

G-L-1

# Review

The Modelling Team  
Department of Design Engineering  
Faculty of Industrial Design Engineering  
Delft University of Technology

# Aim



To pick-up knowledge learned before



To be able to interpret this knowledge in the Modelling Framework



To be better prepared for Modelling



# Contents

IO2081: G-L-1 Review

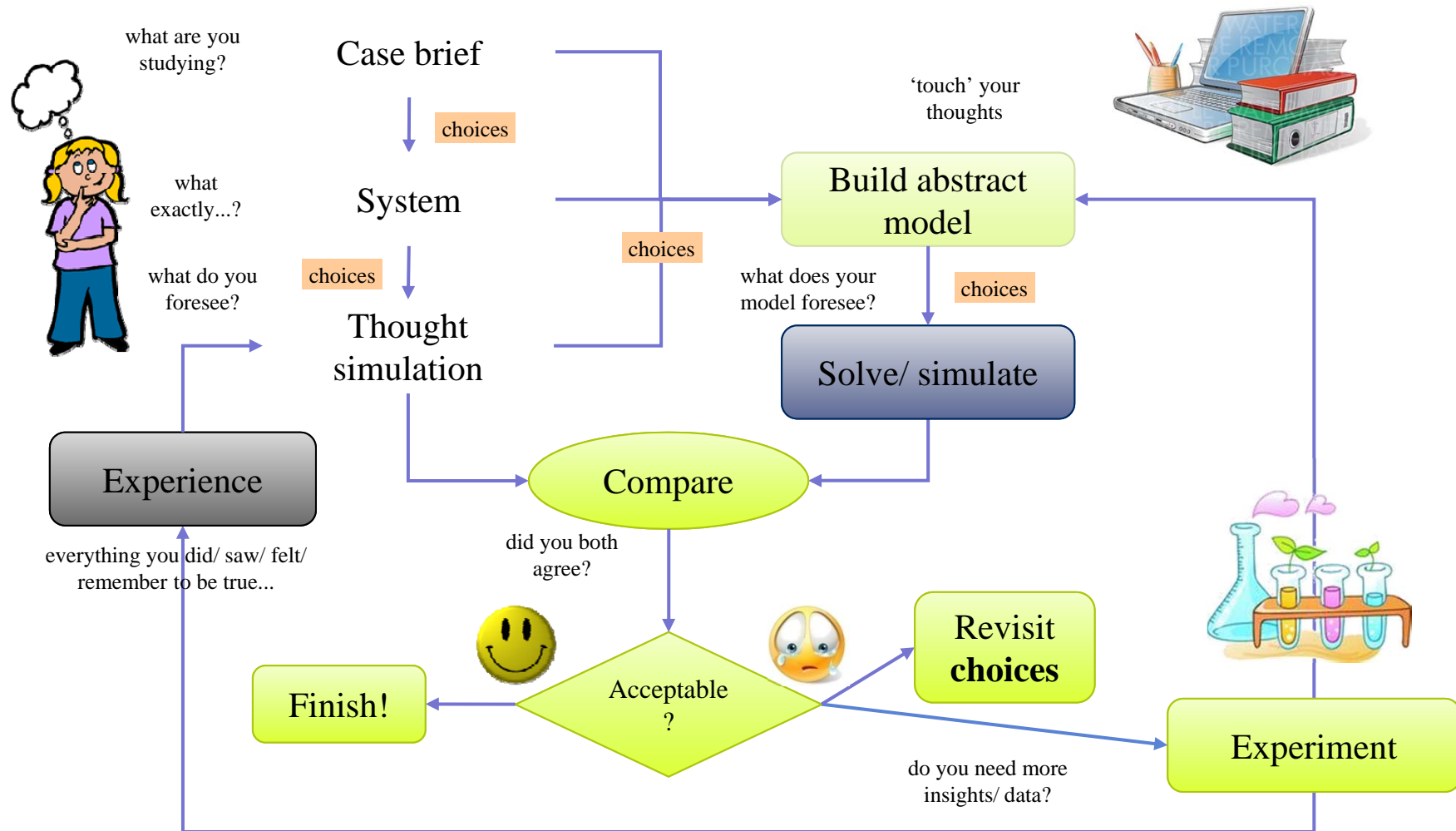
- Our vision
- Statics – Product in Action
- Dynamics – Product in Motion
- Summary
- The workshop

# Our vision





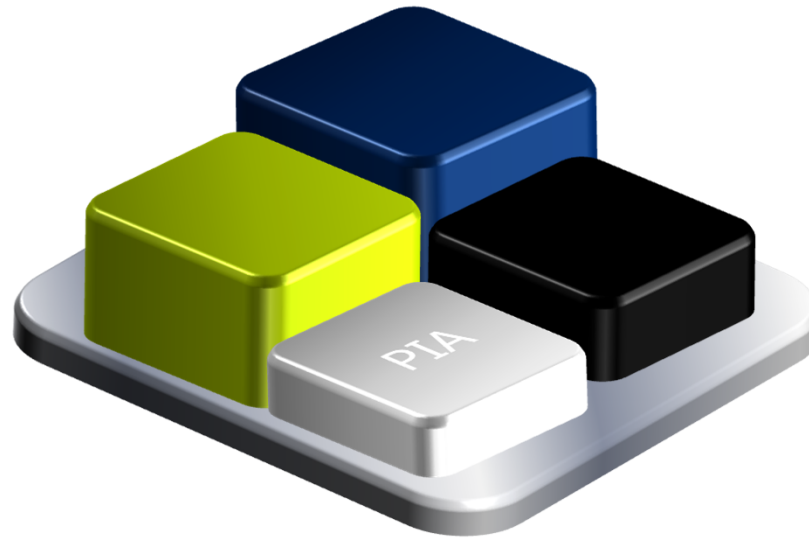
# The Modeling Framework



Courtesy of centech.com.pl and <http://www.clipsahoy.com/webgraphics4/as5814.htm>

