ta3220
Friction-factor problems, part 1

1. [not applicable]
2. An incompressible polymer solution is to be injected at a rate of $800 \mathrm{bbl} /$ day into a cylindrical formation 60 ft thick and infinite in radial extent with permeability 600 md and porosity $22 \%$. ( $1 \mathrm{bbl}=42$ gal.) The polymer solution behaves like a powerlaw fluid with an exponent $\mathrm{n}=0.6$. In the laboratory, this polymer solution has an effective viscosity of 30 cp when injected into the same rock at a rate of $u \equiv(\mathrm{Q} / \mathrm{A})=$ $1 \mathrm{ft} /$ day (i.e., $\mathrm{u}=1 \mathrm{ft}^{3} /\left(\mathrm{ft}^{2} /\right.$ day)). Estimate the effective viscosity the solution as it flows outward in radial (cylindrical) flow at a distance of 1 ft and 50 ft from the wellbore, by following the following steps:
a) Determine $u$ at the two distances from the well from the injection rate.
b) Determine the effective viscosity at these values of $u$.
3. Scale has reduced the diameter of a pipe from $R_{1}$ to $R_{2}=0.5 R_{1}$. If $\Delta \mathbf{P} / L$ were the same as before, how much would this reduce the flow rate Q for
a) laminar flow of a Newtonian fluid
b) laminar flow of a power-law fluid with $\mathrm{n}=0.5$
c) highly turbulent flow of a Newtonian fluid in a tube with given $\mathrm{k} / \mathrm{D}$, where the friction factor is nearly independent of flow rate.
4. A team of underwater explorers, led by Dr. Robert Ballard, found the sunken ship Titanic in 1986. The sinking of the massive, luxurious, supposedly "unsinkable" ocean liner Titanic after hitting an iceberg is part of American mythology. The explorers found that the ship was sunk by a horizontal crack, perhaps 200 ft long, between plates in its hull. At hearings after the sinking, an engineer estimated that, after the ship hit the fatal iceberg, water poured into the ship at an initial rate of 0.189 $\mathrm{m}^{3} / \mathrm{s}\left(24,000 \mathrm{ft}^{3} / \mathrm{hr}\right)$. Suppose the hull were 2 in . thick, that water density and viscosity were $1037 \mathrm{~kg} / \mathrm{m}^{3}$ and 0.001 Pa s , and that the slit were 10 ft below the water line. How wide a slit (2b) would explain this rate of water flow into the ship?

We don't know what roughness factor of $\mathrm{k} / \mathrm{D}$ to apply to this problem; make some reasonable assumption and state it clearly. How much difference does it make what value of roughness one assumes?

crack 200 ft long, located 10 ft below water line


See additional problems on back.
5. a) What $\Delta \mathbf{P}$ (psi) would be required to pump water, with viscosity 1 cp and density $1 \mathrm{~g} / \mathrm{cm}^{3}$, at a rate of $100 \mathrm{~m}^{3} /$ day through a horizontal 5000 - ft section of 1 -in dia. pipe with a roughness height $\mathrm{k}=1 / 1000$ inch?
b) Suppose by introducing a trace of chemical one could suppress turbulence without altering density or viscosity of the water. What would $\Delta \mathrm{P}$ be for the same flow rate Q in this case?
6. Now for another bit of review: An engineer is applying a foam (a Bingham plastic with yield stress $\tau_{0}$ ) in a uniform layer to the outside of a pipe of radius R , held vertically. He wants to know how thick a layer of foam he can apply without having the foam flow down the outer surface. Derive a formula for $\mathrm{R}_{2}$, the maximum outer radius of a foam layer that would not flow, for this problem, in terms of R and fluid properties $\tau_{0}$ and $\rho$. Solve this problem in the following sequence:
a) Derive a momentum balance in cylindrical coordinates for this problem. You do not have to repeat any part of a derivation from BSL that applies directly to this problem; or you can start from the beginning, if you like. Note that there is no pressure difference in the z direction because the foam is exposed to the atmosphere all along its length.
b) Derive an differential equation for the shear stress $\tau_{\mathrm{rz}}$ from the momentum balance.
c) Integrate to obtain an algebraic equation for $\tau_{\text {rz }}$. From the physical description of the problem, determine a boundary condition you can apply to determine the constant of integration in this equation.
d) Apply the condition for onset of shearing of a Bingham plastic to determine the condition for the onset of flow. (Hint: the maximum of $\left|\tau_{\mathrm{rz}}\right|$ occurs at $\mathrm{r}=\mathrm{R}$ ).

## top view


side view


## Friction-factor problems, part 2

1. What is the settling velocity of viscous, heavy-oil droplets of density $0.95 \mathrm{~g} / \mathrm{cm}^{3}$ and diameter 1 mm in water (density $1000 \mathrm{~kg} / \mathrm{m}^{3}$, viscosity 0.001 Pa s )? The crude oil droplets are so viscous that they may be treated as virtual solids.
2. "Ball sealers," round rubber spheres, are used to seal off perforations in some well treatments. The idea is the sphere just flows along with the injected liquid until it reaches the perforation, then squeezes into the perforation and plugs it off. A major problem with ball sealers is that they might float or sink in water rather than going to the perforation of interest. Suppose such a sphere has density $900 \mathrm{~kg} / \mathrm{m}^{3}$ and is in water, with density $1000 \mathrm{~kg} / \mathrm{m}^{3}$ and viscosity 0.001 Pa s . One desires to select particle diameter so that it does not rise any faster than $1 \mathrm{~cm} / \mathrm{s}(0.01 \mathrm{~m} / \mathrm{s})$. What sphere diameter should be selected?
3. An operator has created a fracture 60 ft high, cleanly propped $1 / 8$ in wide with 20-40 mesh sand (particle diameter $\sim 0.6 \mathrm{~mm}$ ). The void fraction (porosity) within the fracture is $30 \%$.
a) What is the permeability of this sand at low Re when packed with this porosity?
b) What would be the total flow rate of liquid through the fracture across its 60 ft height if the liquid were water (viscosity 1 cp , density
 $1.0 \mathrm{~g} / \mathrm{cm}^{3}$ ) and $\Delta \mathbf{P} / \mathrm{L}$ were 10 $\mathrm{psi} / \mathrm{ft}$ ? Use the correlation for "packed beds" from BSL. Do not assume low Re!
