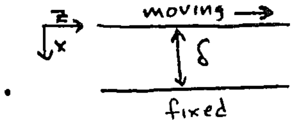
G. Additional Shell-Balance Problems
(from Undergrad exams)

A] Two large, horizontal, parallel plates are separated by a distance δ. The gap is filled with newtonian fluid. The bottom plate is fixed and the top one is moving at a velocity V_p in the z direction.



(a) Define a control volume of thickness Δx, length L in the z direction and width W in the y direction (perpendicular to the page). Do a momentum balance on this control volume. To save time, you do not need to list where each term comes from. (Note: you need not consider pressure terms in this balance.)

(b) Take the limit $\Delta x \longrightarrow 0$ and obtain the differential equation for τ_{xz}. Solve this equation. (You do not need to evaluate the constant of integration yet.)

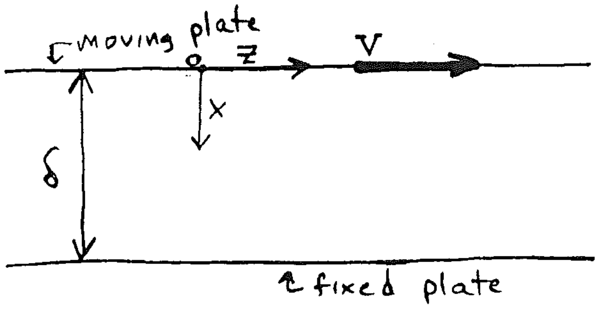
(c) Insert Newton's law of viscosity to obtain a differential equation for v_z.

(d) List two boundary conditions that can be applied to this problem.

(e) Solve the equation for v_z and evaluate the constants.

B] Two infinite, horizontal, parallel plates are separated by a distance δ. An incompressible Bingham plastic fills the gap between the plates. The bottom plate is fixed and the top plate is moving steadily rightward with a velocity $V>0$. Assume steady state.

a) Define a control volume of thickness Δx, length L in the z direction and width W in the y direction (perpendicular to the page). Perform a momentum balance on this control volume. To save time, you do not need to list where each term comes from. (Note: you need not consider pressure terms in this balance.)
b) Let $\Delta x \rightarrow 0$ and obtain a differential equation for τ_{xz}. Solve this differential equation. (You do not need to determine the constant of integration yet.)
c) Insert the relation between τ_{xz} and v_z for a Bingham plastic to obtain a differential equation for v_z.
d) Solve this equation without evaluating the constants of integration yet.
e) State two boundary conditions that apply to this problem.
f) Evaluate the two constants of integration and obtain the equation for v_z.
g) What is the shear force per unit area that must be exerted on the plate at x=0 to maintain its velocity?



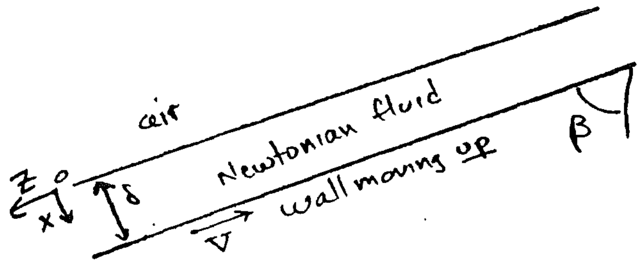
C] Derive the formula for the velocity v_z in a cylindrical tube of a Newtonian fluid that "slips" at the tube wall, i.e., for which the boundary condition at the tube wall ($r=R$) is

$$\tau_{rz} = k\, v_z \qquad \text{at } r=R$$

rather than the usual "no-slip" boundary condition at the wall. If you wish, you may start where this derivation deviates from that for the usual Newtonian fluid given in BSL section 2.2; or you may start at the beginning (with a momentum balance) it you like.

D] a) There is a film of Newtonian fluid of thickness δ on an inclined plane at an angle β to the vertical as shown below. The plane is moving <u>upwards</u> (in the negative z direction) with a velocity $V < 0$ (as defined on the z axis). The upper surface of the film is exposed to air. Derive an equation for the velocity $v_z(x)$ of the fluid for this case. Please use the coordinate system defined in the figure, where the z axis points <u>down</u> along the film. You do not have to repeat any portion of the derivation given in BSL for a falling film that applies to this problem.

b) For what values of V would <u>all</u> of the steamlines of the fluid be pointing upwards, i.e., in the negative z direction?



