Heat [redacted] Transfer in Tubes

[redacted]

1. What is Re? $D\langle v\rangle\rho/\mu = (0.0257)(0.6)(1200)/(0.002) = 9000$

For this range only Fig [redacted] 14.3-2 applies. Fortunately, since T_0 is uniform along inner surface of tube, $\frac{T_{b2}-T_{b1}}{(T_0-T_b)_{ln}} = \ln\left(\frac{T_0-T_{b1}}{T_0-T_{b2}}\right)$

From Fig [redacted] 14.3-2, for Re = 9000, $\frac{T_{b2}-T_{b1}}{(T_0-T_b)_{ln}}\frac{D}{4L}\left(\frac{\hat{C}_p\mu}{k}\right)^{2/3} \doteq 0.004$

$$\rightarrow \ln\left(\frac{T_0-T_{b1}}{T_0-T_{b2}}\right) = (0.004)\frac{(4)(6)}{(0.025)}\left[\frac{(2050)(0.002)}{(0.14)}\right]^{2/3} = 0.404$$

$$\frac{373-273}{373-T_{b2}} = \exp(0.404) \rightarrow T_{b2} = 306\text{ K}$$

2. First transform quantities to SI units

$D = 6\text{ in} = 0.1524\text{ m}$ $\quad R_o = 0.0762\text{ m}$ $\quad R_1 = 0.1016\text{ m}$ $\quad T_{air} = 70°F = 294.3\text{ K}$

$T_{b2} = 300°F = 422.0\text{ K}$ $\quad 100\text{ bbl/hr} = 4.42\cdot10^{-3}\text{ m}^3/\text{s}$ $\quad \mu = 100\text{ cp} = 0.1\text{ Pa}\cdot\text{s}$

$\rho = 0.95\text{ g/cc} = 950\text{ Kg/m}^3$ $\quad \hat{C}_p = 1.5\text{ cal/g}\,°\text{C} = 6285\text{ J/(Kg K)}$ $\quad k = 0.10\text{ Btu/(hr ft °F)} = 0.173\text{ W/mK}$

First need to estimate h for inner pipe surface. What is Re?

$\langle v\rangle = (4.42\cdot10^{-3})/[\pi(0.0762)^2] = 0.242\text{ m/s}$ $\quad Re = D\langle v\rangle\rho/\mu = \frac{(0.1524)(0.242)(950)}{0.1} = 351$;

laminar flow. Can't use Fig [redacted] 14.3-2, because it doesn't go that low in Re.

Can use Eq. [redacted] 14.3-17 if $RePr\,D/L > 10$; i.e., if $(351)[(6285)(0.1)/(0.173)](0.1524)/(1000) = 194 > 10$ ✓

Eq [redacted] 14.3-17: $\frac{h_{ln}D}{k} = \frac{h_{loc}D}{k} = (1.86)(Re\,Pr\,D/L)^{1/3}$ (assume $\mu_b \approx \mu_0$)

$$= (1.86)[(351)(3633)(0.1524)/1000]^{1/3} = 10.77$$

$\therefore\ h_{loc} = (10.77)k/D = (10.77)(0.173)/(0.1524) = 12.23\text{ W/(m}^2\text{K)}$

[Alternate derivation of h_{loc} using Fig [redacted] 14.2-1: $L/(D\,RePr) = 1/194 = 0.0052$
$\rightarrow Nu_{ln} \leq 9.5$, somewhat different from estimate above.]

(BSL Eq. 10.6-30)

Now $U_o = (1/R_o)\left[(1/R_oh_{loc}) + \frac{\ln R_1/R_o}{k^{01}} + 1/R_1h_1\right]^{-1} = \frac{1}{0.0762}\left[\frac{1}{(0.0762)(12.23)} + \frac{\ln(0.1016/0.0762)}{16.3} + \frac{1}{(0.1016)(100)}\right]^{-1} = \frac{1}{0.0762}\{1.07 + 0.0176 + 0.0984\}^{-1}$

Note h at inner surface is main resistance to heat transfer

$U_o = 11.03\text{ W/(m}^2\text{K)}$

BSL Eq. 10.6-31: $dQ = U_o(2\pi R_o dz)(T_{out} - T_b(z))$; T_o is outside temp., constant.

