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Friction-factor problems, part 1

1. There is no flow if $\tau_0 \geq \frac{\Delta\mathcal{P}}{L}\sqrt{\frac{2K}{\phi}}$. Here $\tau_0 = 100$ dyne/cm^2 = 10 Pa.
$\Delta\mathcal{P} = 500$ psi $= 3.45\cdot10^{6}$ Pa.
$$10 \geq \frac{3.45\cdot10^{6}}{L}\sqrt{\frac{2(0.1\cdot10^{-12})}{0.2}} \Rightarrow L \geq 0.345\ \text{m} \quad (34.5\ \text{cm or} \sim 13\tfrac{1}{2}\ \text{in.})$$

Flow stops as depth of penetration approaches 0.345 m.

2. From lecture notes, for PL fluid, $\mu_{eff} \propto u^{n-1} = u^{-0.4}$ here.
We have datum that $\mu_{eff} = 30$ cp (0.03 Pa·s) at $u = 1$ ft/day ($3.53\cdot10^{-6}$ m/s)
(Actually, converting units is kind of pointless here, because μ_{eff} + u will appear only in ratios; units cancel out). Therefore
$\mu_{eff} = 1.98\cdot10^{-4}\, u^{-0.4}$, with μ_{eff} in Pa·s and u in m/s.
(or $\mu_{eff} = 30\, u^{-0.4}$, w/ u in ft/day and μ_{eff} in cp.)
Note $Q = 800$ bbl/d $= 800(42$ gal/bbl$)(1/264.2$ m^3/gal$) = 127$ m^3/d $= 0.00147$ m^3/s

a) At 1 ft radius, $u = Q/A = Q/(2\pi R L) = 0.00147/[2\pi(0.3048)(60\times0.3048)]$
$u = 4.2\cdot10^{-5}$ m/s (really, $4.2\cdot10^{-5}$ m^3/m^2/s.) (11.9 ft/day)
At $r = 50$ ft, u is (1/50) of this value: $8.41\cdot10^{-7}$ m/s

b) At $r = 1$, $\mu_{eff} = 1.98\cdot10^{-4}\, u^{-0.4} = 1.98\cdot10^{-4}(4.2\cdot10^{-5})^{-0.4} = 0.0111$ Pa·s (11.1 cp)
At $r = 50$ $\mu_{eff} = 1.98\cdot10^{-4}(8.41\cdot10^{-7})^{-0.4} = 0.0532$ Pa·s or 53.2 cp

3. a) For laminar Newtonian flow, $Q \sim R^4$. A 2x reduction in $R \rightarrow$ a 16x reduction in Q. $Q_2 = Q_1/16$
b) For laminar power-law flow, $Q \sim R^{(3+1/n)}$. For $n = 0.5$, $Q \sim R^5$. A 2x reduction in $R \rightarrow$ a 32x reduction in Q. $Q_2 = Q_1/32$.
c) If f is constant, as in a rough pipe in highly turbulent flow, $[D/\langle v\rangle^2]$ = constant (BSL eq. 6.1-4), or $\langle v\rangle \sim \sqrt{D}$. $Q = (\pi R^2)\langle v\rangle \sim \pi R^2\sqrt{R} \sim R^{2.5}$. A 2x reduction in $R \rightarrow$ 5.66x reduction in Q. $Q_2 = 0.177\, Q_1$

4. $Q = \langle v\rangle A$; $A = (200\ \text{ft})\times(2b) = 121.9\, b$ (in SI) ; $Q = 0.188$ m^3/s
$\therefore \langle v\rangle = 0.00154/b$.
$Re = \text{"}D\text{"}\, v\rho/\mu$, with "$D$" $= 4R_h = 4b$; $Re = 4b\left(\frac{0.00154}{b}\right)(1037)/(0.001)$
$\Rightarrow Re = 6400$
Re is independent of b! (This is a lucky break for us!) The reason is that, with Q fixed, $\langle v\rangle$ decreases as "D" $= 4b$ increases. The two effects just balance each other, leaving Re constant.
With $Re = 6140$, $K/D = 0.004$, Fig 6.2-2 indicates $f \cong 0.01$. Eq. 6.1-4:
$$f = \frac{1}{4}\frac{D}{L}\frac{\Delta P}{\frac{1}{2}\rho\langle v\rangle^2} = 0.01$$
$D = 4b$ $\qquad$ $L = 6$ in $= 0.1524$ m
$\langle v\rangle = 0.00148/b$
$\rho = 1037$ kg/m^3
$\Delta P = \rho g h$; $\rho = 1037$; $g = 9.8$ m/s^2 ; $h = 10$ ft $= 3.048$ m
$$f = 0.01 = \frac{1}{4}\frac{4b}{0.0524}\frac{(1037)(9.8)(3.048)}{(0.5)(1037)(0.00148/b)^2} = 5.37\cdot10^{8}\, b^3$$
$b = 2.65\cdot10^{-4}$ m $= 0.265$ mm ; Gap width $= 0.53$ mm.

