Failure Criteria

AE1108-II: Aerospace Mechanics of Materials

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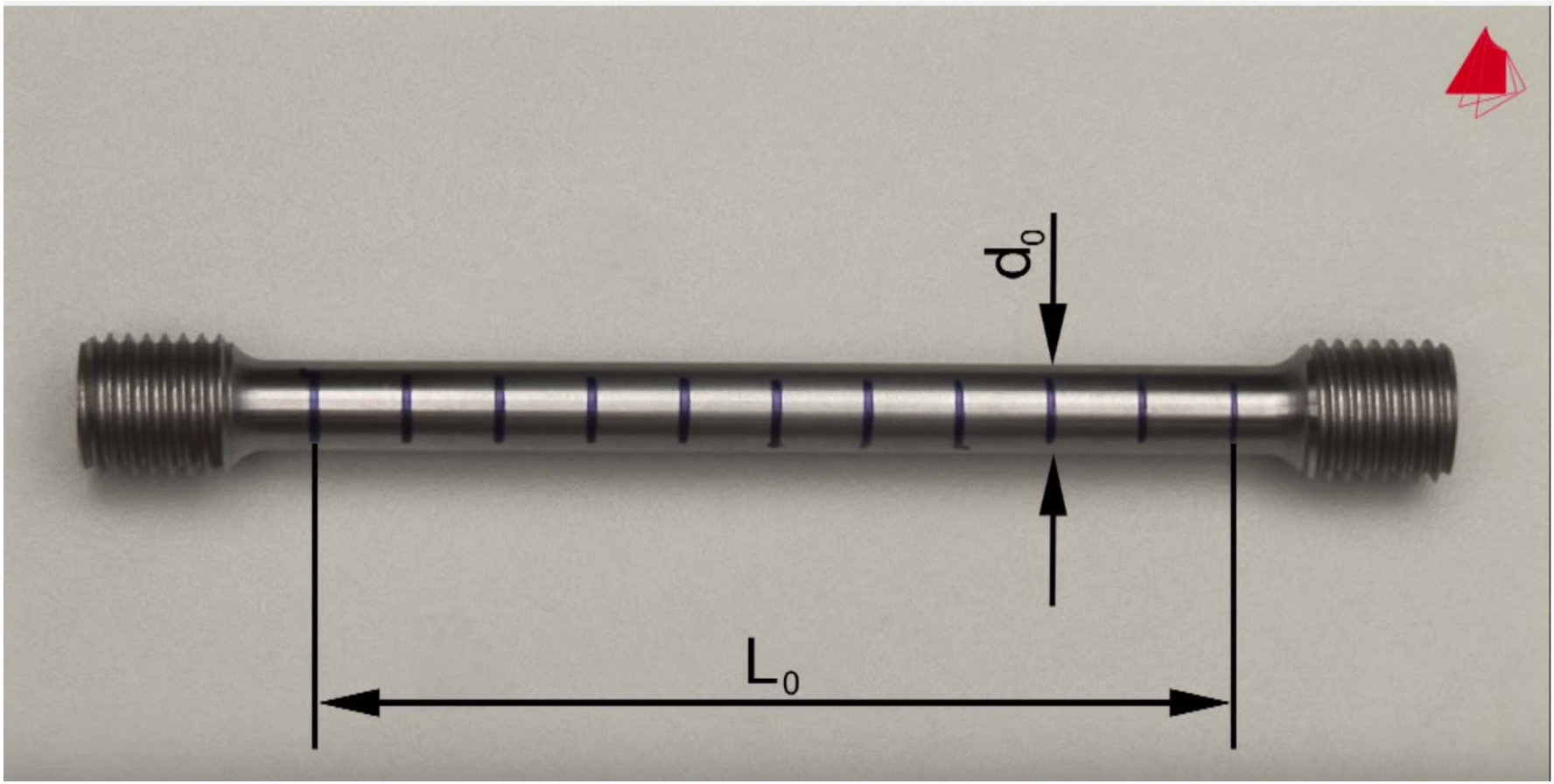
Aerospace Structures
& Materials
Faculty of Aerospace Engineering

Consider a Uniaxial Test Specimen



Images taken from the following youtube video:
https://www.youtube.com/watch?v=D8U4G5kcpcM

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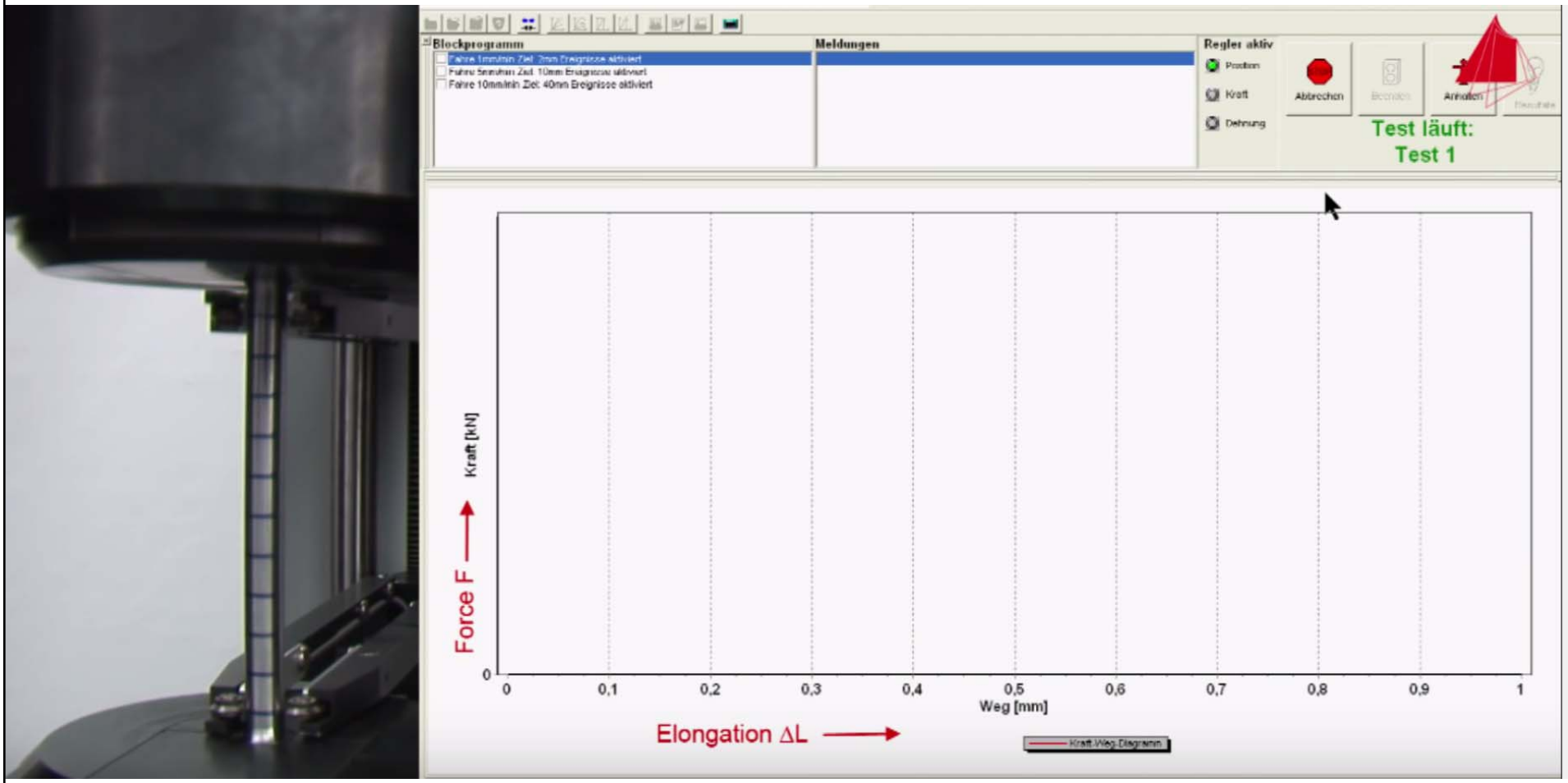
Consider a Uniaxial Test Specimen



