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Unsteady Conduction Probs, Pt. 1

1. a) From Eq. 11.1-10, $q_y = \frac{k}{\sqrt{\pi \alpha t}}(T_1 - T_0)$. Using SI units given, we will get answer in J/m²s (W/m²). Can either convert to "days" at start or at end. Let t [=] sec, t' [=] days

If stay with SI units until end, $q_y = (0.99)/[\pi(\frac{0.99}{2320(761)})t]^{1/2} \cdot [\frac{300-150}{1.8}] = 6.22 \cdot 10^4 \, t^{-1/2}$ W/m².

$Q = (10{,}000) q_y = 6.22 \cdot 10^8 \, t^{-1/2}$ W; but since t is in sec here & we want days,

$Q = (6.22 \cdot 10^8)(t' [(24)(3600)])^{-1/2} = 2.11 \cdot 10^6 (t')^{-1/2}$ in watts, with t expressed in days. If instead of watts (joules/sec) we want units of (joules/day), then $Q = 1.82 \cdot 10^{11} (t')^{-1/2}$ J/day.

If instead were to convert all time units to days at start, then $k = (0.99)(8.64 \cdot 10^4)$ (J/m K day);

$q_y = (8.55 \cdot 10^4)/[(\pi)(\frac{8.55 \cdot 10^4}{(2320)(761)} t')]^{1/2} \cdot (\frac{300-150}{1.8}) = 1.83 \cdot 10^7 (t')^{-1/2}$ J/(m² d), t' expressed in days.

$Q = (10{,}000)(1.83 \cdot 10^7 (t')^{-1/2}) = 1.83 \cdot 10^{11} (t')^{-1/2}$ (J/d), t expressed in days.

b) For part b it is much easier to <u>begin</u> with Q in terms of days (t'). Then

Cum heat loss at any time $t^* = \int_0^{t^*} Q(t')\,dt' = 1.83 \cdot 10^{11} (t')^{-1/2} d(t') = 1.83 \cdot 10^{11} (t'^{1/2})(2) \Big|_0^{t^*}$

$= 3.64 \cdot 10^{11} (t^*)^{1/2}$, t^* expressed in days.

If one used $Q(t)$ instead, cum heat loss $= \int_0^{t^*} (6.22 \cdot 10^8) t^{-1/2} dt$ with both $t^{-1/2}$ in sec$^{-1/2}$ and t in (sec). →

$= (6.22 \cdot 10^8)(2) t^{1/2}$ with heat loss in J and t in sec

$= 3.66 \cdot 10^{11} t^{1/2}$ with t expressed in days.

2. $y = 0.05$ m $\quad \alpha = k/(\rho \hat{C}_p) \quad k = 0.346$ W/cm°C $= 34.6$ W/(m K)

$\rho = 11.34$ g/cm³ $= 11{,}340$ Kg/m³ $\quad \hat{C}_p = 0.031$ cal/(g°C) $= 130$ J/(Kg K)

$\rightarrow \alpha = (34.6)/[(11{,}340)(130)] = 2.35 \cdot 10^{-5}$

$y/\sqrt{4\alpha t} = 0.05/[4(2.35 \cdot 10^{-5})(60)]^{1/2} = 0.67$

From figure handed out in class, $y/\sqrt{4\alpha t} = 0.67 \rightarrow \frac{T-100}{300-100} = 0.17$

$T = 134$°F.

3. a) For these rock properties, $\alpha = k/(\rho \hat{C}_p) = (0.99)/(2320 \cdot 761) = 5.61 \cdot 10^{-7}$ m²/s

$r_{well} = [(4.5)/12][.3048] = 0.1143 \qquad t = (2)(24)(60)^2 = 1.73 \cdot 10^5$

Fig 42 of Carslaw + Jaegar applies.

