10 FRICTION LOSSES IN PIPELINES

10.1 Introduction

The first empirical flow formulae for turbulent flow in pipes running full were derived from field observations in pipes of over 50 mm diameter. The formulae are comparatively simple and may be solved directly. Variations in fluid viscosity, gravity and temperature are not taken into account. One of these formulae is the Hazen-Williams equation which is still very popular in engineering practice.

The Darcy-Weisbach and Colebrook equations have a rational basis for the analysis and computation of head loss. They are universally applicable in pipes with turbulent flow. In their original form these equations have to be iteratively solved which requires a programmable calculator or the use of tables and graphs. Fortunately, the introduction a slight modification in these formulae results in a much simpler and straight-forward calculation that can be executed on any pocket calculator.

One should bear in mind that entrance losses, exit losses and "minor" losses must also be taken into account in pipeline design. These losses are not discussed in this section.

10.2 The Hazen-Williams equation

Although not recommended for final design, the Hazen-Williams equation could be used in first estimates. It reads

$$v = 0.849 \cdot C \cdot R^{0.63} \cdot s^{0.54} \tag{10.1}$$

where v = mean velocity of flow [m/s]
C = friction coefficient
R = hydraulic radius [m]. For a circular pipe running full: R = 0.25 × diameter.
s = head gradient.

The friction coefficients for large diameter pipes (D ≥ 1000 mm) are tabulated below. For diameters less than 1000 mm, C is reduced by (10 - 0.01 D) %.

Table 10.2 Hazen-Williams friction factors C

Type of pipe	Condition			
	New	25 years old	50 years old	Badly corroded
PVC	150	140	140	130
Smooth concrete, AC.	150	130	120	100
Steel, galvanized or bitumen lined	150	130	100	60
Cast iron,ordinary brick	130	110	90	50
Riveted steel,wood stave	120	100	80	45
Old iron,bad condition	60 - 80			

10.3 Darcy-Weisbach and Colebrook equations

The Darcy-Weisbach equation and the Colebrook equation are generally accepted for accurate calculation of energy losses in pipes with turbulent flow. The derivation of these equations can be found in most fluid mechanics text books and is not repeated here.

The Darcy-Weisbach equation is given as:

$$\Delta H = f \frac{L \cdot v^2}{D \cdot 2g} \quad \text{or} \quad s = \frac{\Delta H}{L} = \frac{f \cdot v^2}{D \cdot 2g} \qquad (10.2)$$

where ΔH = loss of head [m]
- f = friction factor, see below
- L = length of pipe [m]
- D = internal pipe diameter [m]
- v = mean flow velocity [m/s]
- g = gravitational acceleration [m/s^2]
- s = hydraulic gradient = ΔH/L

- In UK texts the friction factor is usually defined in a different way.
  In this text f = 0.25 × UK friction factor.
- Head loss and pressure head are commonly expressed in metres water column, [mwc] or [m].
  In practice: 1 mwc = 10^4 Pa = 10 kPa = 0.1 atm = 0.1 bar.

From hydraulic point of view the turbulent flow in a pipe can be wholly rough, smooth or a combination of the two, see Figure 10.3. The last situation occurs in the so called transition zone, for which Colebrook developed an implicit equation for the friction factor f in commercial pipes.

