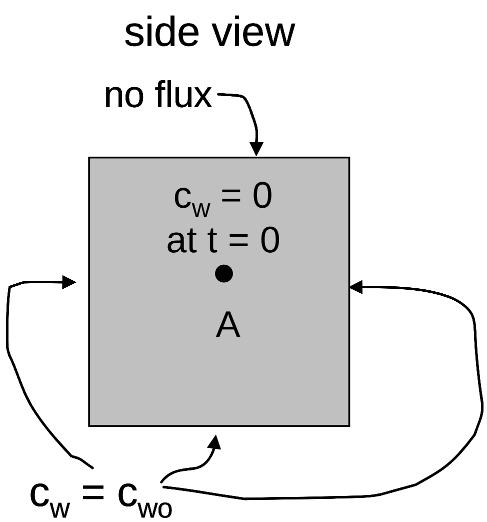
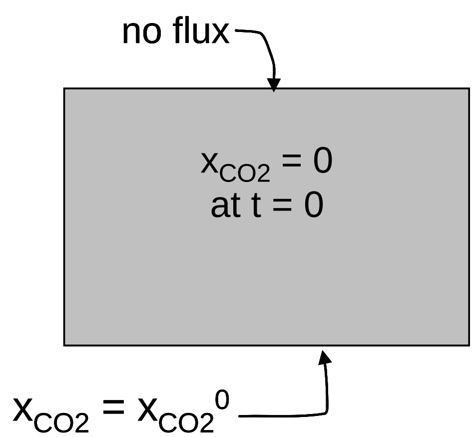
**ta3220 - Transport Phenomena**
**Mass transfer problem set**

1. A barrel previously used to store a toxic waste now contains crude oil. A film of toxic waste coats all surfaces of the barrel <u>except</u> the top; for $t \geq 0$ the oil in contact with these surfaces has a trace concentration $c_{wo}$ of a toxic chemical. Otherwise, at t=0, $c_w$=0 throughout the oil, and for $t \geq 0$ there is no mass flux across the top surface of the oil. The diffusion coefficient of this chemical in oil is $D_{wo} = 4 \times 10^{-10}$ m$^2$/s, and the oil can be assumed to be at rest (not flowing) during the period of interest. The barrel is 1 m high and 1 m in diameter. Estimate the concentration $c_w$ at point "A", in the center of the tank, after 8 years. Assume "simplified" Fick's law applies.

2. A sandstone layer in a reservoir is 1 m thick. There is an impermeable shale layer above and a thief zone of high permeability below. $CO_2$ is injected into the field and quickly fills the thief zone. Thus at time t=0 the oil at the lower surface of the layer absorbs a mole fraction $x_{CO2}{}^o$, in equilibrium with the $CO_2$ below, while the mole fraction of $CO_2$ in the oil within the layer is initially zero. Assuming the "simplified" Fick's law applies to the diffusion process and that the diffusion coefficient for $CO_2$ in oil is $2 \times 10^{-9}$ m$^2$/s, estimate the time required for the mole fraction of $CO_2$ in the oil at top of the layer to reach 75% of $x_{CO2}{}^o$. Assume no $CO_2$ diffuses into the shale above.

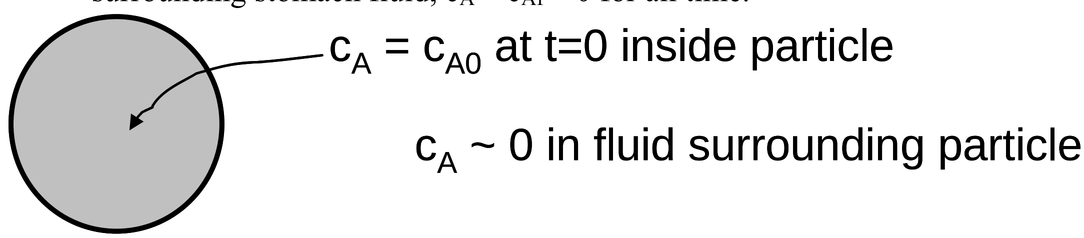



<u>Problem 1</u>




<u>Problem 2</u>

3. Some "time release" pharmaceuticals contain the drug as a solute in a spherical solid particle. Once in the patient's stomach, the drug diffuses to the surface of the particle and is released into the stomach fluids. Suppose the spherical particle has radius R and the initial concentration of drug in the sphere is $c_{Ao}$. In the surrounding stomach fluid, $c_A = c_{Af} \sim 0$ for all time.