(It wouldn't take much of a slit to sink a ship!)
With this gap with, $A \cong 0.32$ m^2 $\langle v\rangle = 5.82$ m/s
In Perry + Chilton's chart, handed out in class, f could be as large as 0.0195.
This gives B: $3.82\cdot10^{-4}$ m $\quad$ ($2B = 0.76$ mm) - not much different.

5. a) $\langle v\rangle = \frac{Q}{\pi R^2}$; $Q = 100\ m^3/day = 1.16 \cdot 10^{-3}\ m^3/s$; $\langle v\rangle = \frac{1.16\cdot 10^{-3}}{\pi (0.0254/2)^2} = 2.28\ m/s$

$Re = D\langle v\rangle \rho/\mu = (0.0254)(2.28)(1000)/(0.001) = 5.8\cdot 10^4$ ($1\ cp = 0.001\ Pa\ s$; $1\ g/cm^3 = 1000\ kg/m^3$)

$K/D = [(0.0254)/1000]/0.0254 = 0.001$. For $K/D = 0.001$ and $Re = 5.8\cdot 10^4$, $f \approx 0.006$ from Fig. 6.2-2 or friction-factor chart handed out in class.

Eq. 6.1-4: $f = \frac{1}{4}\frac{D}{L}\frac{\Delta\mathcal{P}}{\frac{1}{2}\rho\langle v\rangle^2} \rightarrow \Delta\mathcal{P} = f\left(\frac{1}{2}\right)\rho\langle v\rangle^2 4\ L/D = (0.006)\frac{1}{2}(1000)(2.28)^2(4)\frac{(5000)(12)}{1}$ $\boxed{\mathrm{I}}$ $\boxed{\mathrm{II}}$

$\Delta\mathcal{P} = 3.74\cdot 10^6\ Pa$ ($\cong 540\ psi$)

b) The factor f (0.006) in Eq. $\boxed{\mathrm{I}}$ above is replaced by $\frac{16}{Re} = 2.76\cdot 10^{-4}$; $\rightarrow \Delta\mathcal{P} = 1.72\cdot 10^5\ Pa$ ($\cong 25\ psi$). Turbulence increases $\Delta\mathcal{P}$ by a factor of more than 20!

(One could also solve for $\Delta\mathcal{P}$ in laminar flow using Eq. 2.3-19.)

6. The momentum balance is the same as on BSL pp. 48 + 49 (flow through a circular tube), except that $\Delta p = 0$. (Thus $\Delta\mathcal{P} = \Delta p + \rho g L = \rho g L$). Thus the analysis is unchanged up to Eq. 2.3-11: $\tau_{rz} = \frac{\Delta\mathcal{P}}{2L}r + C_1/r$.

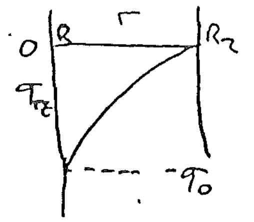
Note that, unlike BSL Sect 4.3 (flow in a tube), one cannot use the BC "τ_{rz} not infinite at $r=0$" because $r=0$ is not with the system for this problem. (In this problem $R \le r \le R_2$).

Instead, since at $r = R_2$ there is contact with air, there is BC

$$\tau_{rz} = 0 \text{ at } r = R_2$$

$$\text{or } 0 = \frac{\Delta\mathcal{P}}{2L}R_2 + C_1/R_2 \rightarrow C_1 = -\frac{\Delta\mathcal{P}}{2L}R_2^2$$

$$\text{and } \tau_{rz} = \frac{\Delta\mathcal{P}}{2L}r - \frac{\Delta\mathcal{P}}{2L}R_2^2/r = \frac{\Delta\mathcal{P}}{2L}R_2\left[r/R_2 - R_2/r\right] \quad \boxed{\mathrm{I}}$$