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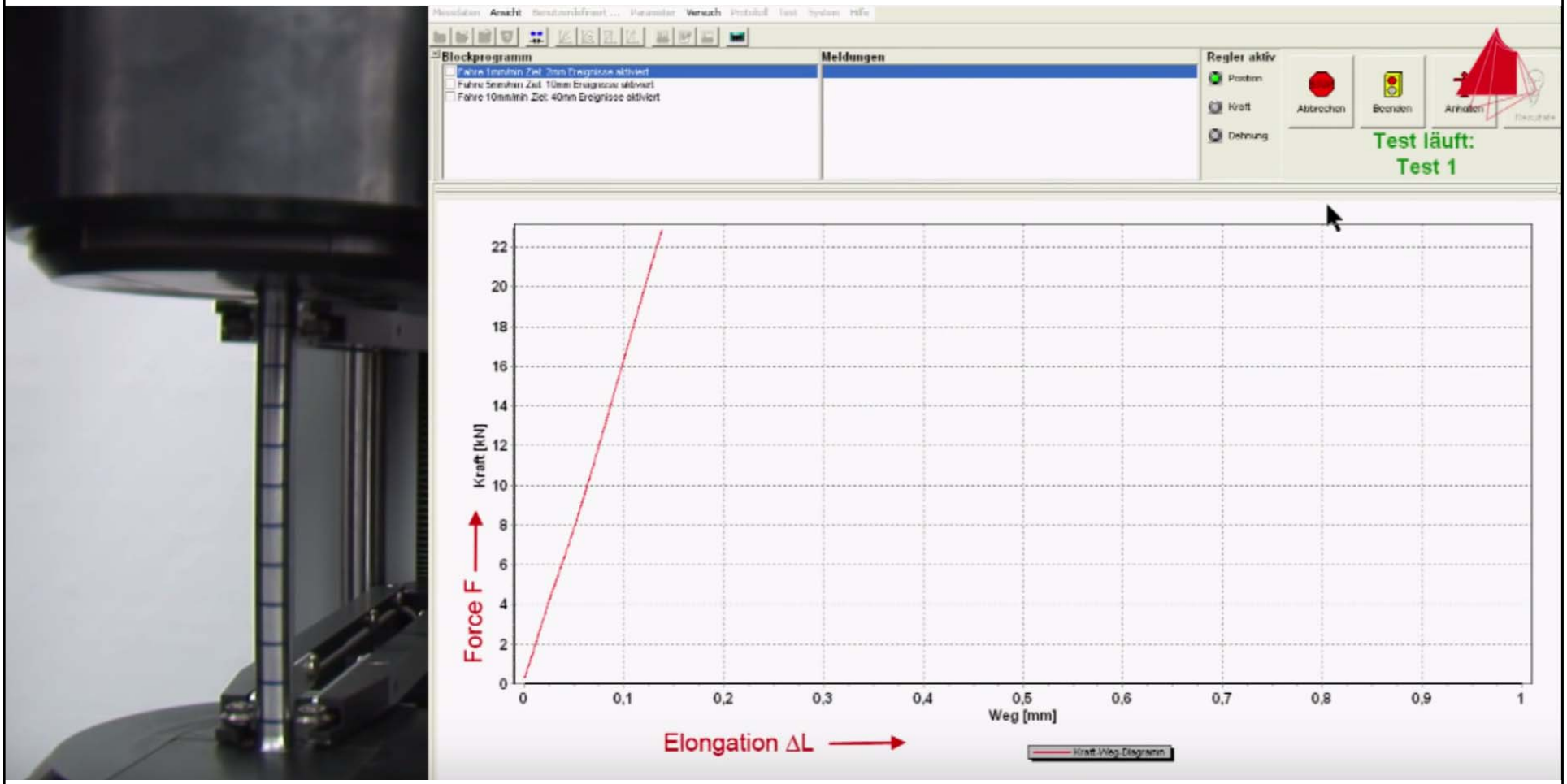
Linear Elastic Part of Test



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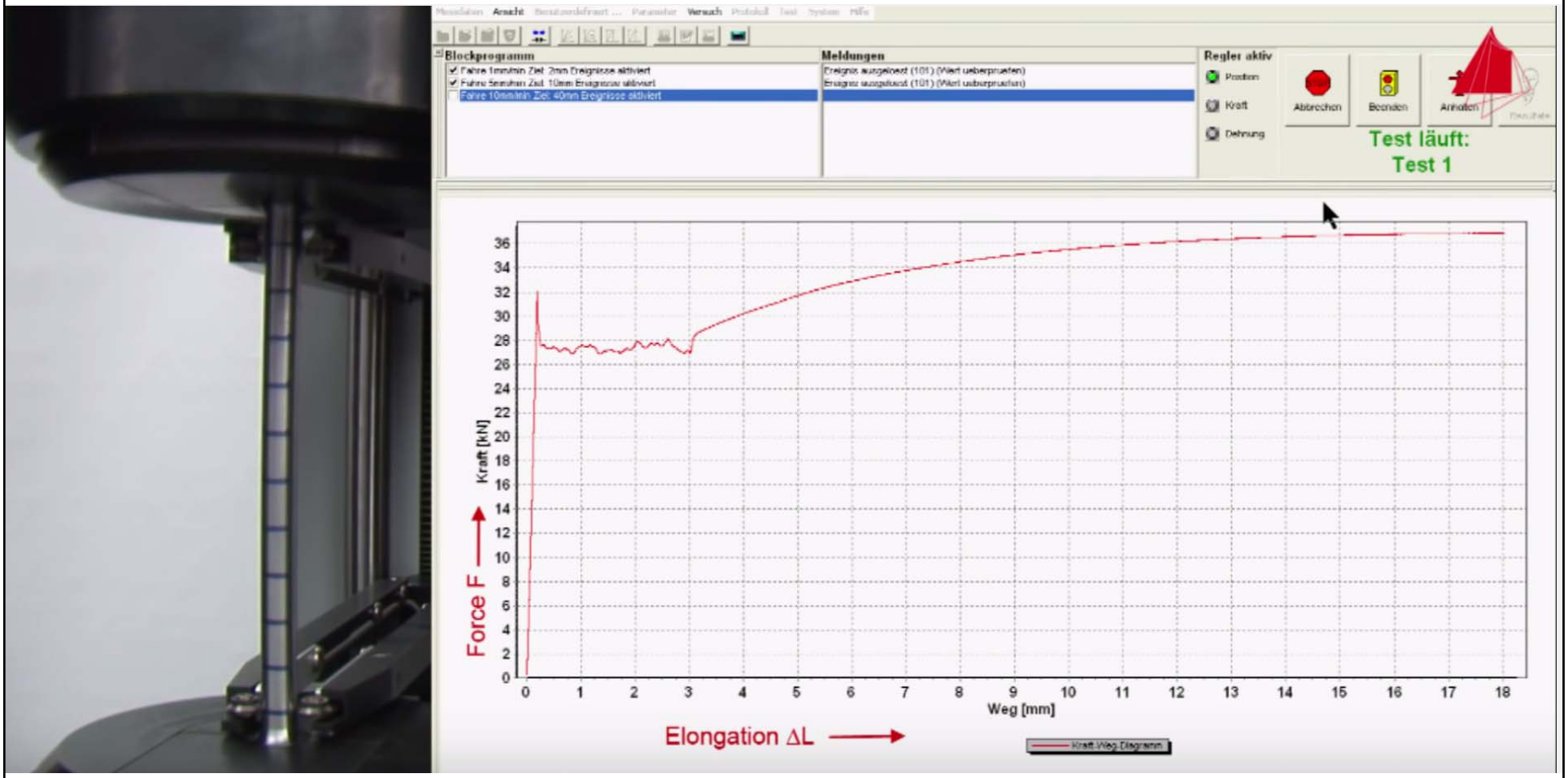
Initial Plasticity and Strain Hardening