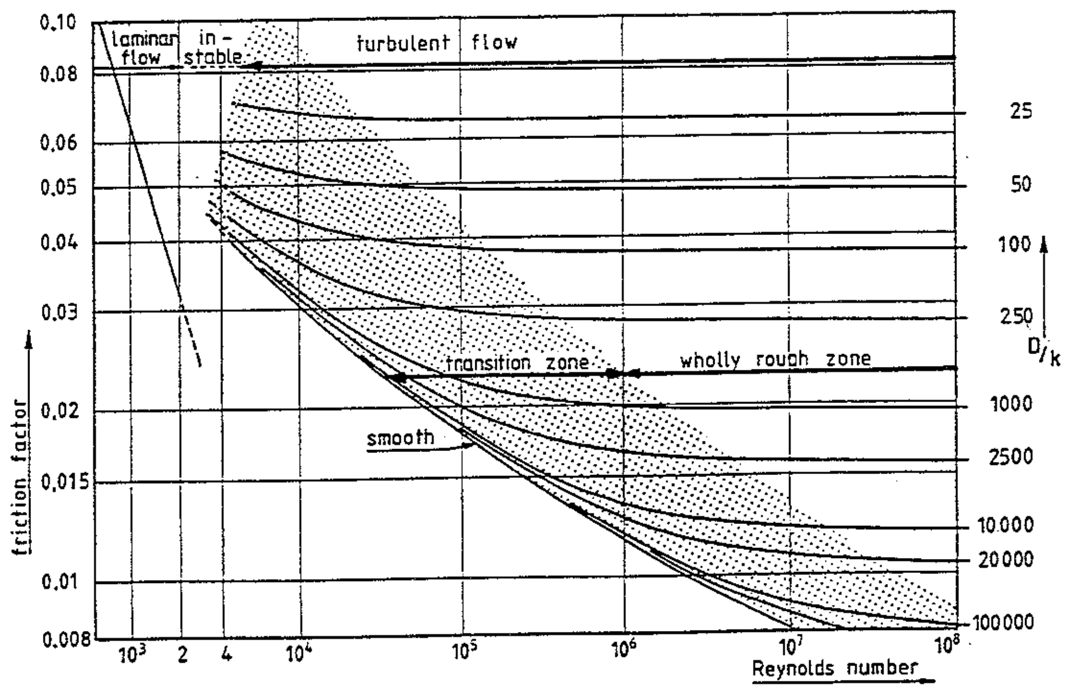

Figure 10.3 Friction factors f for commercial pipes.

The Colebrook equation can be written as:

$$f = 0.25\left(\log\left(\frac{2.51}{Re\sqrt{f}} + \frac{k}{3.72\ D}\right)\right)^{-2} \tag{10.3}$$

where: Re = Reynolds number, here Re = vD/ν. ν = kinematic velocity [m^2 /s]
k = roughness (equivalent sand-grain size) [m]

At high Reynolds numbers the term $2.51/Re\sqrt{f}$ approaches zero, yielding the equation for the wholly rough zone:

$$f = 0.25\left(\log\frac{k}{3.72\ D}\right)^{-2} \tag{10.4}$$

or, after re-arranging :

$$\frac{1}{\sqrt{f}} = 1.14 + 2\log\left(\frac{D}{k}\right) \tag{10.5}$$

In smooth pipes the relative roughness, k/D , is negligibly small thus:

$$f = 0.25\left(\log\frac{2.51}{Re\cdot\sqrt{f}}\right)^{-2} \tag{10.6}$$

or:

$$\frac{1}{\sqrt{f}} = -0.8 + 2\log\left(Re\sqrt{f}\right) \tag{10.7}$$

Equations 10.4 (10.5) and 10.6 (10.7) are in accordance with the measurements of Nikuradse and earlier investigators.

Nikuradse's original experiments used sand with various diameters as artificial boundary roughness. In commercial pipes the roughness is evaluated to the equivalent sand roughness with a sand grain diameter of k metre.

Table 10.3 Roughness k of pipe materials (equivalent sand roughness).

Materials	k [$\times 10^{-3}$ m]	Materials	k [$\times 10^{-3}$ m]
Glass, drawn metals	0.005	Spun concrete, new cast iron, smooth concrete	0.5
Asphalted steel, PVC, AC.	0.02	Concrete, old riveted steel	1
Steel,large dia PVC	0.05	Corroded steel, rough concrete	2
Asphalted cast iron, cement lined	0.1		

The friction factor f is not very sensitive to the value of k assumed. To estimate the friction in AC and PVC pipes after 25 years, f is often multiplied by two.

The kinematic viscosities of water at various temperatures are listed below.

Temperature [C°]	0	5	10	15	20	25	30
Kinematic viscosity ν [$\times 10^{-6}$ m^2/s]	1.79	1.52	1.31	1.15	1.01	0.90	0.81

Due to its implicit nature, the Colebrook equation (eq.10.3) is not easily applicable to engineering problems in the transition and smooth zones. If no computer is at hand, the Moody diagram*) or approximations are commonly used. However, the diagram requires logarithmic interpolation and the approximations may yield large errors.