# Statics Case study: Nut Cracker

Special Thanks to ir. Ernest van Breemen



# Design Brief

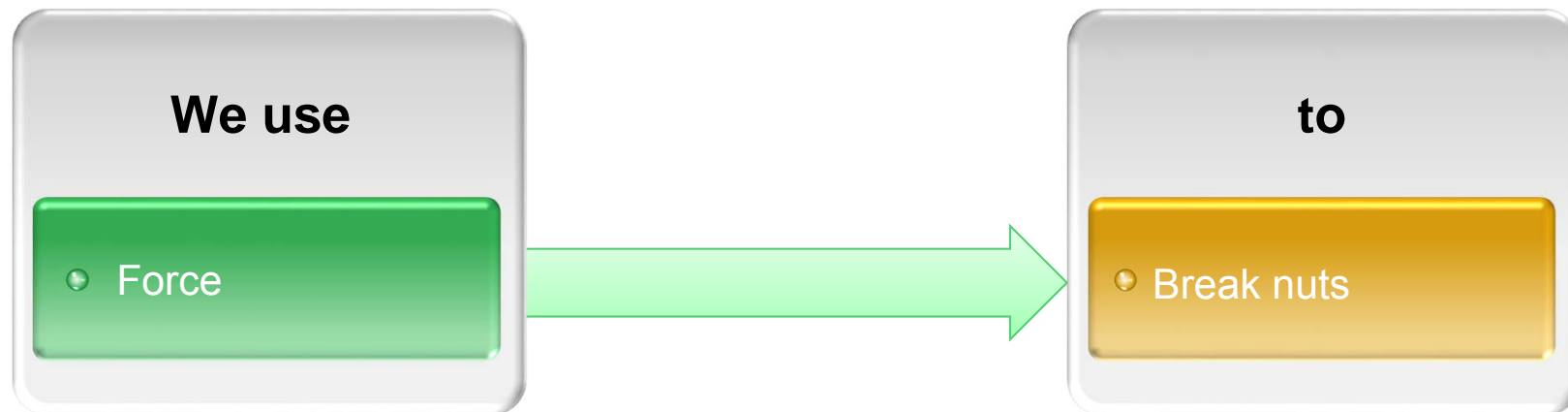
## A portable nut cracker

- Crack nuts by hand force
- Adapt to different sizes of nuts

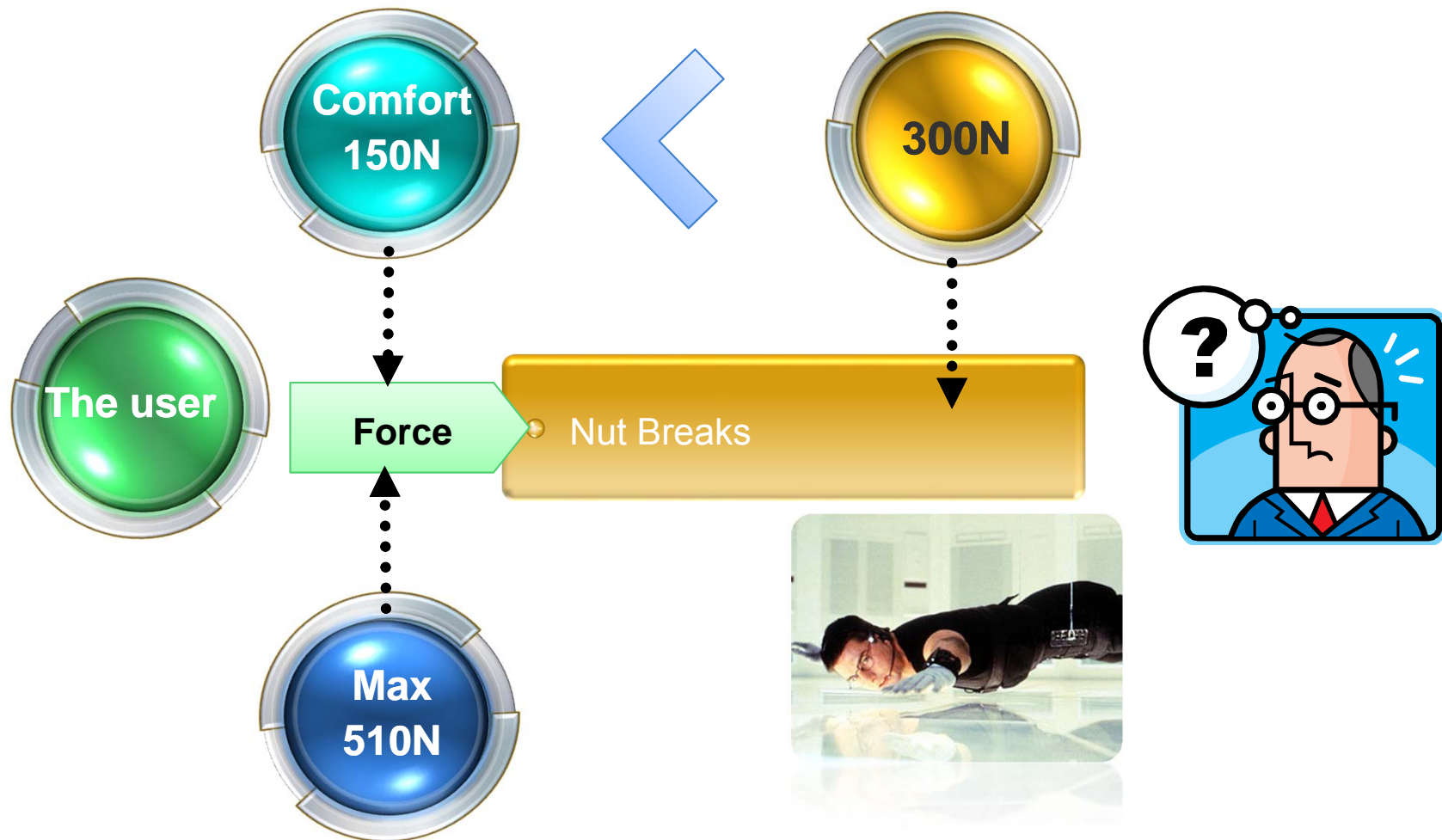