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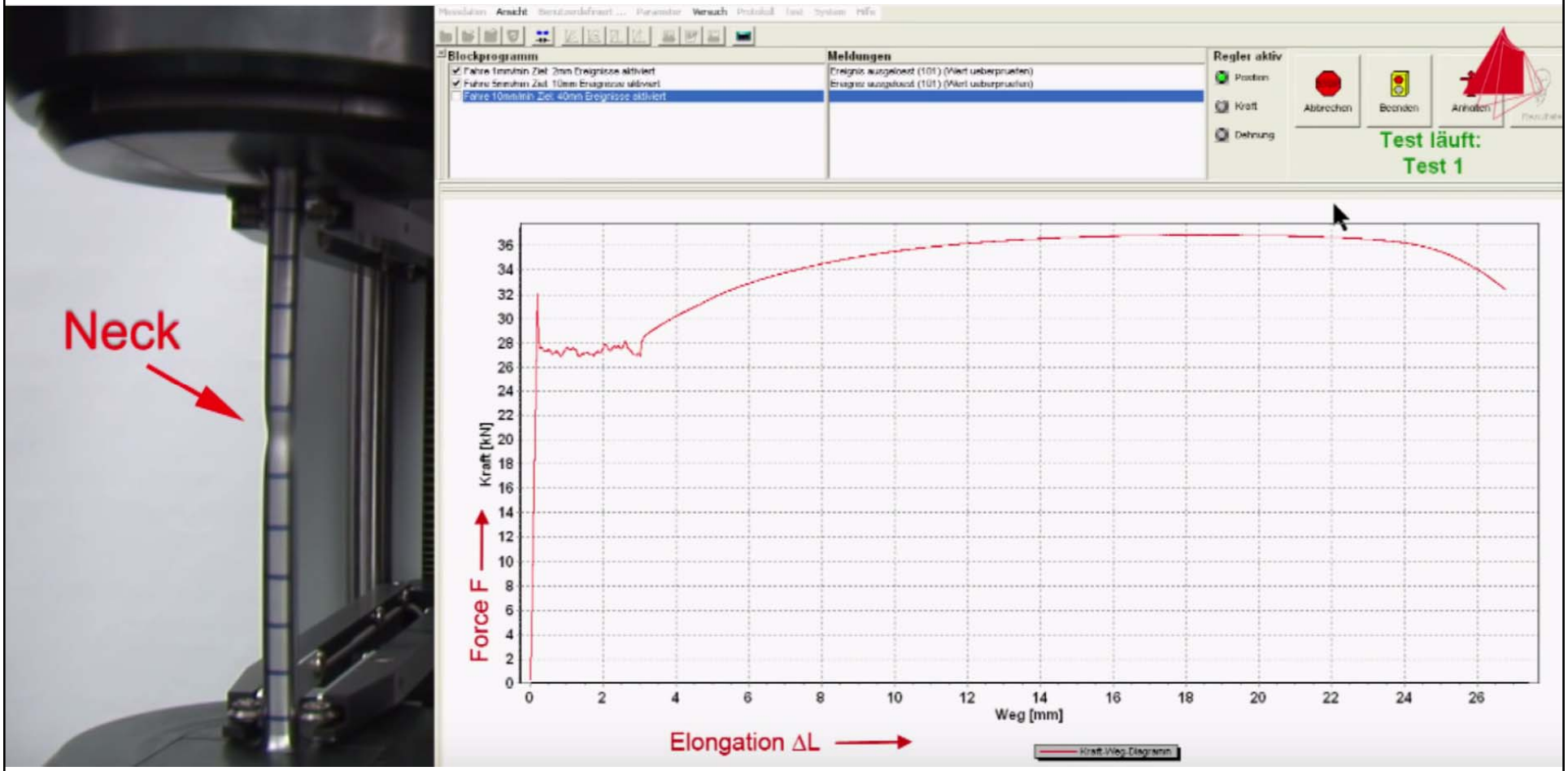
Necking



Images taken from the following youtube video:
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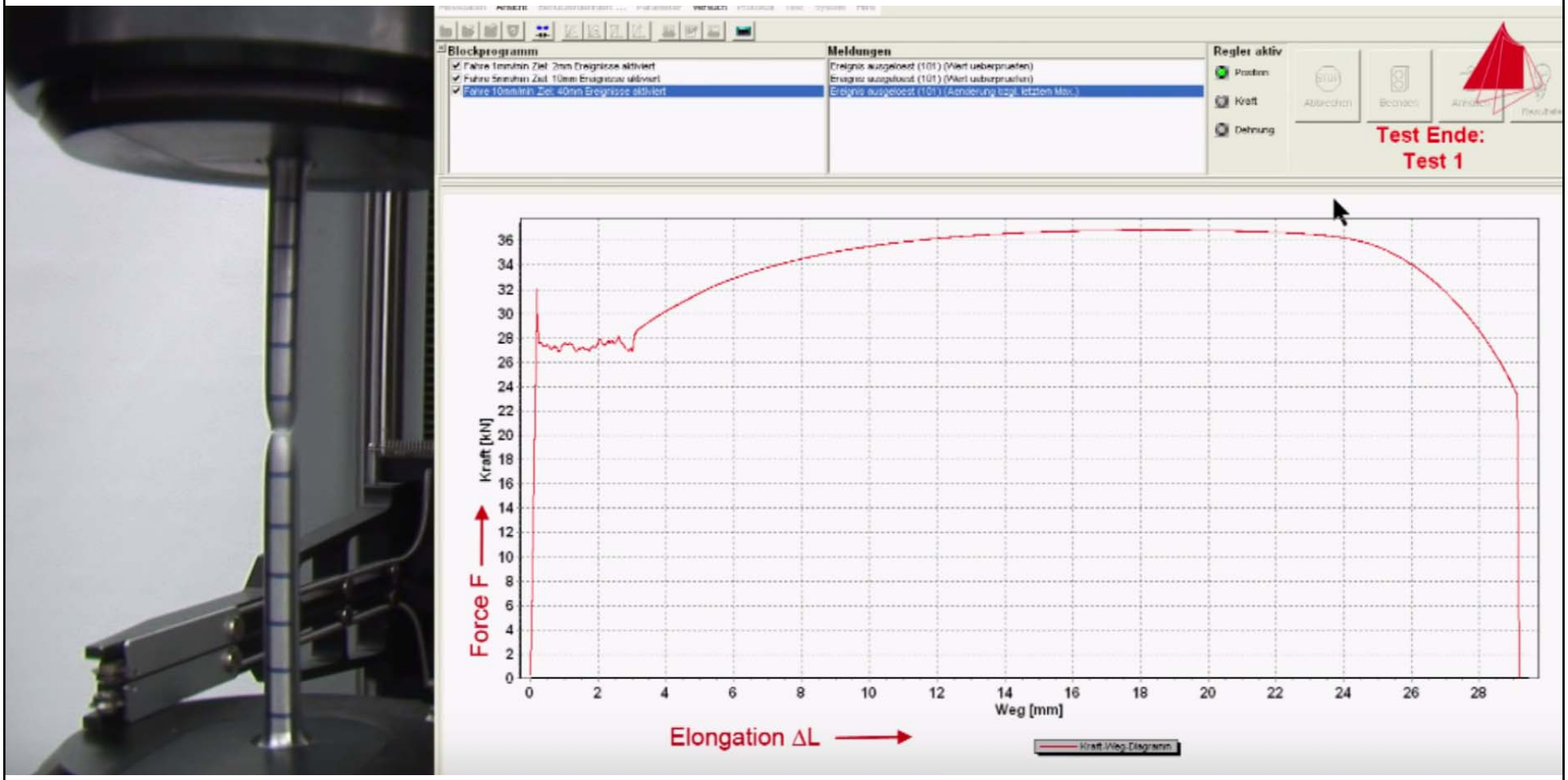
Final Failure



Images taken from the following youtube video:
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Fracture Surface

Why failure at an angle?