Note that this function is less than zero from $r = R$ to $r = R_2$ and $= 0$ at $r = R_2$ (as required by the BC). Therefore $\tau_{rz} < 0$, and the condition for shearing is $\tau_{rz} < -\tau_0$. Since τ_{rz} is at a minimum at $r = R$, the condition for the onset of shearing is

$$\tau_{rz}\big|_{r=R} = \frac{\Delta\mathcal{P}}{2L}R_2\left[R/R_2 - R_2/R\right] = -\tau_0$$

(or, substituting ρg for $\Delta\mathcal{P}$,

$$\frac{\rho g}{2}R_2\left[R/R_2 - R_2/R\right] = -\tau_0$$

(One could solve this equation for R_2 by multiplying by R_2 + solving as a quadratic equation) This is the condition on R_2 for the onset of flow.

The fact that $\tau_{rz} < 0$ in this problem reflects the fact that z momentum is transfered <u>inward</u>, in the negative r direction. The weight of the fluid imparts z momentum to the fluid, which is transported radially <u>inward</u> and leaves the system at the pipe wall.

Alternate derivation: one could write eq. I above as

$$\tau_{rz} = \frac{\Delta \mathcal{P}}{2L}\, r\left[1-(R_2/r)^2\right]$$

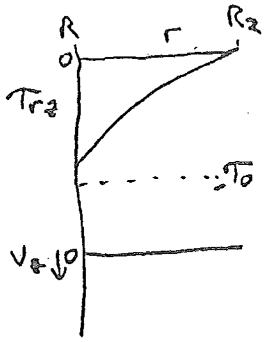
Then the condition for the onset of shearing becomes

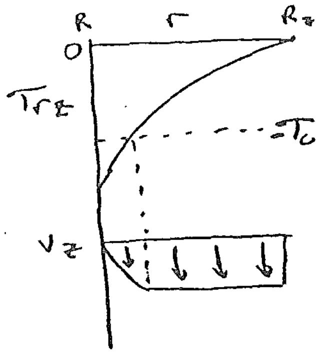
$$\tau_{rz}\big|_{r=R} = \frac{\Delta \mathcal{P} R}{2L}\left[1-(R_2/R)^2\right] = -\tau_0$$

$$-\frac{2L\tau_0}{\Delta \mathcal{P} R} = 1-(R_2/R)^2 \quad \text{or} \quad \left(\frac{R_2}{R}\right)^2 = 1+\frac{2L\tau_0}{\Delta \mathcal{P} R} \Rightarrow \frac{R_2}{R} = \sqrt{1+\frac{2L\tau_0}{\Delta \mathcal{P} R}}$$

$$\text{or} \quad R_2 = R\sqrt{1+(2L\tau_0)/(\Delta \mathcal{P} R)} \quad \text{or} \quad R_2 = R\sqrt{1+(2\tau_0)/(\rho g R)}$$

for $R_2 < R\sqrt{1+2\tau_0/\rho g R}$ $\qquad\qquad$ $R_2 > R\sqrt{1+2\tau_0/\rho g R}$





friction-factor problems, part 2

[redacted] #1 by trial and error method

Given: $\rho = 1.0\ gcm^{-3} = 1000\ kgm^{-3}$
$D = 1.0\ mm = 10^{-3}\ m$
$\mu = 0.001\ Pa.s = 0.001\ kgm^{-1}s^{-1}$
$\rho_o = 0.95\ gcm^{-3} = 950\ kgm^{-3} = \rho_s$
$V_\infty = ?$

The oil droplet is treated as a solid settling in a liquid (water) because of its viscosity.

$$f = \frac{4}{3}\frac{gD}{V_\infty^{2}}\left(\frac{\rho_s - \rho}{\rho}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$\mathrm{Re} = \frac{D\rho V_\infty}{\mu} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Solution :

From equations 1 and 2 :

$$f = \frac{4}{3}\frac{9.81x0.001}{v_\infty^{2}}\left(\frac{950 - 1000}{1000}\right) = \frac{-6.54x10^{-4}}{v_\infty^{2}} \Rightarrow v_\infty = \sqrt{\frac{6.54x10^{-4}}{f}}$$

The negative sign is dropped as the significance is a rising bubble.

$$\mathrm{Re} = \frac{0.001\,x1000\,x\,v_\infty}{0.001} = 1000\ v_\infty$$

Guessing $V_\infty = 0.05$ m/s (arbitrary), therefore,

Re = 1000x0.05 = 50 and f ≅1.4(from chart) giving V_∞ = sqrt(6.54x10^{-4}/1.4) ≅0.0216m/s