10.4 Recommended flow formulae

It is possible to find an explicit approximation of the friction factor in smooth pipes that can be written as $f = 0.25 (\log p)^{-2}$, which allows a combination to be made with the similar equation for the wholly rough zone.
In the smooth situation the friction factor f is very closely approximated by:

$$f = 0.25 \left(\log \frac{5.8}{Re^{0.9}} \right)^{-2} \tag{10.8}$$

Combination with the equation for the wholly rough zone (eq.10.3) yields:

$$f = 0.25 \left(\log \left(\frac{5.8}{Re^{0.9}} + \frac{k}{3.72\, D} \right) \right)^{-2} \tag{10.9}$$

As the difference between the friction factor f calculated after this approximation and the f from the original Hazen-William formula is 0.5% or less, the use of equation 10.9 is recommended.
Although the equation still doesn't look quite attractive, the computation of the explicit expression can be made on a pocket computer having y^x, $\log x$, x^2 and $1/x$ as standard function available.
The equation is also very convenient in pocket computer programmes.

10.5 Design of pipe diameters

In engineering practice, where the flow and the roughness are normally known, the pipe diameter has to be assumed before the head loss can be computed. Next it is decided if the loss is technically and economically acceptable. It is not uncommon that in pressurized irrigation systems the optimum cost for installed pipeline and power requirement is obtained when the hydraulic gradient is a few percent (pipeline diameter < 100 mm) or when the flow is limited to 1.5 - 2 m/s (pipelines > 100 mm).

*) Moody plotted the Colebrook equation similar to Fig 10.3 but on a larger scale.

As a rule of thump in first estimates, the pipe diameter may follow from

$$D = 1.3 \cdot k^{0.03} \cdot Q^{0.44} \tag{10.10}$$

Also, if in a part of the system a certain pressure slope is required or available,

$$D = 0.31 \cdot s^{-0.2} \cdot k^{0.02} \cdot Q^{0.38} \tag{10.11}$$

It is emphasized that these estimative formulae can not be used as flow formulae in final design.
The formulae are applicable to the transport of water and do not involve the change of fluid viscosity at various temperatures, see also example 1.

10.6 Examples

Example 1. Given an irrigation pipeline: internal diameter D = 593 mm, length L = 400 m, roughness k = 0.1 mm, flow Q = 1080 m^3/h.
Find head losses at temperatures of 5° C and 25° C.

Computations: Q = 1080 m^3/h = 0.3 m^3/s, k = 0.1 mm = 0.0001 m, D = 593 mm = 0.593 m.
At 5°C the viscosity is $\nu = 1.52 \times 10^{-6}$ m^3/s

$$v = \frac{4Q}{\pi D^2} = 1.086 \text{ m/s}, \quad Re = \frac{1.086 \times 0.593}{1.52 \times 10^{-6}} = 424000, \quad Re^{0.9} = 116000$$

$$\text{Eq. 10.9:} \quad f = 0.25\left(\log\left(\frac{5.8}{116000} + \frac{0.0001}{3.72 \times 0.593}\right)\right)^{-2} = 0.0155$$

$$\text{Eq. 10.2:} \quad \Delta H = \frac{0.0155 \times 400 \times 1.086^2}{0.593 \times 2 \times 9.81} = 0.62 \text{ m}$$

Similarly, at 25°C and $\nu = 0.9 \times 10^{-6}$:
Re = 715600; $Re^{0.9}$ = 185900; Eq.10.9: f = 0.01476; Eq.10.2: ΔH = 0.60 m

In irrigation pipeline design, the small reduction in head loss at higher temperatures is usually neglected.

Example 2. In a sprinkler irrigation project, pipes with the following internal diameters are available: 581; 461; 369; 291; 231; .... 67.8; 57.0; 45.2 [mm].
Roughness k = 0.03 mm. Design temperature is 10°C, hence $\nu = 1.31 \times 10^{-6}$ m^2/s.
Select a suitable pipe diameter for a flow Q = 800 m^3/h and compute the head loss over L = 680 m.