Why at an angle of ±45°?

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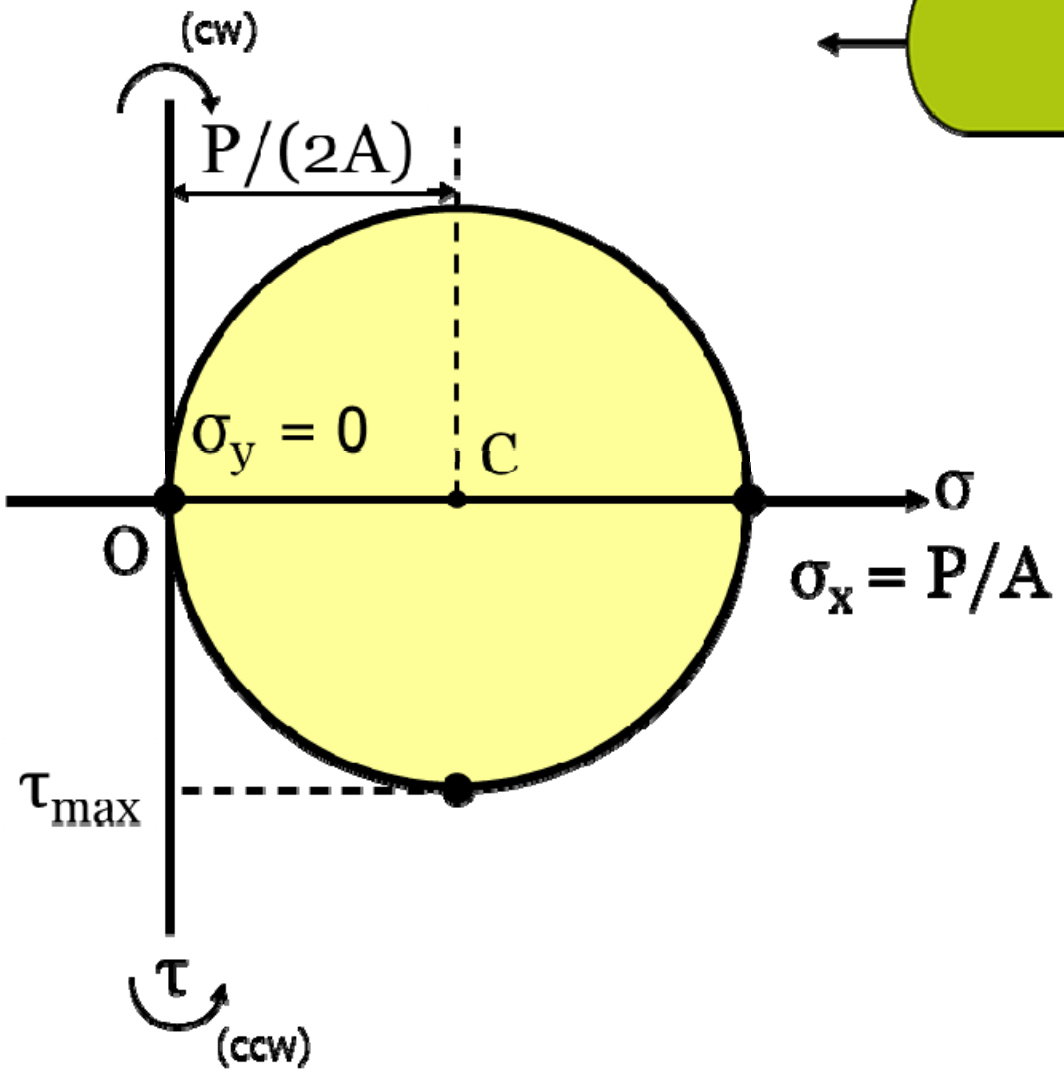
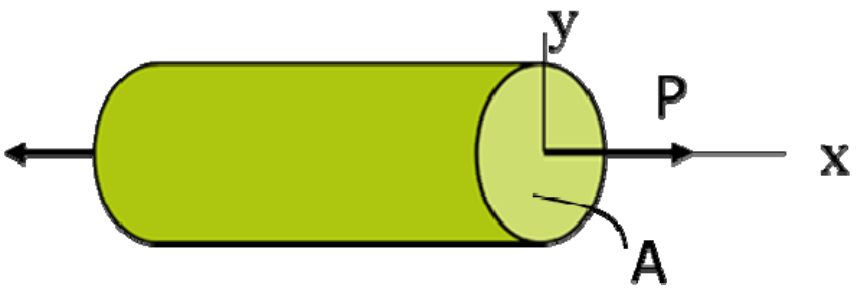
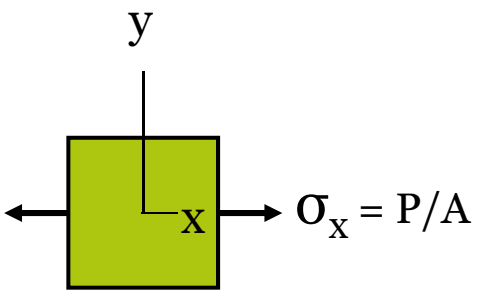
Recall the Mohr Circle for a Uniaxial Test