Recalculating a few times to get a convergent solution:

Re =1000 x 0.0216 = 21.6 and f ≅2.6 giving V_∞ = sqrt(6.54x10^{-4}/2.6) ≅ 0.0159m/s ;

Re =1000 x 0.0159 = 15.9 and f ≅3.4 giving V_∞ = sqrt(6.54x10^{-4}/3.4) ≅ 0.0139m/s ;

Re =1000 x 0.0136 = 13.9 and f ≅3.6 giving V_∞ = sqrt(6.54x10^{-4}/3.6) ≅ 0.0135m/s ;

Re =1000 x 0.0135 = 13.5 and since f ≅3.6 (is same as above), **V_∞ ≅ 0.0135m/s**

1) BSL Eq. 6.1-7: $f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_s-\rho}{\rho}\right)$, or $v_\infty = \left[\frac{4}{3}\frac{gD}{f}\frac{\rho_s-\rho}{\rho}\right]^{1/2}$ $\boxed{\text{III}}$. Unfortunately, f depends on v_∞, which is unknown.

Method 1: trial and error. Guess 1 value of v_∞, read f from Fig 6.3-1, comput v_∞ using Eq III, guess again.

Method 2: guess which of 3 regions of Fig 6.3-1 final answer is in. Plug expression into Eq III, compute v_∞, verify original guess of region was right, or go on to next region.

For instance: guess $Re \lesssim 1$. $f = 24/Re = 24\mu/[D v_\infty \rho]$

$$\text{III} \rightarrow v_\infty^2 = \frac{4}{3}\frac{gD}{24\mu}\frac{Dv_\infty\rho}{1}\frac{(\rho_s-\rho)}{\rho} \rightarrow v_\infty = \frac{4}{3}\frac{gD^2}{24\mu}(\rho_s-\rho) = 2\frac{D^2}{4}(\rho_s-\rho)g/(9\mu)$$

[This is, of course, same as "creeping flow" eq., 2.6-17, which applies for low Re.]

$v_\infty = 2\frac{(0.001)^2}{4}(950-1000)(9.8)/[(9)(0.001)] = -0.027$ m/s (minus sign means particle settles <u>upwards</u> instead of down.)

Check $Re = \frac{(0.001)(0.027)(1000)}{0.001} = 27 > 1$ violates initial assumption. Guess again.

Guess $Re > 1$ and $Re \lesssim 1000$. $f \approx 18.5/Re^{3/5}$.

$$\text{III} \rightarrow v_\infty^2 = \frac{4}{3}\frac{gD}{18.5}\left(\frac{Dv_\infty\rho}{\mu}\right)^{3/5}\frac{\rho_s-\rho}{\rho} \rightarrow v_\infty^{1.4} = \frac{4}{3}\frac{gD^{1.6}}{18.5}\frac{1}{\mu^{0.6}}\frac{\rho_s-\rho}{\rho^{0.4}}$$

$$v_\infty^{1.4} = \left[\frac{4}{3}\frac{(9.8)(0.001)^{1.6}}{(18.5)}\frac{1}{(0.001)^{0.6}}\frac{(-50)}{(1000)^{0.4}}\right] \qquad v_\infty = (0.00223)^{1/1.4} = 0.0127\ \text{m/s}$$

we drop the minus sign before taking the 1/1.4 power, noting that v_∞ is <u>upwards</u>

Check $Re = (0.001)(0.0127)1000/0.001 = 12.7$, < 1000 and > 1. Assumption verified.

Method 3: similar to Example 6.3-1 on p. 187 (and 6.2-2 on p. 183). Although f and Re both depend on v_∞, fRe^2 does not. $fRe^2 = \left[\frac{4}{3}\frac{gD}{v_\infty^2}\frac{\rho_s-\rho}{\rho}\right]\left[\frac{D^2v_\infty^2\rho^2}{\mu^2}\right] = \frac{4}{3}gD^3\rho(\rho_s-\rho)/\mu^2 = \frac{4}{3}\frac{(9.8)(0.001)^3(1000)(-50)}{(0.001)^2}$

$fRe^2 = -653$ Again we drop minus sign after noting that particles settle <u>upwards</u>.

$\therefore f = 653/Re^2$. This expression has a slope of (-2) on Figure 6.3-1 and passes through $f = 653$, $Re = 1$. The intersection of this line with $f(Re)$ gives $Re \approx 15 \rightarrow 15 = \frac{(0.001)v_\infty(1000)}{0.001} \rightarrow v_\infty = 0.015$ m/s. (see Fig. on next p.)

