Determination of Thermal Properties of a Solid (ta3220)

(revised 8 Feb. 2013)

Set-Up

A heating element occupies the entire central axis of a rectangular solid block composed of either concrete or rock salt. The concrete block is approximately 15 x 15 x 30 cm and the salt block is 11.2 x 11.2 x 30 cm. As electric current flows through the heating element, the solid heats up and a temperature profile is set up in the solid. For a given rate of heat release by the element, the temperature profile in the solid over time depends on the thermal properties of the solid. One can infer those properties from the temperature profile.

The diameter of the heating element is 1.2 cm. In this experiment temperature is measured at 5 thermocouples at radial distances from the *outer surface* of the heating element as follows: for the concrete block, 0.5, 1, 2, 4 and 6 cm (distances from central axis 1.1, 1.6, 2.6, 4.6 and 6.6 cm); for the salt block, 1.0,1.3,1.7, 2.2 and 2.8 cm from the outer surface of the heating element, or 1.6, 1.9, 2.3, 2.8, 3.4 cm from the central axis. The thermocouples can be read from the knobs on the switchboard, as will be explained at the laboratory. Note that the thermocouples are separated enough from each other vertically that hole for one element does not significantly distort the conduction of heat to thermocouples further out from the center. The power delivered to the heating element (and therefore the rate of heat release by the heating element) is regulated by a transformer.

Experimental Procedure:

Each lab group will carry out the measurements on one solid block, either the concrete one or the salt block.

1. Determine the readings of the five thermocouples in the block and write them down. The initial temperature distribution should be nearly homogeneous.
2. Set the transformer to 100 and start the stopwatch. The graphs and tables in Appendix A give the relation between the setting and the total rate of release of thermal energy Q.
3. Record the readings of the 5 thermocouples every 2 minutes for 16 minutes; use the table in Appendix C.
4. Reset the transformer to zero (i.e., shut off the heater). Note the exact time at which the heat is turned off (even if it is not exactly 16 minutes).
5. Continue to record the readings of the thermocouples every 2 minutes for at least another 16 minutes, using the table in Appendix C

Theory

We approximate the square concrete block 15 cm on a side as a cylinder of the same volume and mass (radius ≅ 8.5 cm); for the salt block, this gives a radius of 6.3 cm. We assume that the heat loss to air at the outer radius of the block (R_1) is insignificant (stagnant air is a poor heat-transfer medium), and therefore approximate the outer surface as perfectly insulated. We assume the uniform heat flux at the inner surface (surface of the heating element, at radius R_o) is q_o. q_o is the total rate of heat release by the heating element: that is, it is Q (which is known), divided by the surface area of the heating element (a cylinder of radius 0.6 cm, 30 cm long). We likewise approximate the top and bottom surfaces as perfectly insulated; there is therefore no gradient of temperature, and no heat conduction, in the z direction.

The partial differential equation for dimensionless temperature as a function of dimensionless position and dimensionless time is derived in Appendix D. In a previous year, students solved this equation numerically, but you are not required to do so. Plots of dimensionless temperature as a function of dimensionless position and dimensionless time are given in Appendix B. Note that separate plots are needed for the concrete and salt blocks, because the dimensionless outer radii of the blocks differ between the blocks; hence the geometry of the two blocks is slightly different and the mathematical solution also slightly different. Note that dimensionless position of each thermocouple is known at the start, but dimensionless temperature and dimensionless time must be determined from a fit of the model to experimental data. In the process, you determine the thermal properties of the solid.

Another parameter you will need to determine is the heat flux q_o at the surface of the heating element. As noted above, from the setting of the transformer and the chart in Appendix A you can determine the total rate of heat flow from the heating element Q. Next determine the surface area $\mathscr{A}$ of the heating element (it is a cylinder of radius 0.6 cm and its height is 30 cm, the height of the block). The heat flux q_o is then $Q/\mathscr{A}$.

Determining the Properties of the Solids from the Temperature Profile

This section pertains to the 16-min. period of heating of the solids.

As noted, the relation between real time and dimensionless time, and physical temperature and dimensionless temperature, are not known in advance because the properties of the solid are yet to be determined. (You can look up the properties of the solids on the internet, but note that "concrete" can vary widely from one batch to another, and we haven't even specified the composition of the "rock salt." Don't rely on a literature search to tell you the properties of the solids in advance. Once you are done, it might be enlightening to compare your results to the range of properties you find on the internet.) One could take all the temperature data simultaneously and solve for the parameters by minimizing some sort of measure of error between all the data and the model. The following is a simpler method.