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Ductile Failure

- What happens on ±45° plane?
  - Maximum shear stress

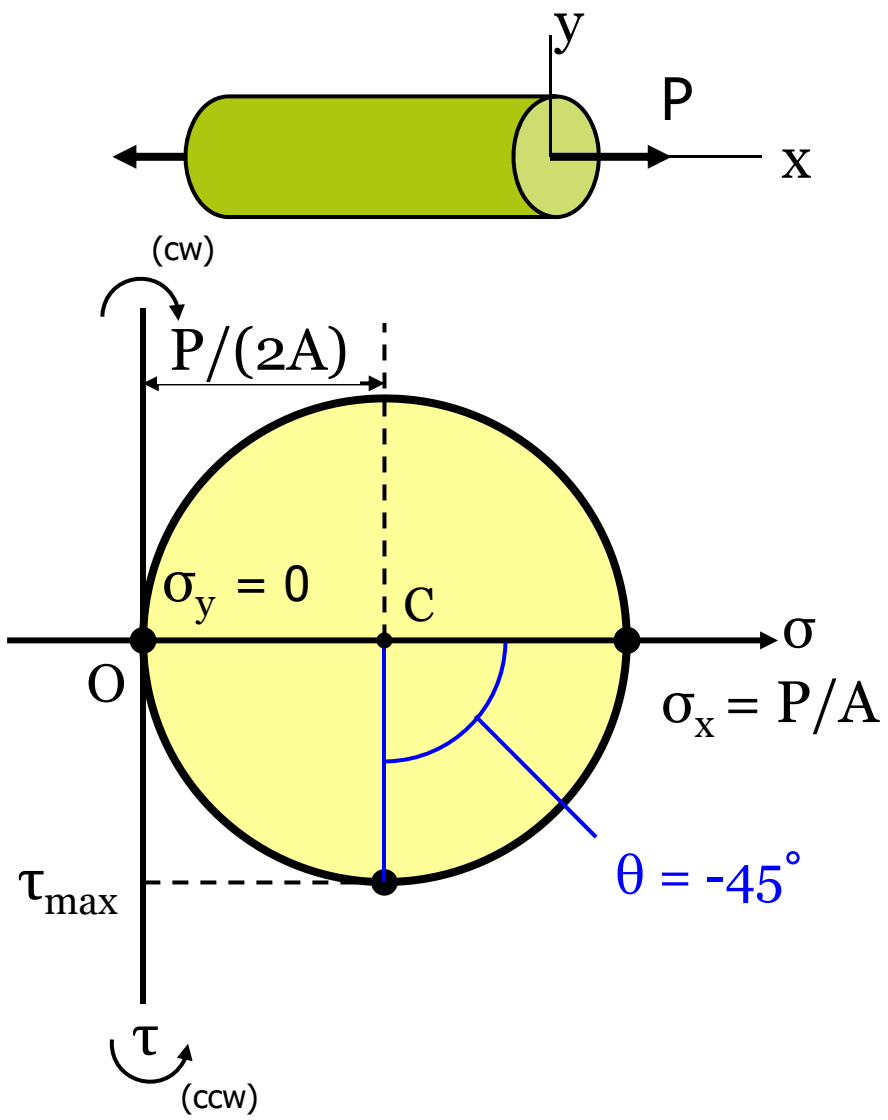
$$\tau_{max} = \frac{\sigma_1}{2}$$

- Yielding occurs when

$$\sigma_1 = \sigma_{yield}$$

- Tresca Yield Criterion

$$\tau_{max} = \frac{\sigma_{yield}}{2}$$



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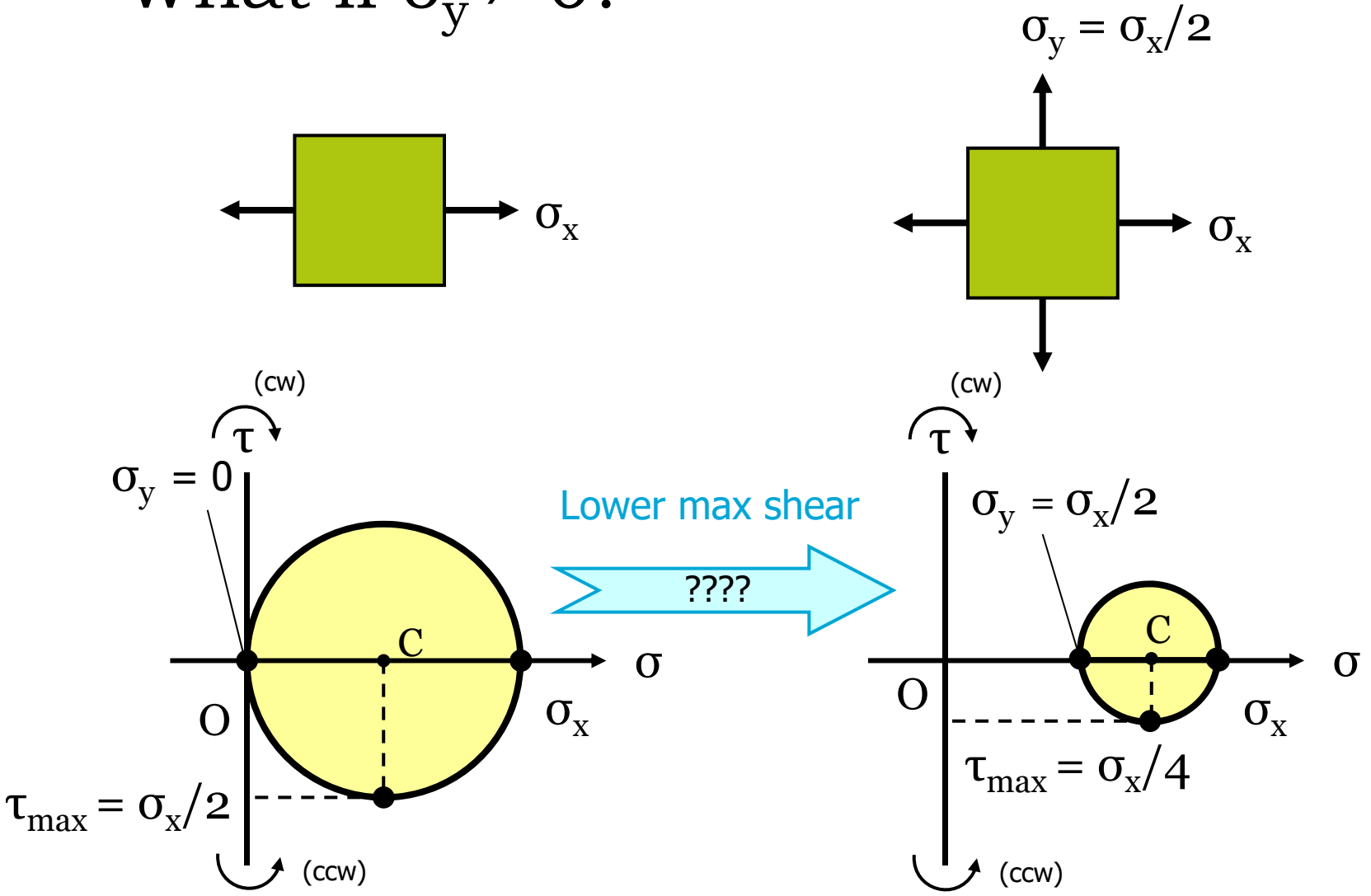
What if $\sigma_y \neq 0$?



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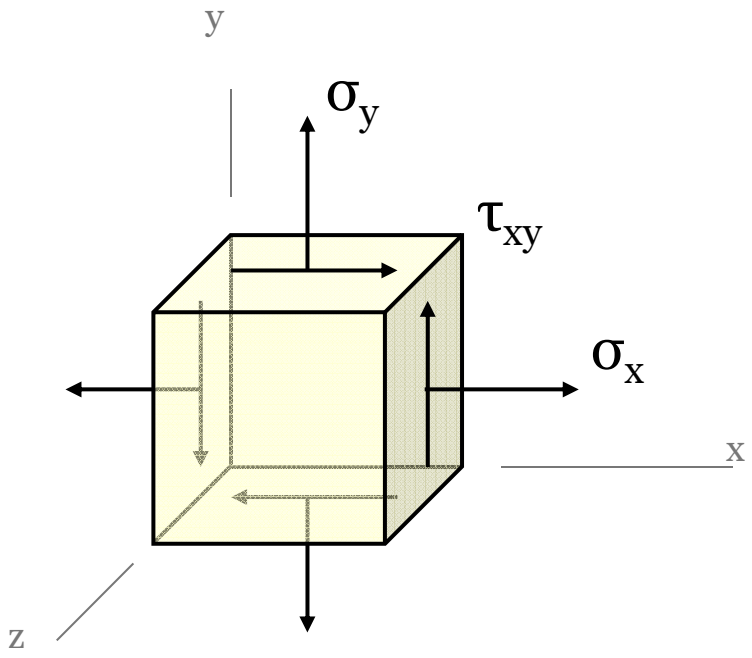
Role of Out-of-Plane Principal Stress