$\log_{10}[\alpha t / r_{well}^2] = \log_{10}[(5.61 \cdot 10^{-7})(1.73 \cdot 10^5)/(0.1143)^2] = \log_{10}(7.42) = 0.87$

My best reading of Fig 42 is $\log_{10}[(r_{well}\, q_{well})/(k(T_{well} - T_0))] = -0.25$

$\frac{(0.1143) q_{well}}{(0.99)(-200/1.8)} = 0.56 \Rightarrow \quad q_{well} = -540$ W/m²

(the negative sign is because q_{well} is really $q_r|_{r=r_{well}}$; since the heat flux is inward, in the negative r direction, $q_{well} < 0$.)

The heat transfer/unit depth $= (2\pi r) q_{well} = 2\pi(0.1143)(-540) = -390$ W/m.

b) $\alpha = k/(\rho \hat{C}_p) = (0.658)/[(983)(4181)] = 1.60 \cdot 10^{-7}$

We want time for $(T - T_0)/(T_1 - T_0) = 0.75$ at $r/R = 0$. Fig 11.1-2 applies.

My best guess is that $\alpha t / R^2 \approx 0.34 \rightarrow t = (0.34)(0.1143)^2/(1.60 \cdot 10^{-7})$

$t = 2.77 \cdot 10^4$ s $\doteq$ 7 hr 42 min.

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Unsteady Conduction Probs, Pt 2

1. This problem uses the "product method" and 3 applications of Fig 11.1-1.
   $\alpha = k/\rho \hat{C}_p = 2.61/((2270)(1000)) = 1.15 \cdot 10^{-6}$ m^2/s ; $t = 6\ (3600) = 2.16 \cdot 10^4$ sec.
   For short dimension, $\alpha t/b^2 = (1.15 \cdot 10^{-6})(2.16 \cdot 10^4)/(0.25)^2 = 0.40$. From Fig 12.1-1,
   $(T_1 - T)/(T_1 - T_0) \hat{=} 0.48$
   For long dimension, $\alpha t/b^2 = (1.15 \cdot 10^{-6})(2.16 \cdot 10^4)/(0.5)^2 = 0.10$. From Fig. 12.1-1,
   $(T_1 - T)/(T_1 - T_0) \hat{=} 0.95$.
   For rectangular solid, $(T_1 - T)/(T_1 - T_0) = (0.48)(0.95)^2 = 0.43$. $T \hat{=} 220°F$.

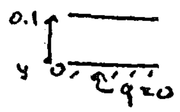
2. We want $\left(\frac{T_1 - T}{T_1 - T_0}\right)_{cube} = \frac{-20}{-100} = 0.2$ at the center. Because the block is cubical,
   $\left(\frac{T_1 - T}{T_1 - T_0}\right)_{cube} = \left(\frac{T_1 - T}{T_1 - T_0}\right)^3_{slab}$ or $\left(\frac{T_1 - T}{T_1 - T_0}\right)_{slab} = (0.2)^{1/3} = 0.58$. $\frac{T_1 - T}{T_1 - T_0} = 0.58$ at $\frac{y}{b} = 0$
   for $\alpha t/b^2 \hat{=} 0.32$. $\alpha = 2.61/[(2270)(1000)] = 1.15 \cdot 10^{-6}$ m^2/s ; $b = 0.5$
   $t = (0.32)(0.5)^2/(1.15 \cdot 10^{-6}) = 6.96 \cdot 10^4$ sec (about 19 hr, 20 min)

One side is insulated; therefore this is equivalent to a slab temporarily heated on both sides with width 0.2 m (i.e., b=0.1 m).

75 min = 4500 s. $\alpha = k/\rho\hat{C}_p = (1.21)/[(3010)(880)] = 4.57\cdot10^{-7}$ m²/s

The solution to this problem is

$\frac{T-T_0}{T_1-T_0} \equiv \Theta = \Theta_1(y,t) - \Theta_1(y,t-t_1)$, where, more precisely, we are interested in Θ at $y=0$.