In earlier years students were told to use the rate of penetration of the heat front at early times to determine the relation between dimensionless time and physical time. There are two problems with this approach: First, this works for one of the blocks (the thermal front takes more than 2 minutes to penetrate to the outer thermocouple), but for the other block the front has penetrated the whole block by the first temperature reading at 2 minutes. It's already too late to see this rate of penetration. Second, it takes a bit of time for the heating element to come up to its steady temperature. This distorts the readings, especially at short times. In other words, the short-time readings are the *least* reliable of all the data, because the system does not immediately fit the heat-flux specified in the inner boundary condition at t = 0.

Therefore, the following approach uses your temperature readings at 16 minutes to estimate the physical properties of the block.

Note that for $t_D > 20$ or 30, the difference in dimensionless temperature between the center and the edge (or between the innermost and outermost thermocouples, or between any two given thermocouples) is nearly independent of time. Check whether your data fit this prediction for the readings late in your experiment. Compare the difference in physical temperature between the innermost and outermost thermocouples after 16 minutes to this difference in T_D from the charts in Appendix B, and use Eq. 5 of Appendix D to calculate thermal conductivity k. An example calculation is given in Appendix E.

Now that you know k, you can take any temperature reading at any time to determine the remaining unknown, the product (ρ C_p). It makes sense to take the measured temperature of the innermost thermocouple at the latest time (16 minutes), because the temperature rise is largest here, so relative errors are probably smallest. (If you suspect that something went wrong with this thermocouple during the experiment, or for any reason you don't trust its reading at 16 minutes, use another thermocouple or another time.) Determine the physical temperature rise at that thermocouple. Given the value of k you estimated in the preceding paragraph, determine T_D at 16 minutes at this position. Look at the charts in Appendix B, and find the dimensionless time t_D that corresponds to this T_D at this position. From that value of t_D, and the real time (16 minutes), estimate α and then (ρ C_p) (you already know k) from Eq. 4 in Appendix D. An example calculation is given in Appendix E.

Now that you know k and α, you can use the dimensionless plots to predict what T(r, t) should be for *all* times. Plot T(r, t) derived from the plots in Appendix B (given your fitted values of α and k) for 4 min., 8 min. and 16 min. and compare them to your measurements at the five thermocouples; that is, put your data as points on the plots of T(r,t) for those times. If you don't get a good fit, adjust the values of k and α and see if you can get a better fit. (Hint: if the T profile of the block as a whole suggests the block is heating up too fast or too slowly, then adjust (ρ c_p); if the temperature difference across the block is too large or too small, adjust k.) Whether or not you get a good fit, discuss possible causes for a poor fit between data and the theory.

The Rest Period After Heating: Superposition

It is an important property of the partial differential equation solved in Appendix D (and many similar equations, for diffusion and Darcy flow among other transport phenomena) that if two functions, call them $T_{D1}(r_D,t_D)$ and $T_{D2}(r_D,t_D)$, each satisfy the equation, then a new function $T_{D3}(r_D,t_D) \equiv [T_{D1} + T_{D2}]$ also solves the equation. (Check this out for yourself.) The trick in the technique called "superposition" is to find a T_{D1} and T_{D2} for which their sum satisfies the *boundary conditions*. We already know it would satisfy the differential equation.

Consider the following choice to describe the after the heating element is turned off. Call the dimensionless time at which the heating element is turned off t_D^o. Let $T_{D1}(r_D,t_D)$ be the solution plotted in Appendix B, i.e. during the period of heating. Let $T_{D2} \equiv [-T_{D1}(r_D,(t_D - t_D^o))]$. Note that T_{D2} has a time scale that starts at zero at the moment the heat is turned off. Then let

$$\begin{aligned} T_{D3} &\equiv T_{D1}(r_D,t_D) \text{ for } t_D < t_D^o \\ &\equiv [T_{D1}(r_D,t_D) - T_{D1}(r_D,(t_D - t_D^o))] \text{ for } t_D > t_D^o \quad . \end{aligned} \tag{1}$$

Because of the principle of superposition (i.e. because of the form of the partial differential equation), T_{D3} satisfies the partial differential equation. Note that it also satisfies the initial and boundary conditions: uniform initial temperature, insulated boundaries above and below and at the outer radius at all times, fixed heat flux for $t_D < t_D^o$ and zero heat flux (Q-Q) for $t_D > t_D^o$. An example calculation for physical temperature during the rest period is given in Appendix E.

