure:** (*cf.* BSL section 2.1)

1. SELECT COORDINATE SYSTEM; DEFINE CONTROL VOLUME
2. STATE BOUNDARY CONDITIONS *
3. PERFORM MOMENTUM BALANCE **
4. THICKNESS → 0 (→ dif. eq. for $\tau$)

a) (optional): solve dif. eq. for $\tau$, apply b.c. - IF b.c. applies to $\tau$ alone

5. RELATE $\tau$ TO dv/dx (apply constitutive equation)
6. SOLVE DIF. EQ. FOR v; APPLY B.C. *
   a) (optional) COMPUTE w (mass rate of flow), etc.

## * BOUNDARY CONDITIONS (*cf.* BSL section 2.1)

1. SPECIFY v AT SOLID SURFACE
   1a) FLUID v = SOLID VELOCITY AT SOLID WALL
2. SPECIFY $\tau$ AT FLUID SURFACE
   2a) IN LIQUID, $\tau = 0$ AT GAS I.F.
   2b) $\tau$, v CONTINUOUS ACROSS LIQUID/LIQUID I.F.
3. $\tau$, v NOT INFINITE ANYWHERE WITHIN CONTROL VOLUME

*"ALL BOUNDARY CONDITIONS ARISE FROM NATURE"*
(i.e., from problem statement)

## ** ELEMENTS OF MOMENTUM BALANCE

MOMENTUM FLUX ($\alpha$ area); called "$\phi$" tensor in BSL (sect. 1.7)

1. CONVECTION OF MOMENTUM THRU SURFACE ($\rho vv$)
2. SHEAR STRESS $\tau$ ON SURFACE ("molecular transport of momentum")
3. PRESSURE PRESSING INWARD ON SURFACE - p

MOMENTUM "GENERATION" or "SOURCE" ($\alpha$ volume)

4. BODY FORCES WITHIN VOLUME

MOMENTUM ACCUMULATION ($\alpha$ volume) (not at steady state!)

5. ACCELERATION OF SYSTEM MASS - $\partial(\rho\, v)/\partial t$

I. B. 1. continued

Key elements in shell balances:

2. Notes on examples in *FTI*

# Newtonian fluid in annulus

An alternate derivation for $v_z(r)$:

The derivation of $v_z(r)$ for Newtonian flow in an annulus in BSL section 2.4 makes use of a clever change of variable in equation 2.4-3. Most students (and probably most professors) would not think of making this change. It is not necessary for solving this problem. You should be able to solve this problem by the straight-ahead method illustrated here. The equation numbers here, after 2.4-2, have no particular relation to the equation numbers in section 2.4.

Start with equation 2.4-2:

$$\tau_{rz} = \left(\frac{P_0 - P_L}{2L}\right) r + \frac{C_1}{r} \qquad (2.4\text{-}2)$$

Equate $\tau_{rz}$ from Newton's law of viscosity with $\tau_{rz}$ from the momentum balance:

$$\tau_{rz} = \left(\frac{P_0 - P_L}{2L}\right) r + \frac{C_1}{r} = -\mu \frac{dv_z}{dr} \qquad (2.4\text{-}3a)$$

$$\frac{dv_z}{dr} = -\left(\frac{P_0 - P_L}{2\mu L}\right) r - \frac{C_1}{\mu r} \qquad (2.4\text{-}4a)$$

$$v_z = -\left(\frac{P_0 - P_L}{2\mu L}\right) \frac{r^2}{2} - \frac{C_1}{\mu} \ln r + C_2 \qquad (2.4\text{-}5a)$$

**B.C.: $v_z = 0$ at $r = R$** (2.4-6a)

$$0 = -\left(\frac{P_0 - P_L}{2\mu L}\right) \frac{R^2}{2} - \frac{C_1}{\mu} \ln R + C_2 \qquad (2.4\text{-}7a)$$

$$C_2 = \left(\frac{P_0 - P_L}{2\mu L}\right) \frac{R^2}{2} + \frac{C_1}{\mu} \ln R \qquad (2.4\text{-}8a)$$

inserting this into Eq. 2.4-5a gives

$$v_z = \left(\frac{P_0 - P_L}{2\mu L}\right) \frac{R^2}{2} \left(1 - \left(\frac{r}{R}\right)^2\right) + \frac{C_1}{\mu} \ln\left(\frac{R}{r}\right) \qquad (2.4\text{-}9a)$$

