"Complex" Heat-Transfer Problems

[redacted]

1. a) In this case we perform a macroscopic balance on the cube.
accumulation: $(\mathrm{Vol}) \cdot \rho \hat{C}_p \frac{dT}{dt}$
heat transfer w/ surroundings: $-(\mathrm{Area})\, h\, (T-T_f)$

$V\rho\hat{C}_p \frac{dT}{dt} = -Ah(T-T_f)$ (note minus sign means since cube is warmer than fluid, it will cool off ($dT/dt<0$)).

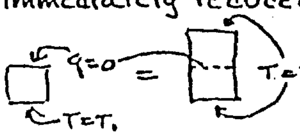
$$\Rightarrow \frac{dT}{dt} = -\frac{Ah}{V\rho\hat{C}_p}(T-T_f)$$

$$\frac{dT}{T-T_f} = \frac{d(T-T_f)}{T-T_f} = d\ln(T-T_f) = -\frac{Ah}{V\rho\hat{C}_p}dt \rightarrow \ln(T-T_f) = -\frac{Ah}{V\rho\hat{C}_p}t + C_1$$

C_1 given by $(T-T_f)$ at $t=0 \rightarrow$ $\ln\left[\frac{T-T_f}{(T-T_f)_{t=0}}\right] = -\frac{Ah}{V\rho\hat{C}_p}t$

$$\ln\left[\frac{T-0}{100-0}\right] = -\frac{[3(0.1)^2](2{,}500)(40)}{(0.1)^3(11{,}340)(125.7)} = -2.10 \; ; \rightarrow T = 12\,°C$$

b) $\alpha = k/(\rho\hat{C}_p) = (34.6)/[(11{,}340)(125.7)] = 2.42\cdot10^{-5}\ m^2/s$
This is a 3D problem. In the left-right dimension, both sides are insulated, so there is no conduction in that direction. In the top-bottom (z) direction, one side is insulated; we assume the other side is immediately reduced to 0°C. Because 1 side is insulated, we double the width in that direction. $2b = 0.2\ m$, $b = 0.1\ m$.



$\alpha t/b^2 = 0.0968$. From Fig 11.1-1, at $y=0$ (corresponding to top, insulated surface, $(T_1-T)/(T_1-T_0) \doteq 0.94$.
For the front-back (y?) dimension, both surfaces are at T_1; $b = 0.05\ m$, $\alpha t/b^2 = (2.42\cdot10^{-5})(40)/(0.05)^2 = 0.38$. From Fig 11.1-1, at $(y/b) = 0$ (plane through middle of cube), $(T_1-T)/(T_1-T_0) \doteq 0.48$.
For cube, slowest spot to cool is line on top, midway between front + back. There, after 40 s., $(T_1-T)/(T_1-T_0) = (0.94)(0.48) = 0.45 = (0-T)/(0-100)$.
$T = 45\,°C$.

c,d) The calculated cooling for internal conduction (part (b)) is slower. For modes in series, that's the best answer. The true cooling rate must be slower yet; i.e., T must be higher than calculated in (b).

2. Dimensions of cube: $(2b)^3\rho = 1 = (2b)^3(11{,}000) \Rightarrow b = 0.0225\ m$ (half-width)
Dimensions of sphere: $\frac{4}{3}\pi R^3\rho = 1 = \frac{4}{3}\pi R^3(11{,}000) \Rightarrow R = 0.0279\ m$
sphere. want $(T_1-T)/(T_1-T_0) = 0.5$ at center. From Fig 12.1-3, $\alpha t/R^2 \doteq 0.14$.
$\alpha = k/(\rho\hat{C}_p) = 35/[(11{,}000)(130)] = 2.45\cdot10^{-5}$; $t = (0.14)(0.0279)^2/(2.45\cdot10^{-5}) = 4.45\,s$.
cube: want $(T_1-T)/(T_1-T_0) = 0.5$. This cube is product of 3 slabs with $b = 0.0225$.
We want $[(T_1-T)/(T_1-T_0)]^3_{slab} = 0.5$, or $[(T_1-T)/(T_1-T_0)]_{slab} = \sqrt[3]{0.5} = 0.794$. From Fig 12.1-1, $(T_1-T)/(T_1-T_0) = 0.794$ at $y/b = 0$ when $\alpha t/b^2 \doteq 0.18$
$t = (0.18)(0.0225)^2/(2.45\cdot10^{-5}) = 3.72$ sec.

3. a) A heat balance on either the cube or the sphere gives

$$-h A (T - T_{oil}) = \rho \hat{C}_p (vol)\, dT/dt = \rho \hat{C}_p (vol)\, d(T - T_{oil})/dt$$

$$\rightarrow \exp\left(-\frac{h A}{\rho \hat{C}_p\, vol}\right) t = \frac{(T - T_{oil})}{(T - T_{oil})_{t=0}} = \frac{-50}{-100} = 0.5$$

For sphere, $A = 4\pi R^2$, $Vol = \frac{4}{3}\pi R^3$

$$0.5 = \exp\left[-\frac{(500)(4\pi (0.0279)^2)\, t}{(11{,}000)(130)((4/3)\pi (0.0279)^3)}\right] = \exp(-0.0376\, t)$$

$t = 18.4$ sec

For cube $A = 6(2b)^2$, $vol = (2b)^3$

$$0.5 = \exp\left[-\frac{500\,(6)(2 \cdot 0.0225)^2}{(11{,}000)(130)(2 \cdot 0.0225)^3}\, t\right] = \exp(-0.0466 t) \quad t = 14.9 \text{ sec}$$

b) The conduction within the solid + convection to the surface are "in series." Therefore the slower estimate is the most accurate. In this case, it's the assumption made in this PS, that convection heat-transfer controls the process.