Energy balance: $dQ = w\hat{C}_p\,dT_b$

$$\rightarrow (U_o 2\pi R_o)/w\hat{C}_p \overset{*}{=} \frac{1}{T_b - T_0}\frac{dT_b}{dz} = -\frac{d[\ln(T_{out}-T_b)]}{dz}\ *$$

$$\ln\left[\frac{T_{out}-T_{b2}}{T_{out}-T_{b1}}\right] = -\frac{U_o 2\pi R_o}{w\hat{C}_p}L = -\frac{(11.03)2\pi(0.0762)}{(4.42)\cdot10^{-3}(950)(6285)}1000 = -0.200$$

$$\frac{294.3 - T_{b2}}{294.3 - 422.0} = \exp(-0.200) = 0.819;\quad T_{b2} = 399\text{ K}\quad (258\,0°F)$$

* Note: $\frac{U_o 2\pi R_o}{w\hat{C}_p} = \frac{U_o 2\pi R_o}{\pi R_o^2\rho v\hat{C}_p} = \frac{U_o 2R_o}{k}\frac{k}{C_p\mu}\frac{\mu}{2R_o v\rho}\frac{4}{2R_o} = \frac{U_oD}{k}\left[Re\,Pr\frac{D_o}{4}\right]^{-1}$

Thus this eq. is $U_o = -Re\,Pr\frac{D}{4k}\frac{d\ln(T_{out}-T_b)}{dz}$

3. We'll use the "hydraulic radius" approximation for the fracture: As presented in lecture, substitute $2h = 4b = 0.004\,m$ for D. First need to compute Re:

$$Re = D\langle v\rangle \rho/\mu = (0.004)(10)(1000)/(0.001) = 40{,}000$$

For this high a value of Re, one can use either Fig 14.3-2 or Eq. 14.3-16. Using Fig 14.3-2,

$$\frac{T_{b2}-T_{b1}}{(T_0-T_b)_{ln}}\left(\frac{D}{4L}\right)\left(\frac{\hat{C}_p\mu}{k}\right)^{2/3}\left(\frac{\mu_b}{\mu_0}\right)^{-0.14} \cong 0.0032 = \left[\frac{-100}{\dfrac{-100}{\ln(150/50)}}\right]\frac{0.004}{4L}\left(\frac{(4190)(0.001)}{0.68}\right)^{2/3}(1)^{-0.14}$$

$L \cong 1.15\ m$.

(Had one used instead Eq. 14.3-16, $\frac{h_{ln}D}{k} = (0.026)(Re)^{0.8}\left(\frac{\hat{C}_p\mu}{k}\right)^{1/3}\left(\frac{\mu_b}{\mu_0}\right)^{0.14} = (0.026)(40{,}000)^{0.8}\left(\frac{(4190)(0.001)}{0.68}\right)^{1/3}$

$\frac{h_{ln}D}{k} = 229$. From Eq "III" fr. lecture, $\frac{h_{ln}D}{k} = 229 = \frac{T_{b2}-T_{b1}}{(T_0-T_b)_{ln}}\,Re\,Pr\,D/4L$

$\rightarrow 229 = \ln(150/50)\,(Re\,Pr\,D/4L) = \ln\left(\frac{150}{50}\right)(40{,}000)(6.16)\frac{0.004}{(4)L}$; $L = 1.18\ m$)

4. The rate of heat transfer here is given by Eq. 10.6-29

$$Q_0 = 2\pi L (T_{water} - T_{air}) \left[\frac{1}{r_0 h_0} + \frac{\ln(r_1/r_0)}{k^{01}} + \frac{\ln(r_2/r_1)}{k^{12}} + \frac{1}{r_2 h_a} \right]^{-1}$$

Only the terms in brackets are affected by the strategies. At the start,
$r_0 = 1.27$ cm, $r_1 = 1.905$, $r_3 = 2.25$ cm, or 0.0127 m, 0.01905 m, 0.0225 m.
(No need to change units in $\ln(r_1/r_0)$ terms, etc.)
$k^{01} = 16.3$ W/m K, $k^{12} = 1$ W/m K, $h_3 = 50$ W/m² K.
Therefore all the terms in the brackets are known except h_0, which is "h_{ln}" for water flowing inside the tube.