Ref. <http://www.lizandlaura.com/fun-and-games/fortune/?read=1301456242>

# The basic thought



# How about the force?



# What shall we do?

*Give me a place to stand,  
and I shall move the earth.*

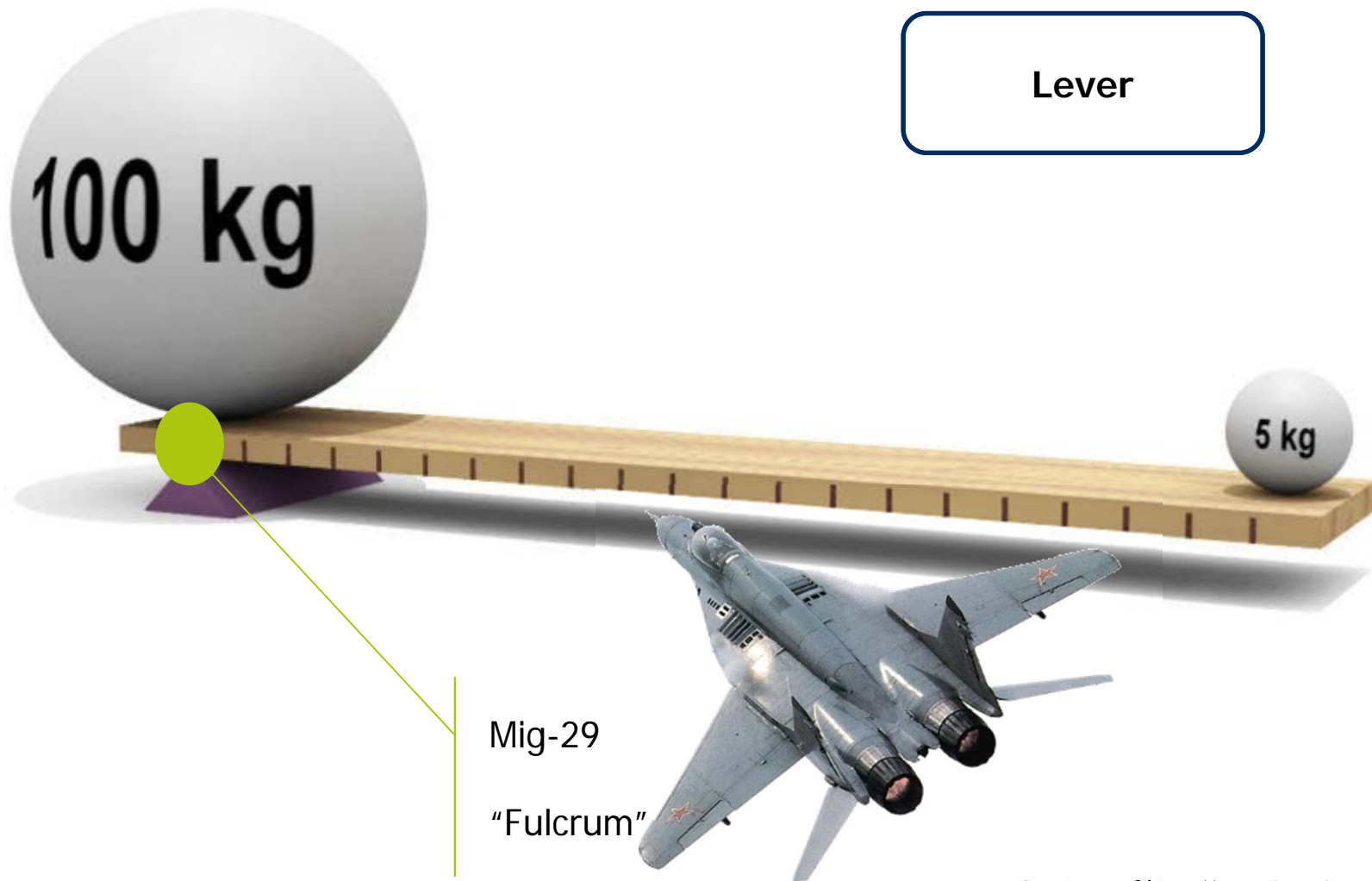


**Archimedes of Syracuse**

Courtesy of <http://en.wikipedia.org/wiki/Archimedes>