Using Eq. 1, plot temperature T(r,t) for 4, 8 and 16 min. *after* the heat is turned off, and plot the actual data on these same plots. Comment on how good the fit is, and if the fit is not perfect discuss the possible causes of imperfection in the fit.

If the experiment goes smoothly, t_D^o should be close to 16 min. (It will probably be a little later, because you'll be taking temperature measurements right at 16 min.) But whatever the time at which you turn off the heating element, of course use *that* time, not 16 min., for t_D^o in your calculations. See example in Appendix E.

Analogy to Well-Testing

The procedure you follow here is similar to that of "well testing" to determine flow properties of a geological formation, i.e. an aquifer or oil or gas reservoir. The differential equation for unsteady Darcy flow of a slightly compressible fluid is identical to that for unsteady heat conduction in a solid. In well testing, too, one uses superposition: measuring pressure after shutting off injection or production from the well, just as here you measure temperature after shutting off the heater. One difference is that in this experiment we measure temperature as a function of position and time within the solid. In a well test, the analogous measurements would be pressure in the formation as a function of position and time. We don't have pressure measurements from within the formation. Instead, in a well test one measures pressure within the well as a function of time. It is as though in this experiment one measured temperature in the heating element as a function of time instead of within the solid. One other difference is that in our heat-conduction experiment the outer radius is nearby. In a well test, the pressure might not begin to change at all at the outer radius of the reservoir during the test. The time for the temperature front in our experiment to penetrate to the outer radius is a few minutes. In a well test, the pressure wave may not penetrate to the outer reservoir radius at all during the test.

Report:

Each student must do a report. The report must contain the following:

1. A brief description of the apparatus and the procedure and purpose of the experiment. Nothing elaborate is needed; one-half page would suffice if you are concise.
2. The raw data for the solid block you worked with.
3. A description of how the data were used to fit α and k, including the fits to the theoretical curves.
4. Plots of (dimensional) temperature as a function of (dimensional) time corresponding to 4, 8 and 16 minutes of heating and 4, 8 and 16 minutes into the rest period. On this plot should appear the values computed from the dimensionless plots and the measured values at the five thermocouples. This implies 3 plots of T(r) for each value of t for the period of heating (with measured data included on the plots) and 3 plots of T(r) for each value of t for the period after the heat is turned off.

It's easiest to interpret this plot (i.e., compare it to the plots in Appendix B) if you plot all the thermocouples at each time (i.e., T(r) for fixed t), and see how T varies with r at a given time, rather than plot one thermocouple as a function of time.

5. A discussion of any mismatch between the measurements and the model, including the magnitude of the mismatch and possible causes.

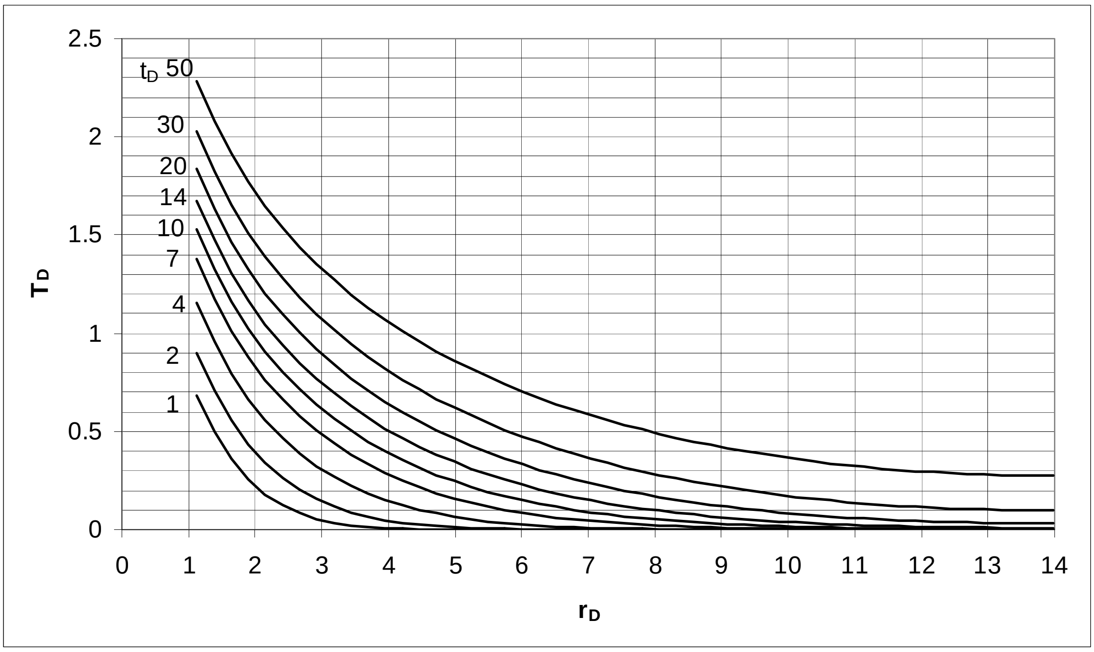
Appendix A: Chart for Determining Rate of Heat Release at Heating Element