E] An engineer is applying a foam (a Bingham plastic with yield stress τ_o) in a uniform layer to the <u>outside</u> of a pipe of radius R, held vertically. He wants to know how thick a layer of foam he can apply without having the foam slide down the outer surface. Derive a formula for R_2, the maximum outer radius of a foam layer that would not flow, for this problem, in terms of R and fluid properties τ_o and ρ. Solve this problem in the following sequence:

a) Derive a momentum balance in cylindrical coordinates for this problem. You do not have to repeat any part of a derivation from BSL that applies directly to this problem; or you can start from the beginning. Note that there is no pressure difference in the z direction because the foam is exposed to the atmosphere along its length.

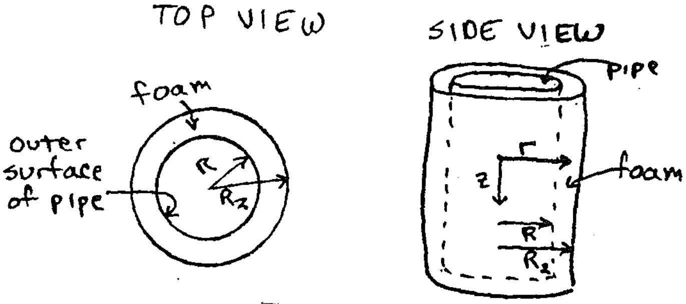
b) Derive an differential equation for the shear stress τ_{rz} from the momentum balance.

c) Integrate to obtain an algebraic equation for τ_{rz}. From the physical description of the problem, determine a boundary condition you can apply to determine the constant of integration in this equation.

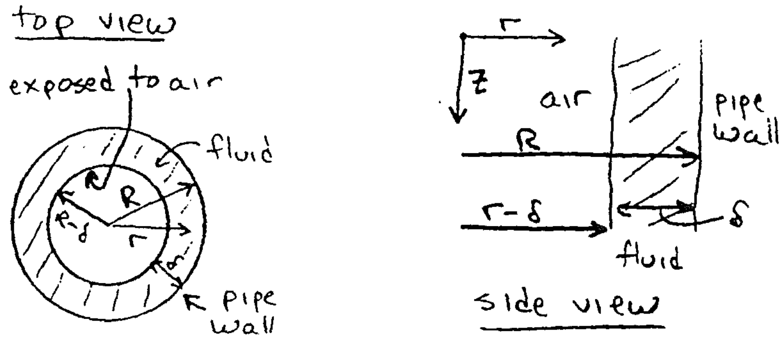
d) Apply the condition for onset of shearing of a Bingham plastic to determine the condition for the onset of flow. (Hint: the maximum of τ_{rz} occurs at $r=R$). (Note: the final equation is not explicit in R_2 (i.e., $R_2 > \ldots$) but implicit in R_2 (i.e., $f(R_2) > 0$.)

(FIGURE ON NEXT P.)

Figure for problem "E"



F] A film of Newtonian fluid of thickness δ is flowing down the inside of a cylindrical pipe of radius R as shown below. The pipe is oriented vertically. The inner surface of the film is exposed to air, and there is no pressure difference applied to the fluid; it flows under the force of gravity alone. Derive a formula for the steady-state velocity v_z of the fluid as a function of r, $(R - \delta) \leq r \leq R$, for given fluid properties ρ and μ and geometrical parameters R and δ. You do not have to repeat any part of any derivation we have done in class or in BSL that applies directly to this problem. However, <u>do not assume that the film is approximately rectangular</u>; work the problem in cylindrical coordinates.
(30 points)



G] A Newtonian fluid fills the gap between two parallel plates held vertically. The plate on the left side (x = 0) is held fixed, while the opposite plate (x = δ) moves downwards with velocity V. (V > 0 here because the z axis points downward.) The fluid has density ρ and viscosity μ. There is no applied pressure force driving the flow. Derive an equation for the steady-state velocity $v_z(x)$. You do not need to repeat any part of any derivation in BSL that applies directly to this problem.