# The lever

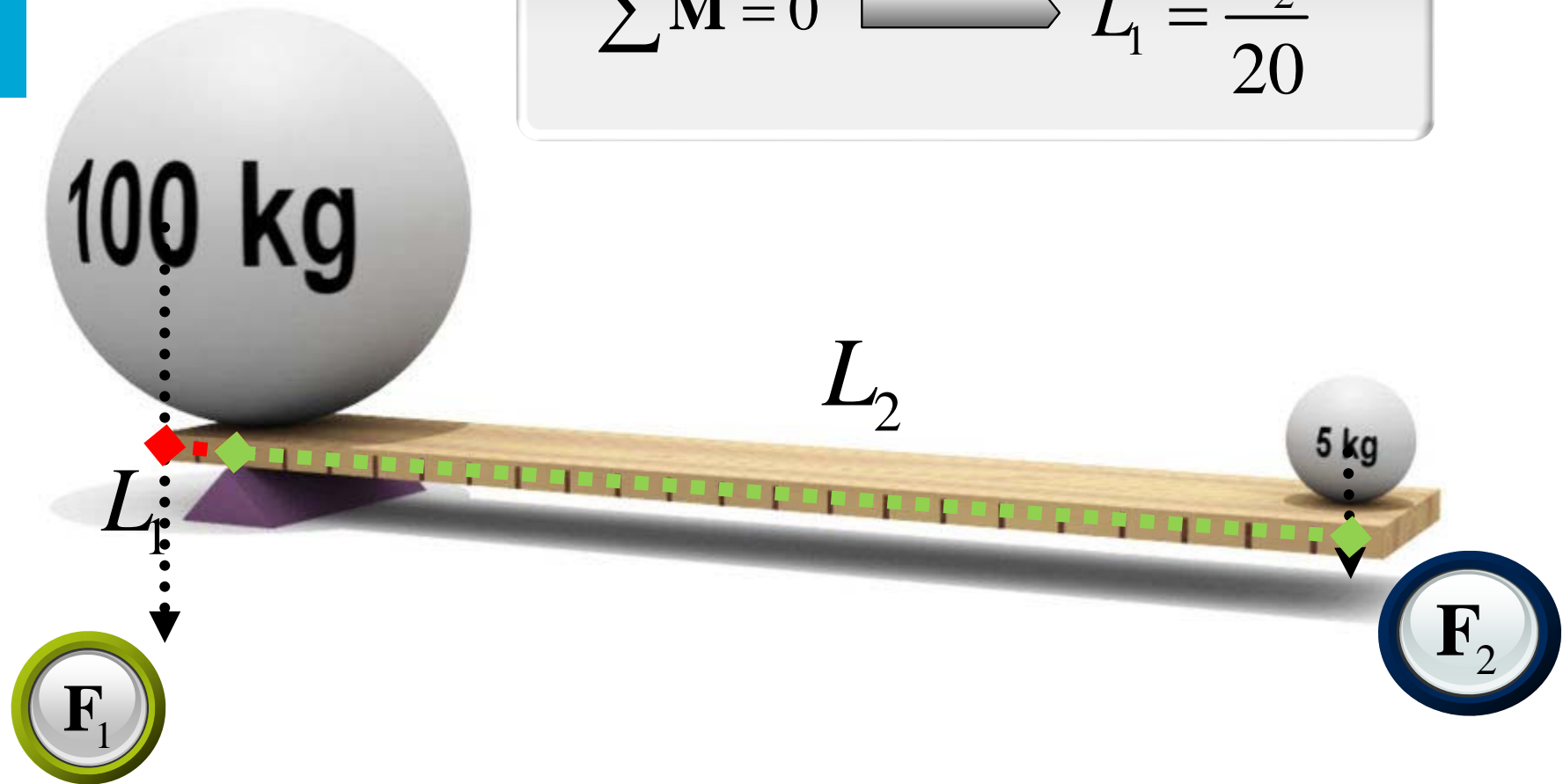
Lever



Courtesy of <http://en.wikipedia.org/wiki/Lever>

## Physics behind

$$\sum M = 0 \longrightarrow L_1 = \frac{L_2}{20}$$



Courtesy of <http://en.wikipedia.org/wiki/Lever>



# The Experience

1