Plane Stress State



Out-of-Plane Direction (z-axis)

$$\sigma_z = 0$$

$$\tau_{zx} = \tau_{zy} = 0$$

Direction where shear stress = 0
(definition of principal stress direction)

$$\sigma_z = 0 = \text{Principal Stress}$$

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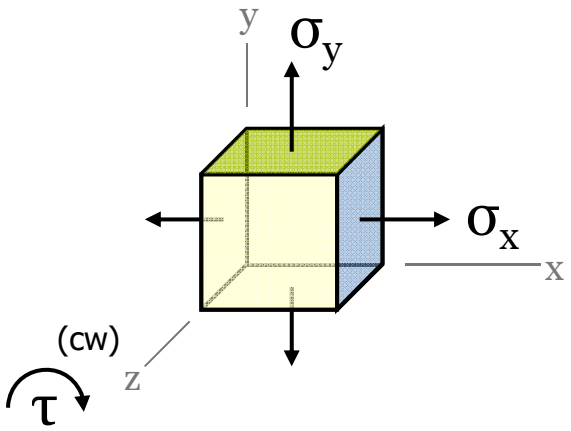
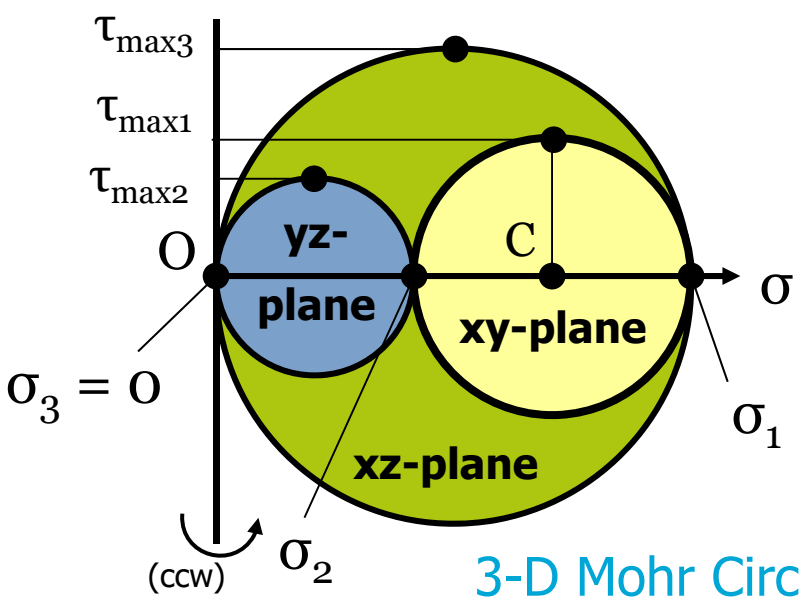
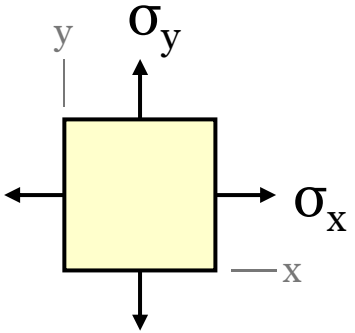
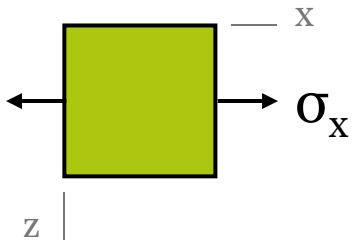
How Can We Visualize?

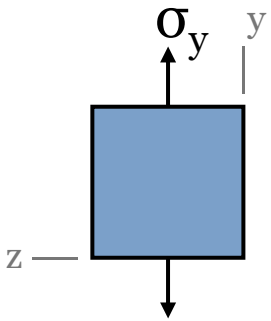




3-D Mohr Circle







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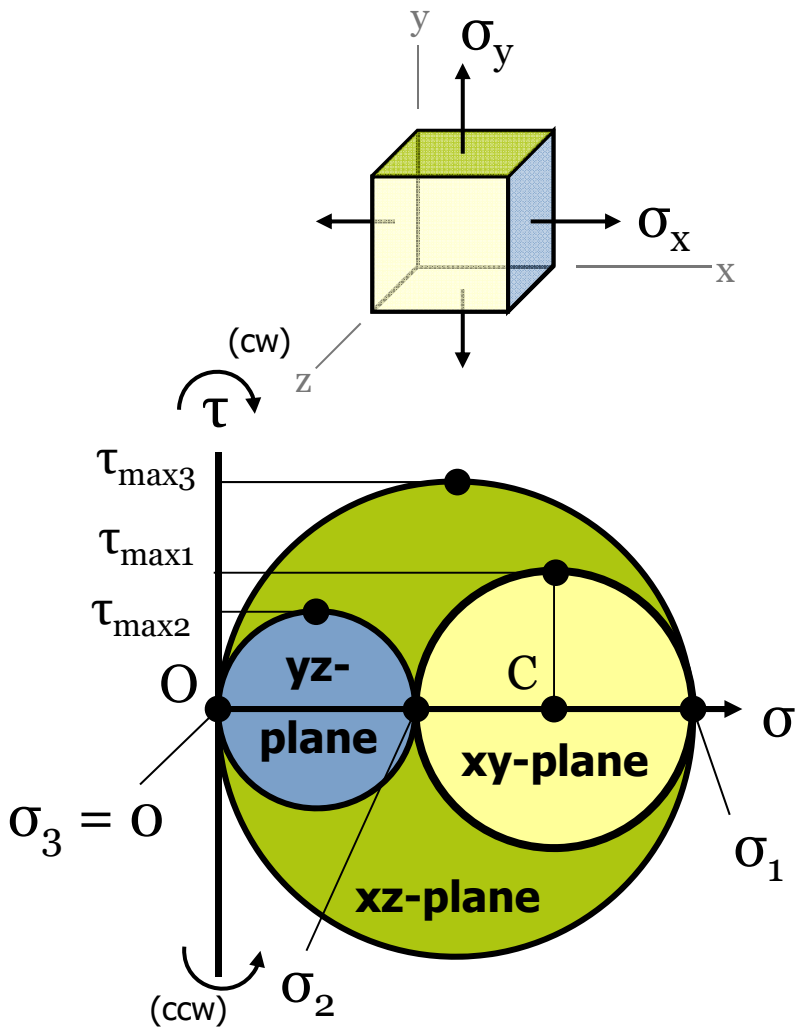
3 Possible Values for Max Shear



$$\tau_{max1} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max2} = \frac{\sigma_2 - \sigma_3}{2}$$