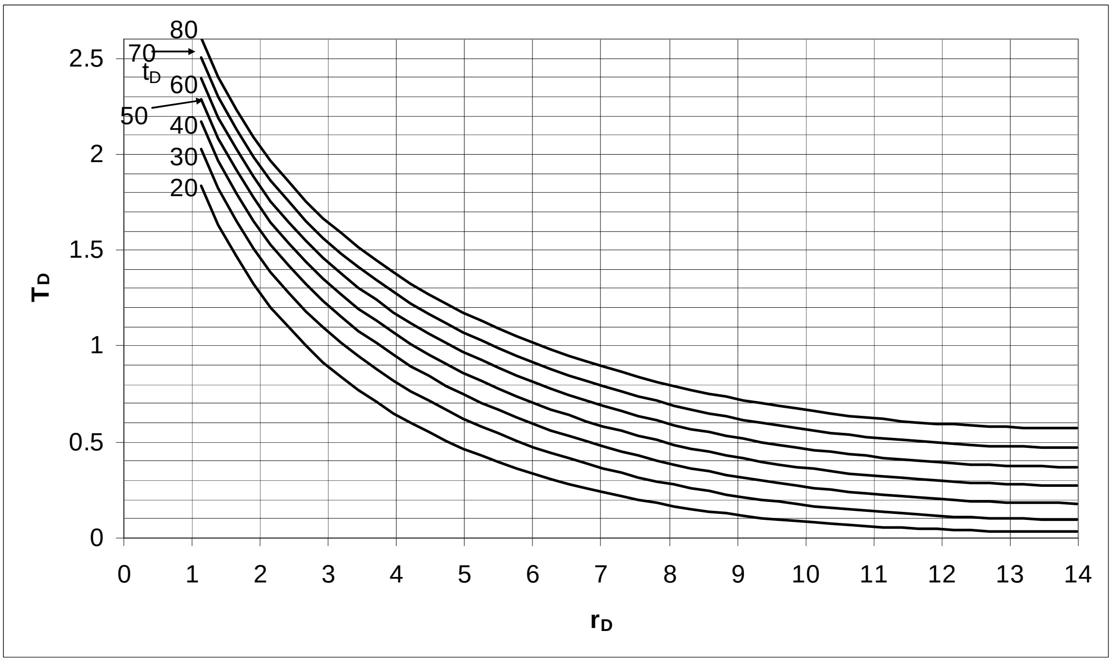
Verwarmingselementen praktikumproef

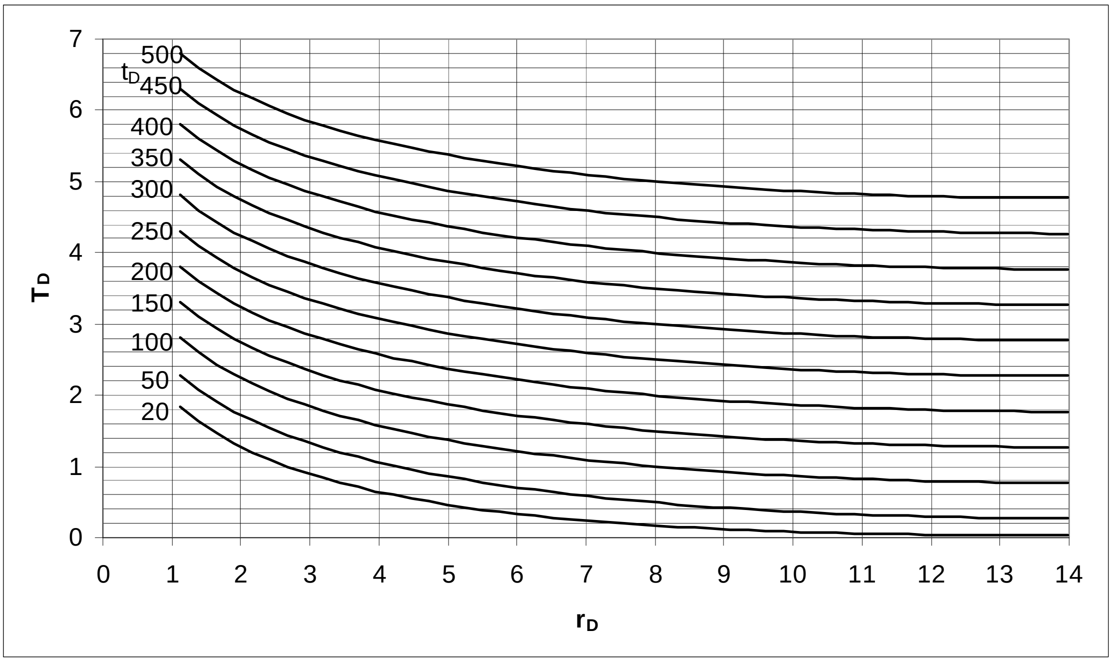
Gemeten weerstand: R1= 88.81 ohm

R1			
U (Volts)	I x10 (mA)	P (Watt)	R (Ohm)
0	0	0	
20	21.5	4.3	93.02326
40	43.3	17.32	92.37875
60	66.1	39.66	90.77156