Use the theory of the lever

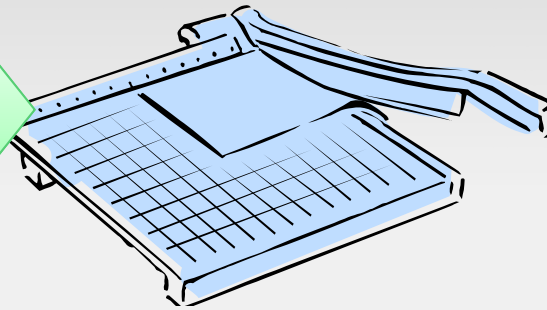
To zoom force at least two times larger

2

Design a place to stand

# The first idea

Something similar



- It should work

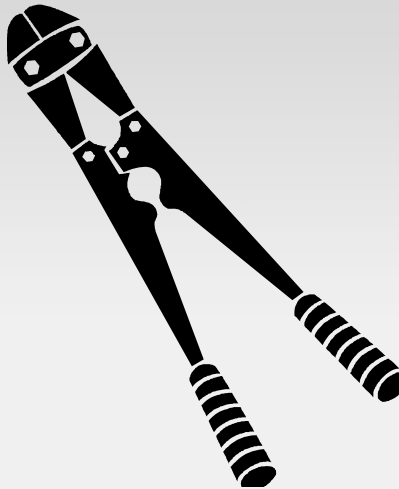
But

- Too large

- Not potable

# Then

Something similar