**B.C.: $v_z = 0$ at $r = \kappa R$** (2.4-10a)

$$0 = \left(\frac{P_0 - P_L}{2\mu L}\right) \frac{R^2}{2} \left(1 - \left(\frac{\kappa R}{R}\right)^2\right) + \frac{C_1}{\mu} \ln\left(\frac{R}{\kappa R}\right) \qquad (2.4\text{-}11a)$$

$$\frac{C_1}{\mu} \ln\left(\frac{1}{\kappa}\right) = -\left(\frac{P_0 - P_L}{2\mu L}\right) \frac{R^2}{2} \left(1 - \kappa^2\right) \qquad (2.4\text{-}12a)$$

$$C_1 = -\frac{\mu}{\ln\left(\frac{1}{\kappa}\right)} \left(\frac{P_0 - P_L}{2\mu L}\right) \frac{R^2}{2} \left(1 - \kappa^2\right) \qquad (2.4\text{-}13a)$$

3.6

Plug this expression for $C_1$ into Eq. 2.4-9a:

$$v_z = \left(\frac{P_0 - P_L}{2\mu L}\right)\frac{R^2}{2}\left(1 - \left(\frac{r}{R}\right)^2\right) - \left(\frac{\mu}{\ln\left(\frac{1}{\kappa}\right)}\left(\frac{P_0 - P_L}{2\mu L}\right)\frac{R^2}{2}\left(1-\kappa^2\right)\right)\frac{1}{\mu}\ln\left(\frac{R}{r}\right) \tag{2.4-14a}$$

Group terms together with common factor:

$$v_z = \left(\frac{P_0 - P_L}{4\mu L}\right)R^2\left(1 - \left(\frac{r}{R}\right)^2\right) - \left(\frac{P_0 - P_L}{4\mu L}\right)R^2 \ln\left(\frac{R}{r}\right)\frac{1}{\ln\left(\frac{1}{\kappa}\right)}\left(1-\kappa^2\right) \tag{2.4-15a}$$

$$v_z = \left(\frac{(P_0 - P_L)R^2}{4\mu L}\right)\left(1 - \left(\frac{r}{R}\right)^2 - \frac{\left(1-\kappa^2\right)}{\ln\left(\frac{1}{\kappa}\right)}\ln\left(\frac{R}{r}\right)\right) \tag{2.4-17a}$$

This is Eq. 2.4-14 of BSL.

I. B. 4. Limits to laminar flow in terms of Re



## III. F. Assumptions Behind Derivations in BSL Ch. 2 (laminar flow of Newtonian fluid)

- "slow," rectilinear flow (except flow around sphere)
- incompressible ($\rho$ constant), Newtonian ($\mu$ constant) fluid
- steady state ($v(\underline{x})$ independent of t)
- ignore entrance and exit effects
- no slip at walls (v = wall velocity at wall; "BC type 1")
- fluid = continuum

Assumptions break down when flow too fast: "turbulence"
Quantify conditions for breakdown in "Reynolds number:"

$$\mathrm{Re} \equiv \frac{(\text{length})(\text{velocity})(\text{density})}{\text{viscosity}} \sim \frac{\text{inertial effects}}{\text{viscous effects}} \sim \frac{\rho V^2}{(\mu V/L)}$$

| Type of Flow | Def. of Re | Trans. to turbulence | Reference |
|---|---|---|---|
| Horizontal, infinite plates (BSL Fig. 1.1-1) | $\frac{YV\rho}{\mu}$ | 3000 | Schlichting, Boundary Layer Theory, 1979, p. 590 |
| Falling Film | $\frac{4\delta\langle v_z\rangle\rho}{\mu}$ | 20 | BSL, p. 46 maybe also in FT I? |
| Slit | $\frac{4B\langle v_z\rangle\rho}{\mu}$ | 2300 | White, *Fluid Mechanics* 1979, p. 433 |
| Circular Tube | $\frac{D\langle v_z\rangle\rho}{\mu}$ | 2100 | BSL, p. 52 |
| Annulus | $\frac{2(R-\kappa R)V\rho}{\mu}$ | 2000 | BSL, p. 56 |
| Flow Around Sphere | $\frac{D_{sph}V_{fl}\rho_{fl}}{\mu_{fl}}$ | 0.1 * | BSL, p. 61 |

* computed $V_\infty \equiv V_{fl}$ is within about 10% of true value for Re $\leq 1$

*These values apply only to Newtonian Fluids!*