80	87.8	70.24	91.11617
100	111.3	111.3	89.84726
120	132.8	159.36	90.36145
140	155.5	217.7	90.03215
160	177.5	284	90.14085
180	199.4	358.92	90.27081

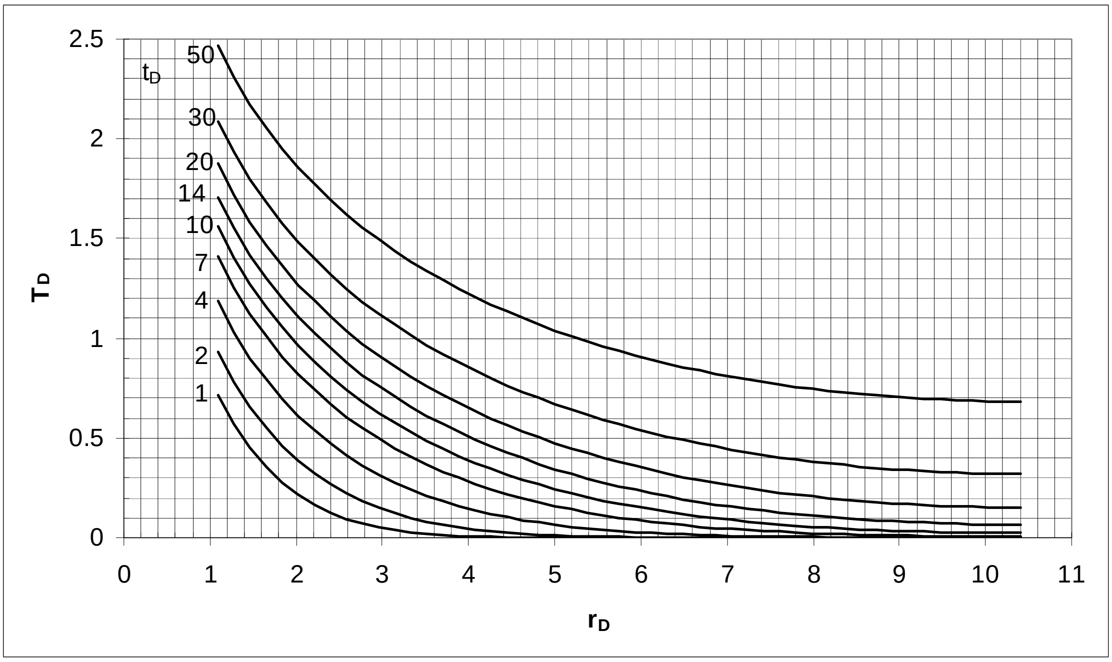
Appendix B: Dimensionless Temperature Plots for Concrete Block