a) Suppose the mass transfer between the surface of the spherical particle and the surrounding fluids controls the process. That is, $c_A$ is uniform in the particle, and mass transfer is given by $N_{Ar}=(k_x/c)\,(c_A-c_{Af})$, with $k_x$ and c constant. Derive an equation for the time at which the concentration of component A in the particle is 10% of its initial concentration.

b) Now suppose that diffusion within the particle controls the process, i.e., that transfer from the surface to the surrounding fluid is instantaneous. The diffusion coefficient for A in the particle is $D_A$ and is constant, and the "simplified" Fick's law applies. Give an equation for the time until concentration in the center of the particle reaches 10% of the initial concentration.

c) Give a formula for how to estimate which assumption is more accurate, (a) or (b), as a function of the parameters of the problem.

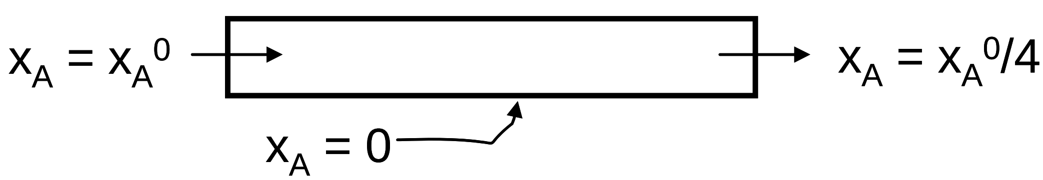
4. An engineer adopts the following model for acidization near the wellbore: A dilute solution of acid A in water flows into a tube 100 μm in diameter at a velocity of 100 ft/day ($3.5 \times 10^{-4}$ m/s) with a mole fraction of acid $x_A^0$. The acid is completely consumed as it contacts the tube wall (i.e., $x_A=0$ at the tube wall). The engineer has looked up the following solution properties, although he's not sure which are relevant:

| | | |
|---|---|---|
| $\rho = 1000$ kg/m$^3$ | $\hat{C}_p = 4180$ J/(kg K) | $\mu = 0.001$ Pa s |
| $D_{AB} = 4 \times 10^{-9}$ m$^2$/s | $k = 0.60$ W/(m K) | $c = 55.6$ kgmol/m$^3$ |

You may assume that these properties do not change as the acid is consumed. Because the acid is dilute, you may assume that the "simplified" Fick's law applies.

a) How far does the acid solution travel down the tube until 75% of the acid is consumed?

b) How would the answer to part (a) change if $D_{AB}$ were $4 \times 10^{-11}$ m$^2$/s?



5. A layer of solid ("scale") is slowly being deposited on the inner surface of a pipe. Fluid supersaturated in chemical "A" enters the pipe at concentration $c_{Ab1}$. At the wall (r=R) the concentration is maintained at a lower, uniform concentration $c_{Ao}$ by solid deposition on the wall. The pure solid deposited by solute A has molar density (i.e., density divided by molecular weight) $c_A$*. Derive a mathematical expression for dR/dt, the rate at which R is being reduced by deposition of solid at this point in time, given the system properties below. You may assume that the pipe radius is reduced uniformly along its length, i.e., that R is independent of z even as R is being reduced, and that the "simplified" Fick's law applies. (Reminder - you only need derive dR/dt for this point in time. You do not need to integrate for R as a function of time.)

Hint: proceed in three steps. First, determine the concentration at the outlet, and therefore the amount of "A" that is being removed from the solution per unit time. Then determine how much of solid "A" that corresponds to (i.e., being deposited on wall), per unit time. Then figure out how much of a reduction in radius that amount of solid would correspond to.

Properties of Liquid with Dissolved Chemical A

$\rho$ = 1000 kg/m$^3$ $\hat{C}_p$ = 4190 J/(kg K) $\mu$ = 0.001 Pa s

$\mathcal{D}_{AB}$ = 1 x 10$^{-8}$ m$^2$/s k = 0.67 W/(m °K)

Properties of Flow System

current inner pipe diameter = 0.1 m volumetric flow rate = 0.002 m$^3$/s

pipe length = 10 m