Determining h_0: What is Re? $Re = \frac{D \langle v \rangle \rho}{\mu} = \frac{0.0254(1)(1000)}{(0.001)} = 2.54 \cdot 10^4$.

For Re > 20,000, one can use Eq. ~~13.~~ 14.3-16:

$$\frac{h_{ln} D}{k} = 0.026 (Re)^{0.8} (Pr)^{1/3} \left(\frac{\mu_b}{\mu_0}\right)^{0.14} \quad \boxed{\mathrm{I}}$$

We ignore μ_b/μ_0 terms, $Re = 2.54 \cdot 10^4$
$Pr = (\hat{C}_p \mu / k) = (4190)(0.001)/(0.68) = 6.16$

$$= (0.026)(2.54 \cdot 10^4)^{0.8} (6.16)^{1/3} = 159.2$$

$h_{ln} = h_0 = (159.2)(0.68)/(0.0254) = 4260$ W/m² K

Instead, one could have used Fig. 14.3-2. $Re = 2.54 \cdot 10^4 \rightarrow 0.0034 \approx \frac{h_{ln} D}{k} Re^{-1} Pr^{-1/3}$

$\rightarrow (0.0034) = \frac{h_{ln}(0.0254)}{(0.68)} (2.54 \cdot 10^4)^{-1} (6.16)^{-1/3} \rightarrow h_{ln} = 4240.$

Now one can evaluate the bracketed term

$$\left[\frac{1}{(0.0127)(4260)} + \frac{\ln(1.905/1.27)}{16.3} + \frac{\ln(2.5/1.905)}{1} + \frac{1}{(0.025)(50)} \right]^{-1}$$

$$\left[\underbrace{0.0184}_{\text{convective heat transfer inside tube}} + \underbrace{0.0248}_{\text{conduction through tube}} + \underbrace{0.272}_{\text{conduction through scum}} + \underbrace{0.8}_{\text{convective heat transfer on outside}} \right]^{-1} = (1.1152)^{-1} = 0.897$$

Note that biggest term, representing largest resistance, is due to poor convective heat transfer outside tube. Note also that Q_0 is proportional to [bracketed term]$^{-1}$; so anything that can reduce the individual terms within improves heat transfer.

Now for the individual strategies.

a) Eliminating the scum leaves the first 2 terms unchanged. The third term is eliminated, and the fourth becomes $\frac{1}{r_1 h_1} = \frac{1}{(0.01905)(50)} = 1.050$. The whole term is
$[0.0184 + 0.0248 + 1.050]^{-1} = 0.915$. Heat transfer is more efficient by a factor of $0.915/0.897 = 1.02$ (2% better).

b) Replacing the pipe makes the second term $\frac{\ln(1.905/1.27)}{100} = 0.00405$ and getting rid of the scum has the same effect on terms 3 + 4 as in (a).
$[0.0184 + 0.00405 + 1.050]^{-1} = 0.932$. Increase factor $= 0.932/0.897 = 1.04$.

c) Only the 4th term changes, from $\frac{1}{(0.025)(50)}$ to $\frac{1}{(0.025)(500)} = 0.08$.
$[0.0184 + 0.0248 + 0.272 + 0.08]^{-1} = 2.53$. Factor $= (2.53)/(0.897) = 2.82$.

d) Increasing water flow by 10x changes h_0. Eq. $\boxed{\mathrm{I}} \rightarrow$
$h_{ln} D/k = (0.026)(2.54 \cdot 10^5)^{0.8}(6.16)^{1/3} = 1004$
$\rightarrow h_{ln} = (1004)(0.68)/(0.0254) = 2.69 \cdot 10^4$. $\rightarrow \frac{1}{r_0 h_0} = \frac{1}{(0.0127)(2.69 \cdot 10^4)} = 0.00292$
$[0.00292 + 0.0248 + 0.272 + 0.8]^{-1} = 0.909$
Factor $= 0.909/0.897 = 1.013$ (1.3% better).