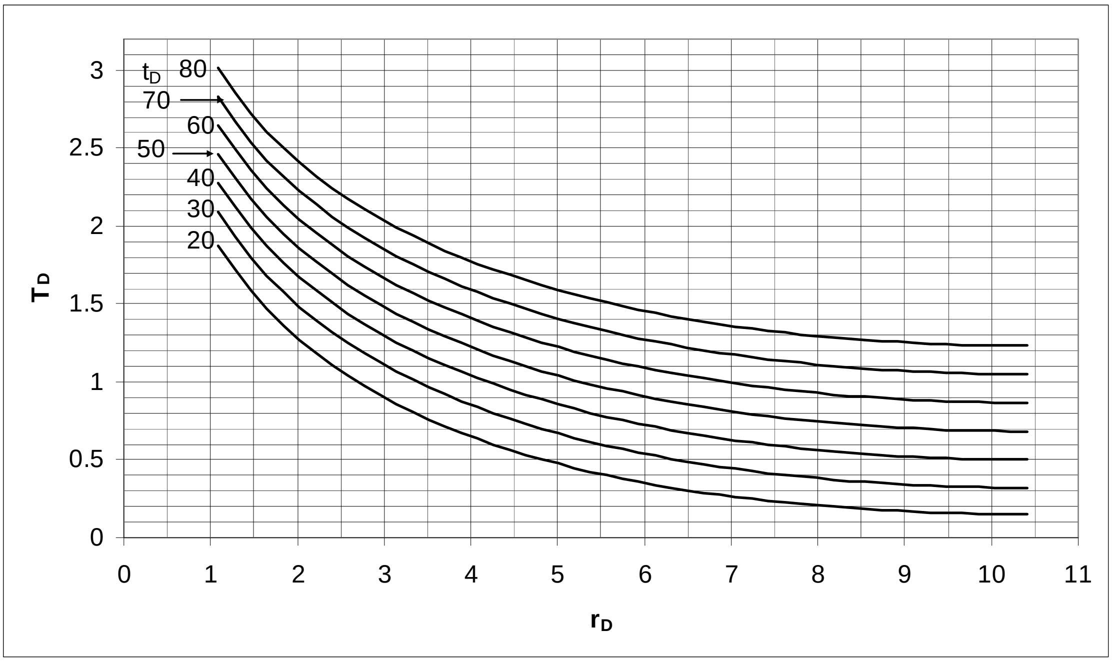




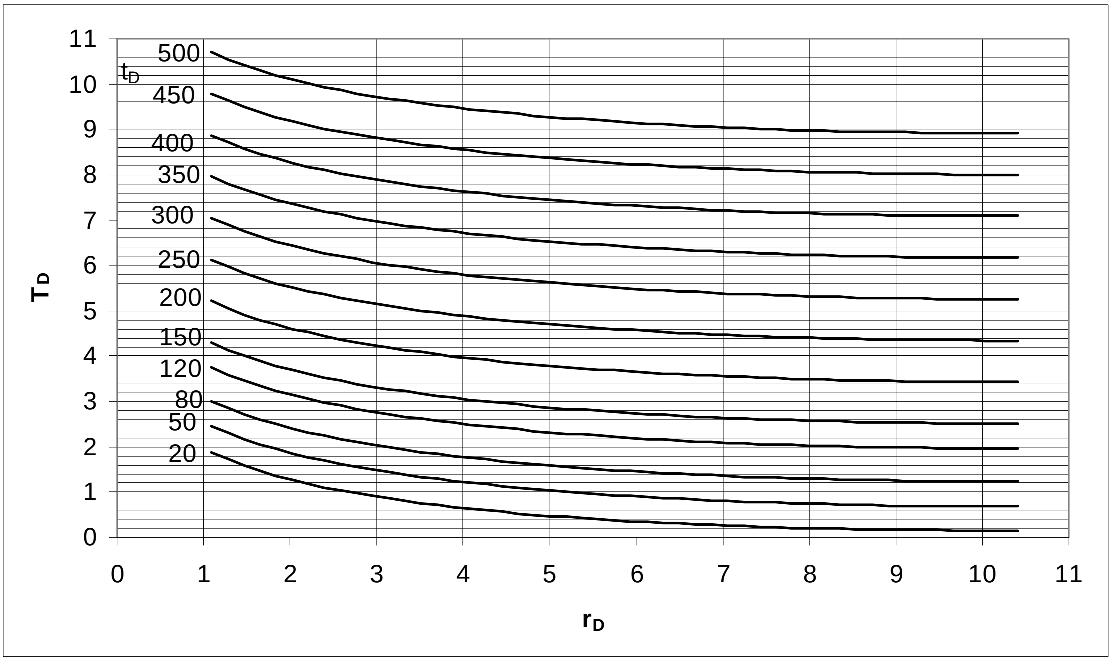


Appendix B2: Dimensionless Temperature Plots for Salt Block





t_D
80
70
60
50
40
30
20
T_D
r_D



t_D
500
450
400
350
300
250
200
150
120
80
50
20
T_D
r_D

Appendix C: Table for Entering Raw Data

Raw temperature data for period of heating

position	initial	2 min	4 min	6 min	8 min	10 min	12 min	14 min	16 min
1									
2									
3									
4									
5									