$$\tau_{max3} = \frac{\sigma_1 - \sigma_3}{2}$$

τ_{max} is the largest of these three

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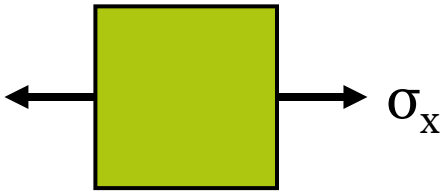
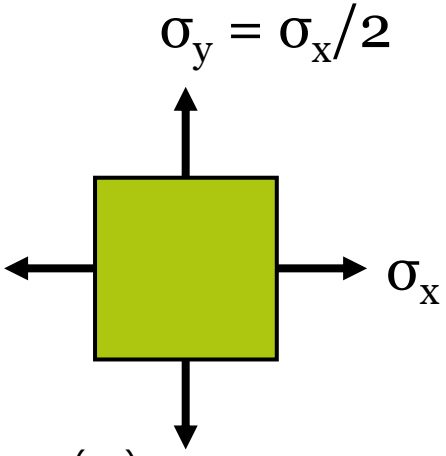
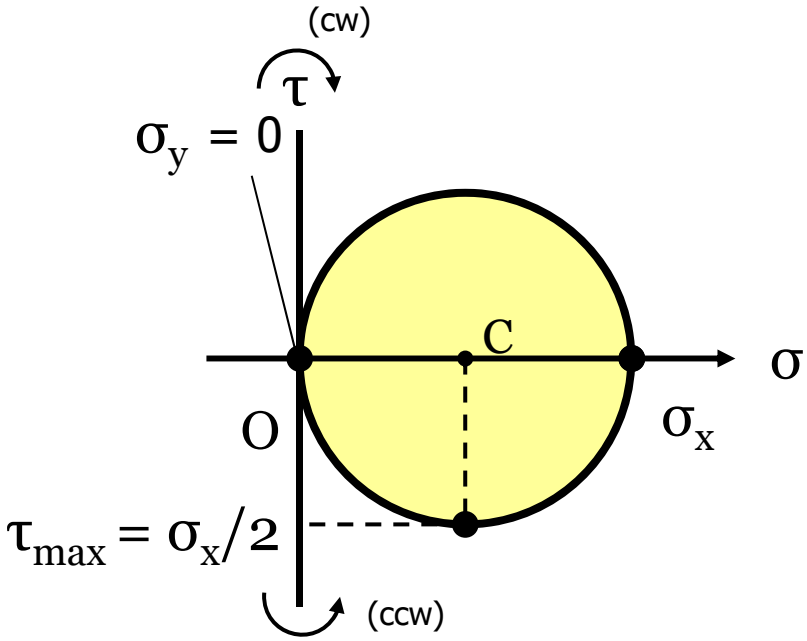
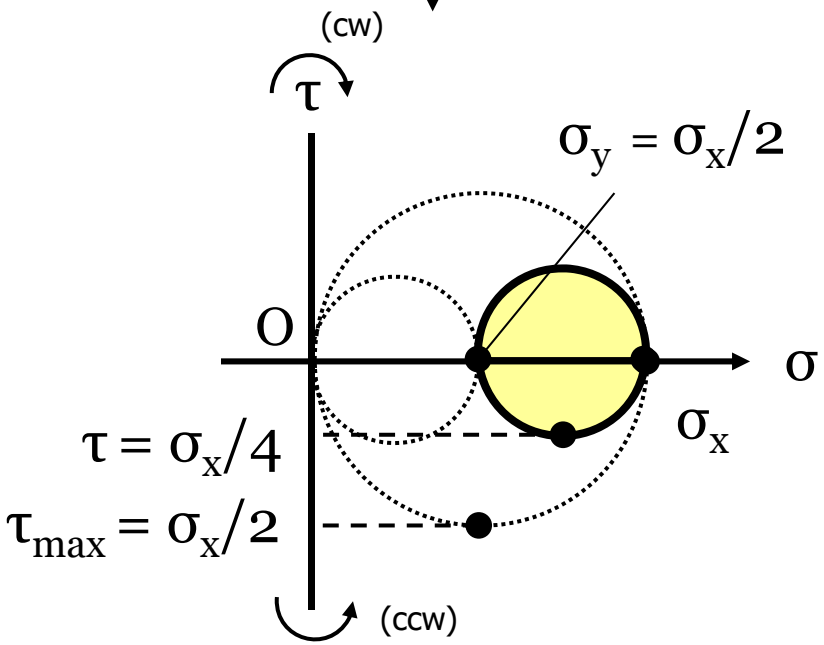
What if $\sigma_y \neq 0$?









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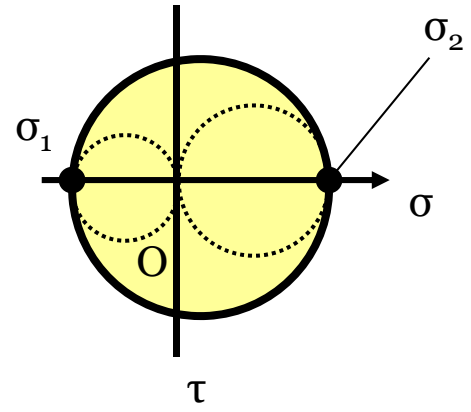
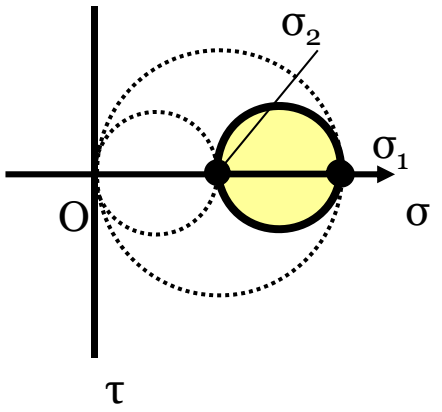
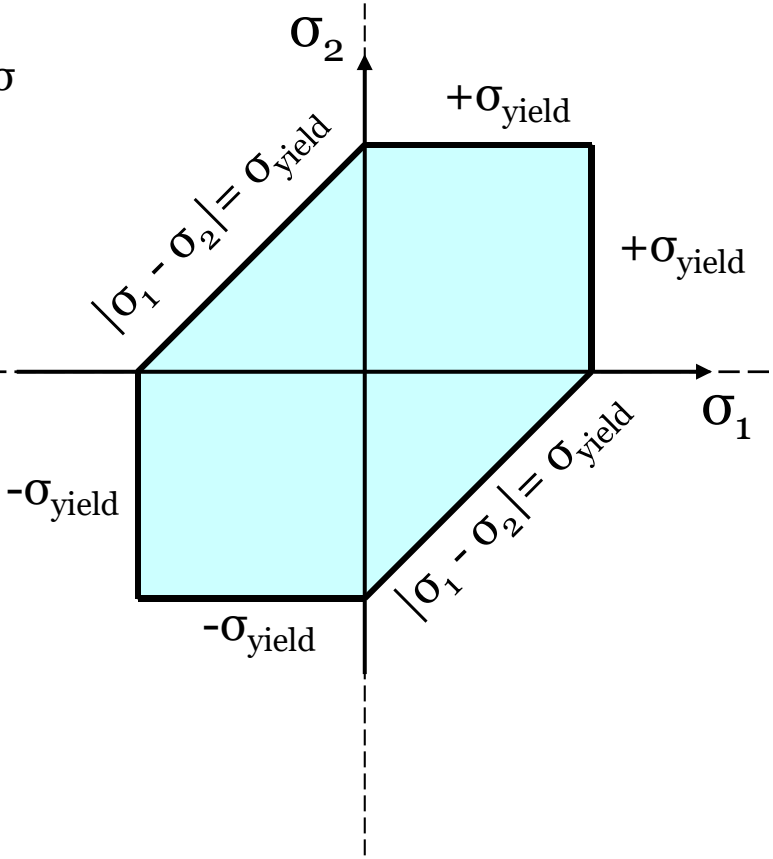
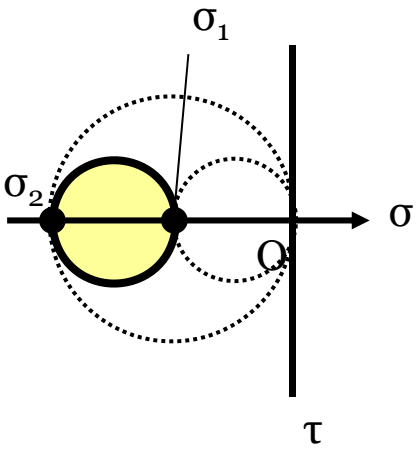
Tresca Yield Criterion