Roughly the same answer as by method 2. See figure, next p.

2. Don't know D, so don't know Re.

$Re = Dv\rho/\mu \propto D$ $\quad Re = \frac{(0.01)(1000)}{0.001}D = 10^4 D$

$f = \frac{4}{3}[gD/(v^2)]\frac{\rho_s-\rho}{\rho} \propto D$ (Eq. 6.1-7) $\quad f = \left[\frac{4}{3}\frac{9.8}{(0.01)^2}\frac{(900-1000)}{1000}\right]^{-1} D = 13,067 D$ (neg sign just means sphere rises.)

trial + error. Guess $D = 0.01$. $Re = 100$. Fig 6.3-1 $\rightarrow f = 1.1$; $1.1 = 13,067D \rightarrow D = 8.4\cdot10^{-5}$

$D = 8.4\cdot10^{-5} \rightarrow Re = 0.84 \rightarrow f \approx 30 \rightarrow D = 2.3\cdot10^{-3}$

$2.30\cdot10^{-3} \rightarrow \quad = 23 \rightarrow f \approx 2.5 \rightarrow$ " $= 1.9\cdot10^{-4}$

$1.9\cdot10^{-4} \rightarrow \quad = 1.9 \rightarrow f \approx 16 \rightarrow\ \approx 0.00122$

$0.00122 \rightarrow \quad = 12.2 \rightarrow f \approx 4 \rightarrow\ = 3.06\cdot10^{-4}$

$3.06\cdot10^{-4} \rightarrow Re = 3.06 \rightarrow f \approx 11 \rightarrow D = 8.42\cdot10^{-4}$

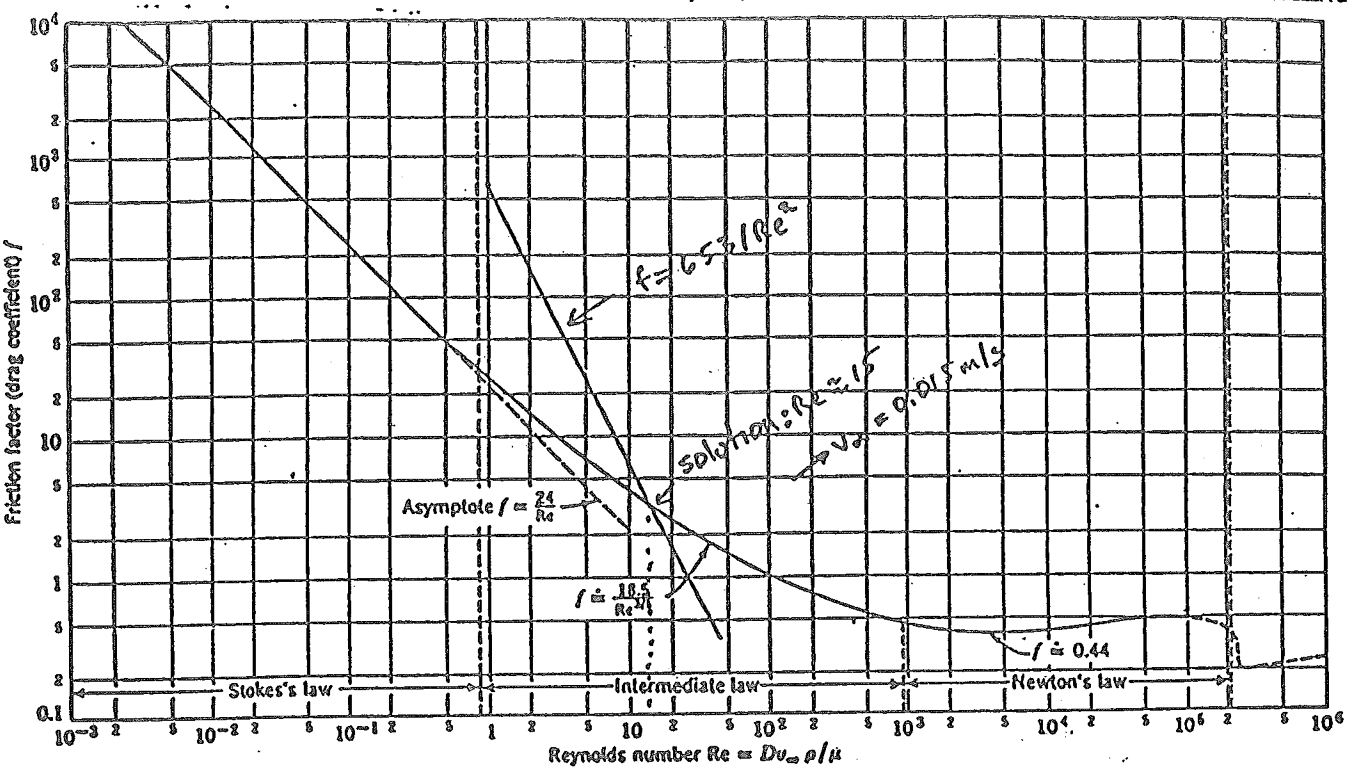
$\rightarrow Re = 8.42 \rightarrow f \approx 5.5 \rightarrow D = 6.96\cdot10^{-4}$