Raw temperature data for period after heating

position	initial	2 min	4 min	6 min	8 min	10 min	12 min	14 min	16 min
1									
2									
3									
4									
5									

NOTE: for salt block, the thermocouples are numbered 6 to 10.

Appendix D: Derivation of Governing Equation (following pages)
(adapted from 2009 lab report of Quirijn Noordoven, Marinus Dalm, Erika Deviese, Yolanda Kolenberg, with thanks)

Assumptions:

- Constant heat flux q_0 from surface at r = R_0 ;

$$q_0 = \frac{Q}{2\pi R_0 H} \tag{1}$$

  q_0 = heat flux [W/m^2]
  R_o= radius heating element [m]
  H = height heating element [m]

- Perfectly insulated boundary at r = R_1
- Assume no heat flux (perfect insulation) at the top and bottom flat surfaces.
- Uniform and constant properties k, ρ, C_p within the block.

Partial differential equation for radial (cylindrical) conduction:

$$\alpha \frac{1}{r}\frac{\partial}{\partial \mathrm{r}}\left(\mathrm{r}\ \frac{\partial T}{\partial r}\right) = \frac{\partial \mathrm{T}}{\partial \mathrm{t}} \tag{2}$$

where

α	Thermal diffusivity [m^2/s]
r	radius [cm]
T	temperature [K]
t	time [s]

With the following initial and boundary conditions:

T=T_0 at t=0 for all R

$-k\dfrac{\partial T}{\partial r} = q_0$ at r = R_0 for t>0 [constant heat flux from heater]

$-k\dfrac{\partial T}{\partial r} = 0$ at r = R_1 for t>0 [perfectly insulated outer boundary]

k ≡ Thermal conductivity [W m^{-1}K^{-1}]

<u>Now define dimensionless variables</u>

Let $r_D \equiv \dfrac{r}{R_0}$; $1 \le r_D \le \dfrac{R_1}{R_0}$ (3)

Equation (2) becomes now: $\dfrac{\alpha}{R_0^{\ 2}}\dfrac{1}{r_D}\dfrac{\partial}{\partial \mathrm{r_D}}\left(r_D\ \dfrac{\partial T}{\partial r_D}\right) = \dfrac{\partial T}{\partial t}$

Let $t_D = \dfrac{t}{t^*}$ (t* not yet defined)

Rewrite equation (2): $\frac{\alpha}{R_0{}^2}\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\ \frac{\partial T}{\partial r_D}\right)=\frac{\partial T}{t^*\partial t_D}$ → $\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\ \frac{\partial T}{\partial r_D}\right)=\frac{R_0^2}{\alpha t^*}\frac{\partial T}{\partial t_D}$

Let $t^* \equiv \frac{R_D{}^2}{\alpha}$

$$t_D \equiv \frac{\alpha t}{R_0{}^2} \qquad (4)$$

Rewrite equation (2): $\frac{1}{R_D}\frac{\partial}{\partial r_D}\left(r_D\ \frac{\partial T}{\partial r_D}\right)=\frac{\partial T}{\partial t_D}$

Let $T_D \equiv \frac{T-T_0}{T^*}$ [T* not yet defined]

The radial boundary conditions become:

$-k\frac{T^*}{R_0}\frac{\partial T}{\partial r}=q_0$ at $r = R_0$ for $t_D > 0$

$-k\frac{T^*}{R_0}\frac{\partial T_D}{\partial r_D}=0$ at $r = R_1$ for $t_D > 0$

Let $T^* \equiv \frac{R_0 q_0}{k}$ and

T_D becomes now: $T_D = \frac{(T-T_0)k}{R_0 q_0}$ (5)

Substitute equation (1) in eq. (5)

$$T_D = \frac{(T-T_0)k(2\pi R_0 H)}{R_0 Q}$$

Equation (2) becomes now: $\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\ \frac{\partial T_D}{\partial r_D}\right)=\frac{\partial T_D}{\partial t_D}$

with initial and boundary conditions

$T_D=0$ at $t_D=0$

$-\frac{\partial T_D}{\partial r_D}=1$ at $r_D=1$

$-\frac{\partial T_D}{\partial r_D}=0$ at $r_D=\frac{R_1}{R_0}$

Appendix E: Example calculations

1. Determining k from long-time temperature difference.

At long times, (t_D > about 20 or 30), the difference in T_D between any to fixed positions hardly changes further with time. Check that this is true in your data. For instance, the difference in temperature between the innermost and outermost thermocouples should be roughly constant in the later stages of the period of heating.