- It should work

- Portable

But

- Too large

# The breakthrough

Something similar

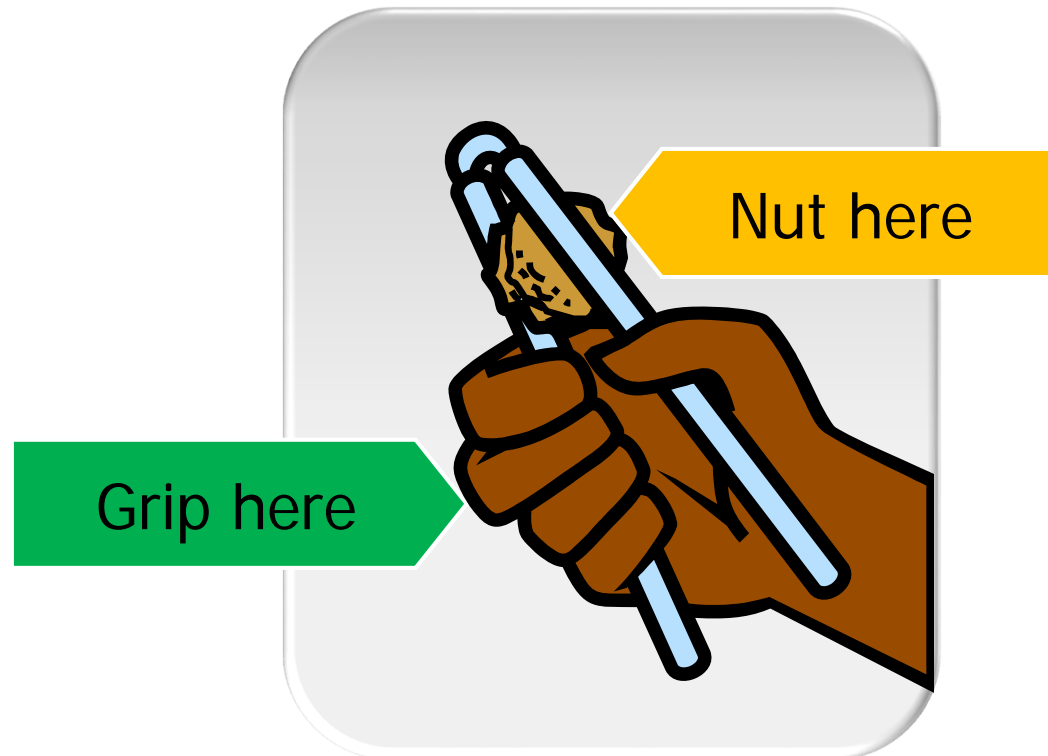


- It should work

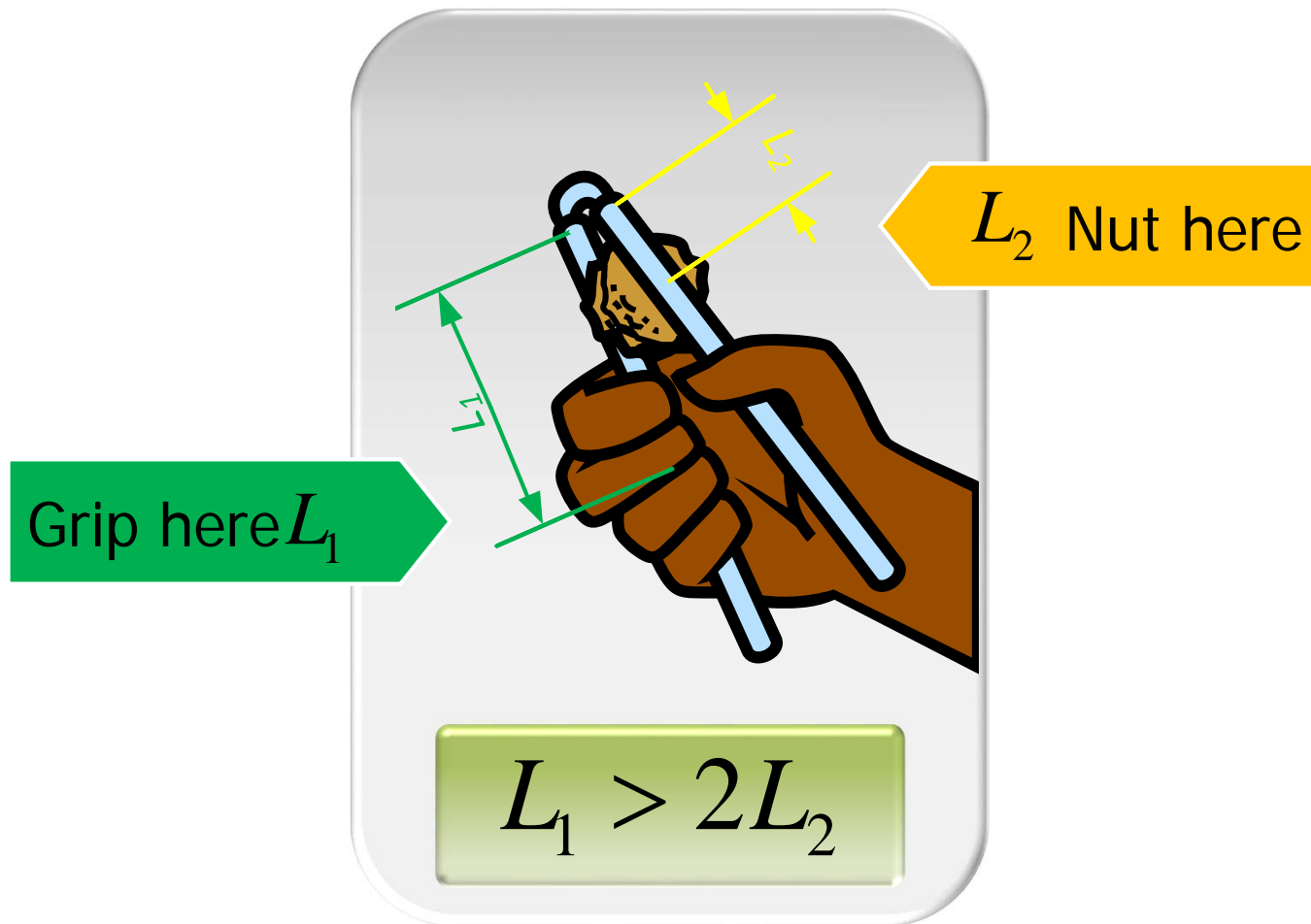
- Potable

- Small

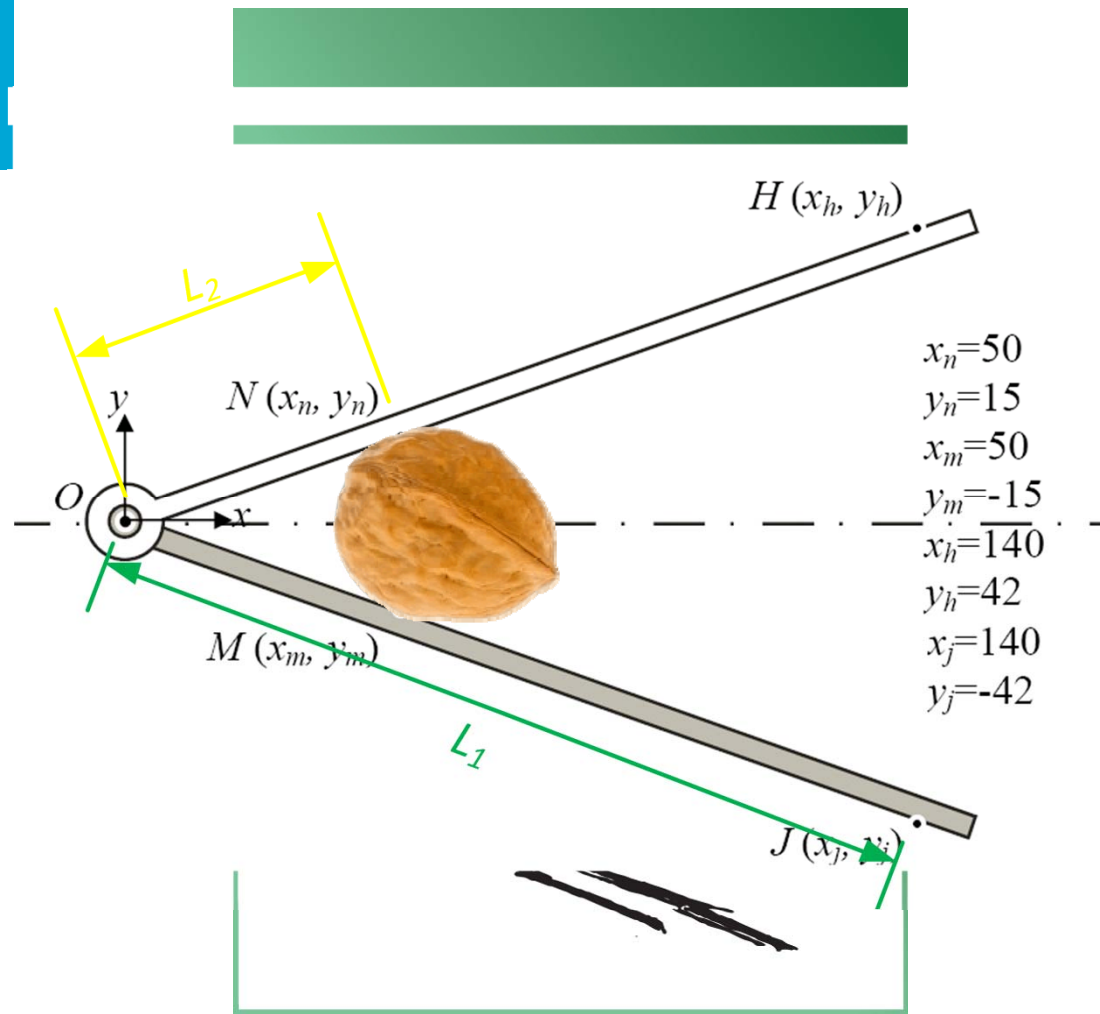