$\rightarrow$ " $6.96 \rightarrow \quad \approx 6.2 \rightarrow \quad 4.74\cdot10^{-4}$

$4.7 \rightarrow \quad 7.5 \rightarrow \quad 5.74\cdot10^{-4}$

$Re = 5.7 \rightarrow f \approx 7 \rightarrow D \approx 5.35\cdot10^{-4}$ converging on $\sim 5.5\cdot10^{-4}$ m

Figure for Problem 1:



3\. a) Eq. 6.4-9 applies at low Re: $v_0 = \frac{Q}{A} = \frac{\Delta\wp}{L}\frac{D_p^2}{150\mu}\frac{\epsilon^3}{(1-\epsilon)^2}$. Darcy's law: $\frac{Q}{A} = \frac{\Delta P}{L}\frac{K}{\mu}$
$\rightarrow K = \frac{D_p^2}{150}\frac{\epsilon^3}{(1-\epsilon)^2}$. For this case, $K = [(0.6\cdot10^{-3})^2/150][(0.3)^3/(0.7)^2] = 1.32\cdot10^{-10}\ m^2$

(about 130 Darcy).

b) Eq. 6.4-12 applies at all Re:

$$\frac{\Delta\wp}{L} = \frac{150\mu v_0}{D_p^2}\frac{(1-\epsilon)^2}{\epsilon^3} + 1.75\frac{\rho v_0^2}{D_p}\frac{(1-\epsilon)}{\epsilon^3}$$

$\frac{\Delta P}{L} = 10$ psi/ft $= 2.26\cdot10^5$ Pa/m $\mu = 1$ cp $= 0.001$ Pa s $\epsilon = 0.3$

$\rho = 1$ g/cm³ $= 1000$ Kg/m³ $D_p = 0.6$ mm $= 6\cdot10^{-4}$ m

$$(2.26\cdot10^5) = \frac{(150)(0.001)v_0}{(6\cdot10^{-4})^2}\frac{(0.7)^2}{(0.3)^3} + (1.75)\frac{(1000)v_0^2}{(6\cdot10^{-4})}\frac{0.7}{(0.3)^3}$$

$$0 = 7.5617\cdot10^7 v_0^2 \quad + 7.5617\cdot10^6 v_0 \ - 2.26\cdot10^5$$

$$v_0 = \frac{-7.5617\cdot10^6 \pm \sqrt{(7.5617\cdot10^6)^2 + 4(7.5617\cdot10^7)(2.26\cdot10^5)}}{2(7.5617\cdot10^7)} = \frac{-7.562\cdot10^6 \pm 1.1204\cdot10^7}{2(7.5617\cdot10^7)}$$

positive root applies: $v_0 = 0.0296$ m/s $= Q/A$

If fracture is 60 ft (18.29 m) high and 1/8 in ($3.175\cdot10^{-3}$ m) wide, its cross-section area is 0.0581 m².

$\therefore Q = A(Q/A) = (0.0581)(0.0296) - 0.00172$ m³/s

(this is about 149 m³/day, or 935 bbl/day.)

Alternate methods of solution: a) trial and error, using Fig 6.4-1. b) One could guess Re < 10 and use Eq. 6.4-9, or Re > 1000, and Eq. 6.4-11. In this case final result has $Re \equiv (D_p\rho v_0/\mu)(1/(1-\epsilon)) = (6\cdot10^{-4})(1000)(0.00721)/[(0.001)(0.3)]$ $= 14.4$; a little too large for Re < 10. c) One could derive a graphical method like Examples 6.2-2 and 6.3-1.