Pick any large value of t_D (say 500). Find the positions r_D of your innermost and outermost thermocouple, and read T_D off the chart in Appendix B for the two thermocouple locations. Subtract the two values. Suppose the result is 2 units in T_D. Compare this to your actual temperature difference between the same two thermocouples late in the experiment; suppose that difference is 20°C. Thus $(T-T_o) = 10\ T_D$. Then, from equation 5 in Appendix D,

$$T_D = \frac{(T - T_0)k}{R_0 q_0} \rightarrow T_D = \frac{10 T_D k}{R_0 q_0} \rightarrow 0.1 = \frac{k}{R_0 q_0} \quad .$$

From this you can determine k.

2. Determining ($\rho\ C_p$) from long-time temperature difference.

Now that you know k, you know the relation between T_D and $(T-T_o)$. You can use any thermocouple measurement to estimate the relation between t_D and t, but it makes sense to use the 16-min. measurement at the innermost thermocouple (unless you have some reason not to trust this measurement), because its value is largest and therefore small reading errors have the smallest effect.

Suppose, as above, that $(T-T_o) = 10\ T_D$. Suppose your innermost thermocouple is at $r_D = 3$. Suppose the actual $(T-T_o)$ reading at the innermost thermocouple at 16 minutes is 40°C. Because, as you've already calculated, $(T-T_o)=10\ T_D$, $T_D = 4$. We want to find the time t_D where $T_D=4$ at $r_D = 3$. If the block is the salt block (see charts in Appendix B), then I estimate (last chart in the appendix) that t_D is between 150 and 200; much closer to 200, though, and I would estimate $t_D \cong 188$ at 16 minutes (960 sec) - this is the value of t_D where the chart gives $T_D = 4$ at $r_D = 3$. In other words, $t_D \cong 188/960 = 0.196\ t$ (where t is in seconds, as required using SI units), and from Eq. 4 in Appendix D you can calculate ($\rho\ C_p$) for the block.

Using $(T-T_o)=10\ T_D$ and $t_D = 0.196\ t$ (in this hypothetical example), you can convert all your data into dimensionless form, and compare them to the charts, or convert any curve for dimensionless variables into dimensional quantities.

3. Using superposition during the rest period.

Continue with the values from the previous examples: $(T-T_o)=10\ T_D$ and $t_D = 0.196\ t$. For the data during the period of heating, you use the charts directly to estimate what temperature should be at each thermocouple.

For times after the heat is turned off, you use superposition. You are instructed to turn off the heater right at 16 minutes, but whatever happens you must note the actual time at which you turn off the heater. Suppose someone was asleep and you actually turned off the heater after 18 minutes instead. Suppose again that the inner thermocouple is at $r_D = 3$. After 4 minutes (240 s) of rest, 22 minutes (18+4 min., 1320 s) total, the dimensionless time since the start is 1320 x 0.196 = 258. The dimensionless time since the heater was turned off is 240 x 0.196 = 47. We look up those two values off the chart (continuing to use the salt block as an example). I estimate

$TD\ (r_D=3,\ t_D=258) \cong 5.4$

$TD\ (r_D=3,\ t_D=47) \cong 1.43$

and the actual dimensionless temperature is 5.4-1.43 ≅ 4.0. The physical temperature rise is 10 x 4.0 or 40°C.

After 16 minutes of rest, total time is 18+16=34 min. (2040 s); t_D = 400. The time of rest is 16 min., 960 s, t_D = 188. From the charts, I get

$TD\ (r_D=3,\ t_D=400) \cong 7.9$

$TD\ (r_D=3,\ t_D=188) \cong 4.0$

and the actual dimensionless temperature is 7.9-4.0 ≅ 3.9. The physical temperature rise is 10 x 3.9 or 39°C. You can repeat this procedure for every thermocouple at each requested time in the period of rest.