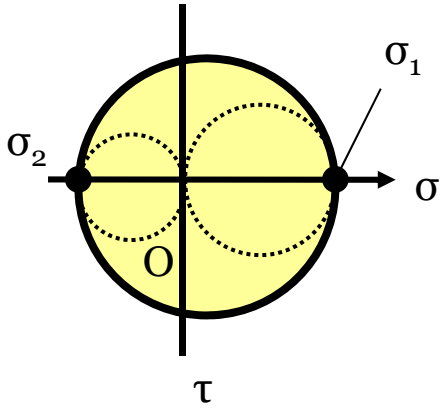
$$\tau_{max} = \frac{\sigma_{yield}}{2}$$











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Other Failure Criteria

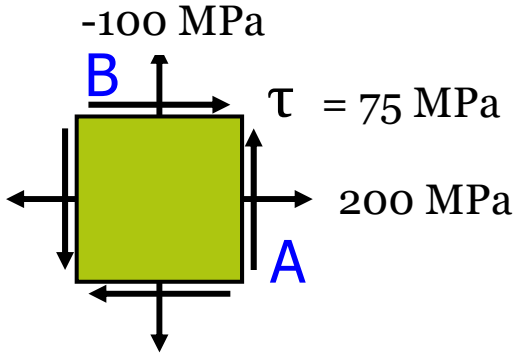
- Von Mises yield criterion (ductile materials)
- Brittle failure criterion (brittle materials)
- Tsai-Wu-Hill failure criterion (composite materials)
- Many, many more!!!!

In this course we will limit ourselves to Tresca yield criterion

Tresca Yield Criterion Example

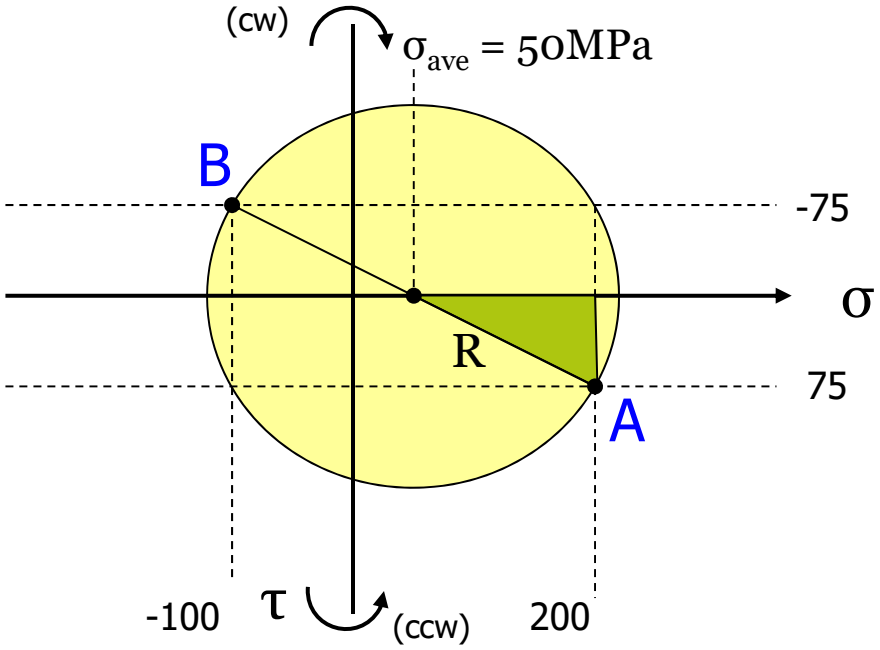
For the given plane stress state at a critical point in a 2024-T3 Al structure, determine if yielding has occurred according to the Tresca Yield Criterion

$\sigma_{yield} = 345$ MPa



Solution: Step 1: Find principal stresses (Mohr's circle)

$$R = \sqrt{(75)^2 + (200-50)^2} = 168MPa$$

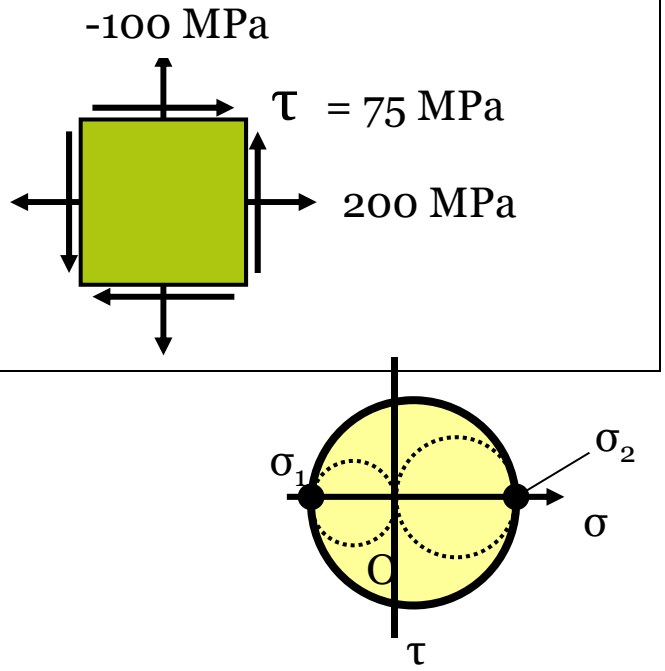


$$\sigma_1 = \sigma_{ave} + R = 218MPa$$
$$\sigma_2 = \sigma_{ave} - R = -118MPa$$
$$\sigma_3 = 0MPa \text{ (plane stress)}$$

Tresca Failure Criterion Example

For the given plane stress state at a critical point in a 2024-T3 Al structure, determine if yielding has occurred according to the Tresca Criterion

$\sigma_{yield} = 345$ MPa



Solution: Step 2: Determine Max Shear Stress

$\sigma_1 = 218MPa \quad \sigma_2 = -118MPa \quad \sigma_3 = 0MPa$

$$\tau_{max\,1} = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = \left|\frac{218 - -118}{2}\right| = 168MPa$$

$$\tau_{max\,2} = \left|\frac{\sigma_2 - \sigma_3}{2}\right| = \left|\frac{-118 - 0}{2}\right| = +59MPa$$

$$\tau_{max\,3} = \left|\frac{\sigma_1 - \sigma_3}{2}\right| = \left|\frac{218 - 0}{2}\right| = 109MPa$$

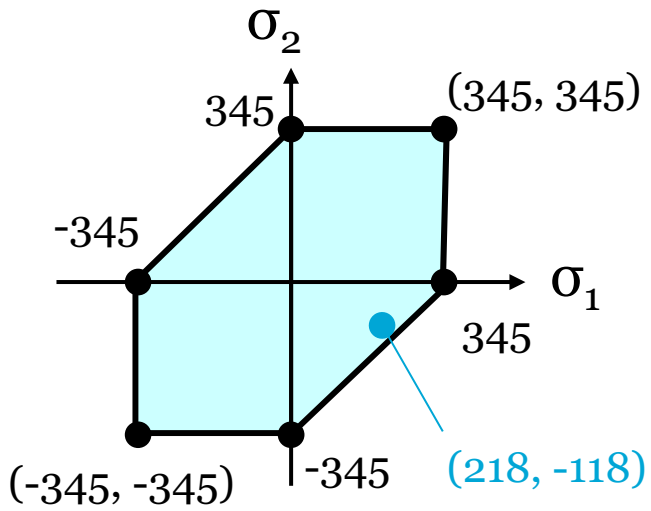
Tresca Failure Criterion Example

$\sigma_1 = 218MPa$ $\quad \sigma_{yield} = 345\ \text{MPa}$

$\sigma_2 = -118MPa$

$\sigma_3 = 0MPa$ (plane stress)

$\tau_{max} = 168MPa$



Step 3: Tresca Criterion

$$\tau_{max} \leq \frac{\sigma_{yield}}{2}$$

$168 \leq 345/2 = 172.5$

$169MPa \leq 172.5MPa$ satisfied, failure does not occur

But it is pretty darn close!