Computations: Q = 800 m^3/h = 0.222 m^3/s. k = 3×10^{-5} m
Use eq.10.10 for the first estimate of a suitable diameter: $D = 1.3 \times (3 \times 10^{-5})^{0.03} \times 0.222^{0.44} = 0.49$ m.
From the available diameters a 581 mm pipe or a 461 mm pipe would probably be the best choice.
For D = 461 mm (= 0.461 m) :

$$v = \frac{4Q}{\pi D^2} = 1.33 \text{ m/s}, \quad Re = \frac{1.33 \times 0.461}{1.31 \times 10^{-6}} = 4.685 \times 10^5 \quad Re^{0.9} = 126944$$

$$\text{Eq. 10.9:} \quad f = 0.25\left(\log\left(\frac{5.8}{126944} + \frac{0.00003}{3.72 \times 0.461}\right)\right)^{-2} = 0.0141$$

$$\text{Eq. 10.2:} \quad \Delta H = \frac{0.0141 \times 680 \times 1.33^2}{0.461 \times 2 \times 9.81} = 1.885 \text{ m}$$

Similarly, for a 581 mm pipe: D = 581 mm = 0.581 m; v = 0.838 m/s;
Re = 715600; $Re^{0.9}$ = 185900; Eq.10.9: f = 0.01476; Eq.10.2: ΔH = 0.60 m.

Example 3. In a sprinkler irrigation project, pipes with the following internal diameters are available:
581; 461; 369; 291; 231; .... 67.8; 57.0; 45.2 [mm].
Roughness k = 0.03 mm. Design temperature is 10°C, hence $\nu = 1.31 \times 10^{-6}$ m²/s
Select a suitable pipe diameter for a flow Q = 7.2 m³/h and compute the head gradient (friction slope).

Computations: $Q = 7.2$ m³/h $= 0.002$ m³/s. $k = 3 \times 10^{-5}$m
Use eq. 10.10 to estimate a suitable diameter: $D = 1.3 \times (3 \times 10^{-5})^{0.03} \times 0.002^{0.44} = 0.062$ m.
From the available diameters a 57 mm pipe or a 67.8 mm pipe would probably be the best choice.
For D = 57 mm (= 0.057 m) :

$$v = \frac{4Q}{\pi D^2} = 0.7837 \text{ m/s}, \qquad Re = \frac{0.784 \times 0.057}{1.31 \times 10^{-6}} = 34100, \qquad Re^{0.9} = 12000$$

Eq. 10.9: $$f = 0.25 \left(\log\left(\frac{5.8}{12000} + \frac{0.000\,03}{3.72 \times 0.057}\right)\right)^{-2} = 0.0244$$

Eq. 10.2: $$s = \frac{0.0244 \times 0.784^2}{0.057 \times 2 \times 9.81} = 0.0133 = 1.33\%$$

Similarly, for a 67.8 mm pipe:
D = 67.8 mm = 0.678 m; v = 0.554 m/s; Eq.10.9: f = 0.01476; Eq.10.2: s = 0.575%

Example 4. In a sprinkler irrigation project, pipes with the following internal diameters are available:
581; 461; 369; 291; 231; .... 67.8; 57.0; 45.2 [mm].
k = 0.03 mm. Design temperature is 10°C $\Rightarrow \nu = 1.31 \times 10^{6}$ m²/s
Select a suitable pipe diameter for a flow Q = 300 m³/h over a length L = 2.2 km if the available head over this reach is ΔH = 10.6 m.

Computations: $Q = 300$ m³/h $= 0.0833$ m³/s, $k = 3 \times 10^{-5}$m, $s_{available} = 10.6/2200 = 0.0048$
Use eq. 10.11 for a preliminary estimate :
$D = 0.311 \times 0.0048^{-0.2} \times (3 \times 10^{-5})^{0.02} \times 0.0833^{0.38} = 0.286$ m.
From the available diameters a 291 mm pipe is selected : D = 291 mm (= 0.291 m)

$$v = \frac{4Q}{\pi D^2} = 1.255 \text{ m/s}, \qquad Re = \frac{1.255 \times 0.291}{1.31 \times 10^{-6}} = 278000, \qquad Re^{0.9} = 79400$$

Eq. 10.9: $$f = 0.25 \left(\log\left(\frac{5.8}{79400} + \frac{0.000\,03}{3.72 \times 0.291}\right)\right)^{-2} = 0.01565$$

Eq. 10.2: $$\Delta H = \frac{0.01565 \times 2200 \times 1.252^2}{0.291 \times 2 \times 9.81} = 9.46 \text{ m} \quad (< 10.6 \text{ m})$$

Make sure that in these computations the flow velocity remains low enough to avoid adverse effects like cavitation, water hammer, etc.