# Thought simulation



# Thought simulation



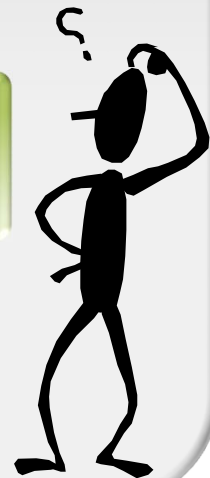
# First design with polished stainless steel



$$L_1 = \sqrt{140^2 + 42^2} \approx 146$$

$$L_2 = \sqrt{50^2 + 15^2} \approx 52$$

$$L_1 > 2L_2$$

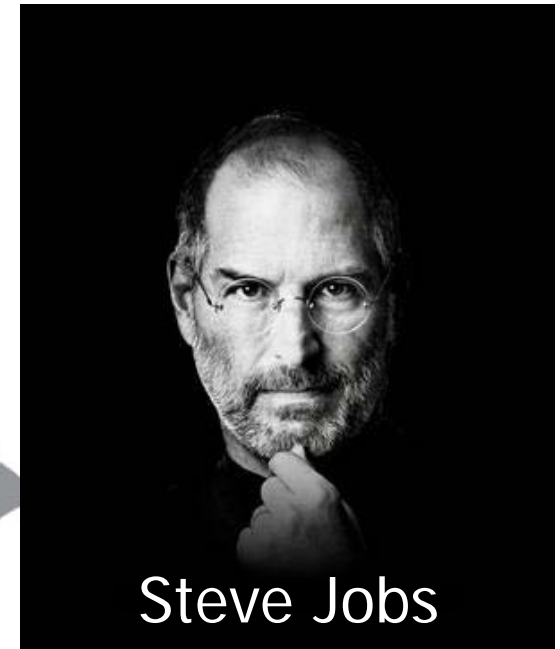
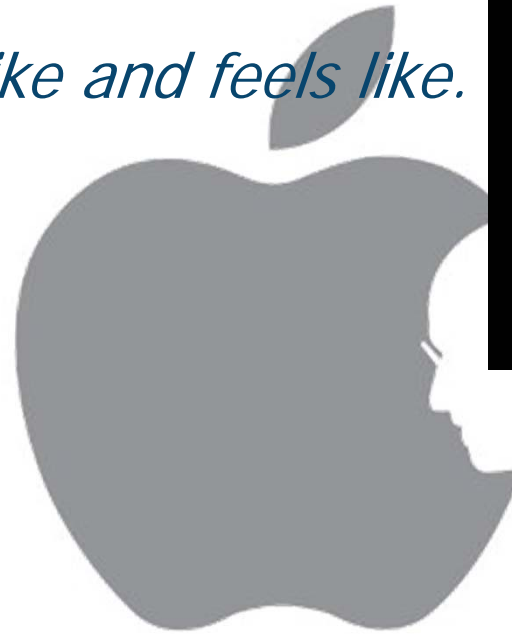