At $t_1 = 75$ min = 4500 s, $\alpha t_1/b^2 \cong 0.2$

For $t < 75$ min, we can pull data directly off Fig 12.1-1

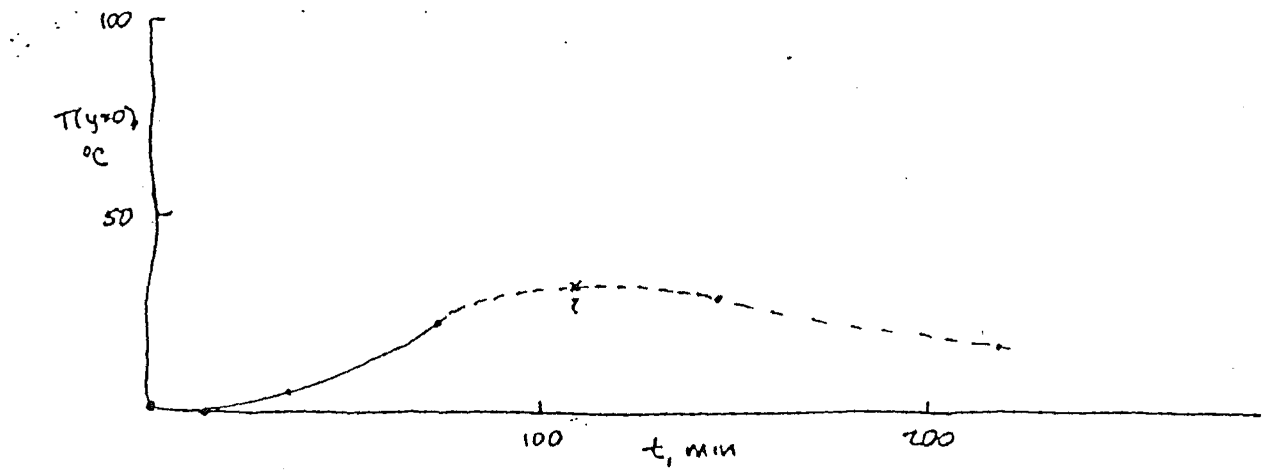
$\frac{\alpha t}{b^2}$	t		$\frac{T-T_0}{T_1-T_0}$	T
≤ 0.04	≤ 875	(14½ min)	0	0°C
0.1	2190	36½ min	0.05	5°C
0.2	4380	73 min	0.23	23°C

For $t > 75$ min, both terms must be included.

For $\alpha t/b^2 = 0.4$, $\Theta_1^{(t)} \cong 0.53$; Since $\alpha t_1/b^2 \cong 0.2$, then $\alpha[t-t_1]/b^2 \cong \alpha t/b^2 - 0.2$. Thus when $\alpha t/b^2 = 0.4$, $\alpha[t-t_1]/b^2 = 0.2$; $\Theta_1(t-t_1) \cong 0.23$. $\Theta = 0.53 - 0.23 \cong 0.3$. $T = 30°C$. This time corresponds to $t = 0.4b^2/\alpha = 8760$ sec or 146 min.

For $\alpha t/b^2 = 0.6$, $\Theta_1(t) \cong 0.71$; $\Theta_1(t-t_1) = \Theta_1(\frac{\alpha t}{b^2} = 0.4) \cong 0.53$.
$\Theta = 0.71 - 0.53 = 0.18$; $T = 30°C$. $t = (0.6)b^2/\alpha = 13{,}130$ s = 219 min.

A rough plot of these results gives



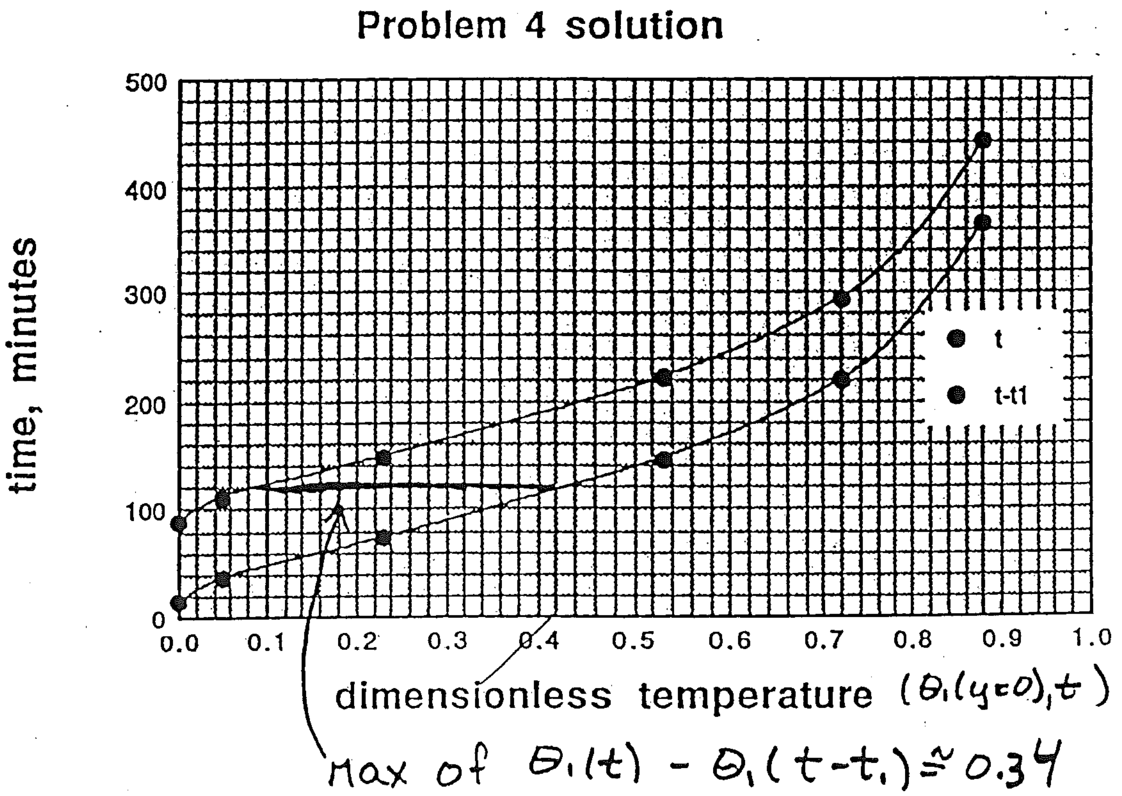
Getting more precise is difficult because there are so few lines on Fig 12.1-1. I might guess that at $\alpha t/b^2 = 0.3$, $\Theta_1(t) = 0.38$, $\Theta_1(t-t_1) = \Theta_1(\frac{\alpha t}{b^2} = 0.1) \cong 0.05$
$\rightarrow \Theta \cong 0.38 - 0.05 = 0.33$. This corresponds to $t = 0.5\,b^2/\alpha = 6570$ s $\cong$ 109½ m

I would guess max T is between 30 and 35°C, occurring between about 100 and 130 min.

Another, possibly better way to plot the results is as follows. Plot $\theta_1(y,t)$ and $\theta_1(y,t-t_1)$ on the same axes. θ is the difference between the two plots

$\alpha t/b^2$	t (sec)	t (min)	$\theta_1(y=0,t)$
≤ 0.04	≤ 875	$14\frac{1}{2}$	0
0.1	2190	$36\frac{1}{2}$	0.05
0.2	4380	73	0.23
0.4	8750	146	0.53
0.6	13,130	219	0.72
1.0	21,880	365	0.88

$\theta_1(y=0, t-t_1)$ is simply the same curve displaced by 75 min:



Since the maximum of $\theta(y=0,t) = \theta_1(0,t) - \theta_1(0,t-t_1)$

$$\simeq 0.34 = \frac{T-T_0}{T_1-T_0} = \frac{T-0}{100-0}$$

the maximum T at the center is $34°C$.