# Design is how it works

*"Make it look good!" That's not what we think design is.*

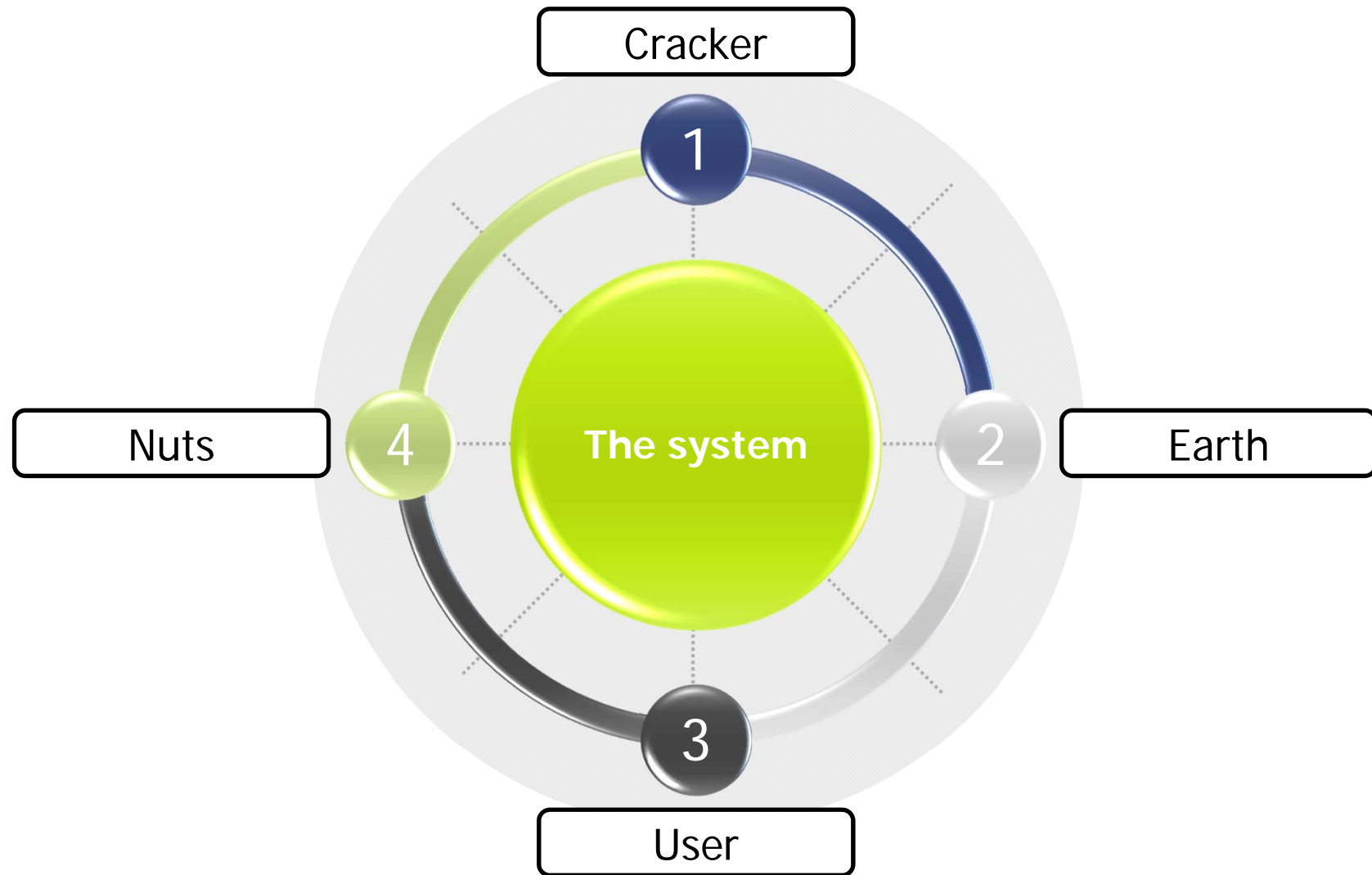
*It's **not** just what it looks like and feels like.*

*Design is how it works.*





# Components in the system?

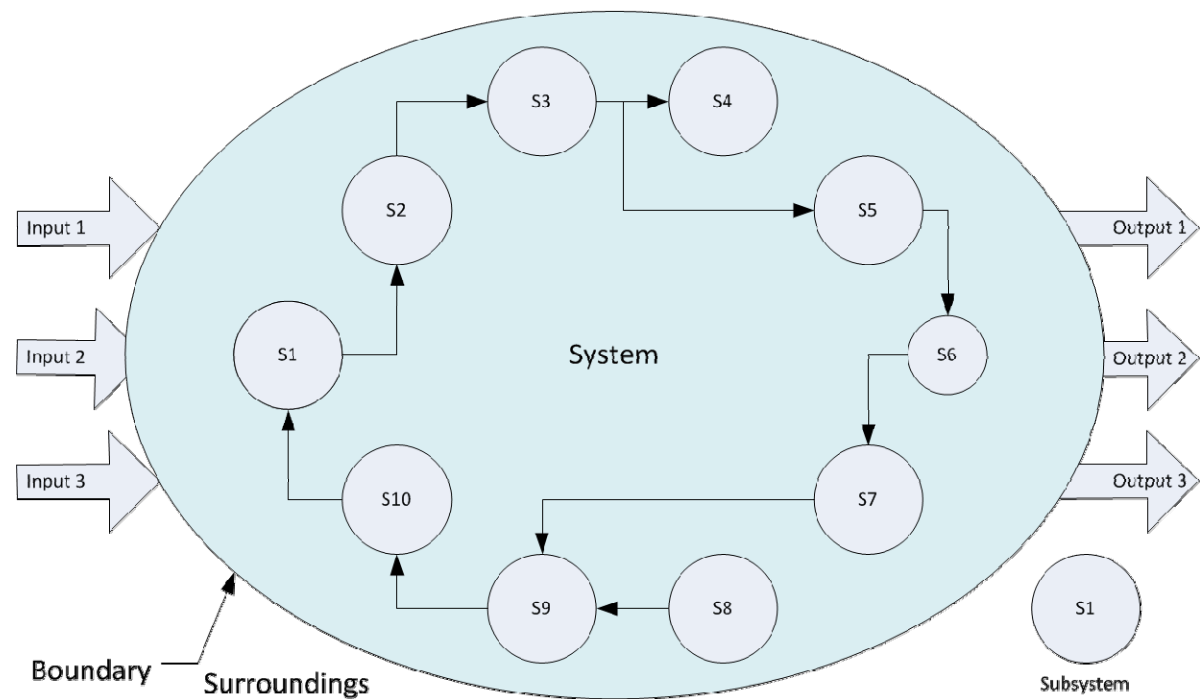


# What is a system?

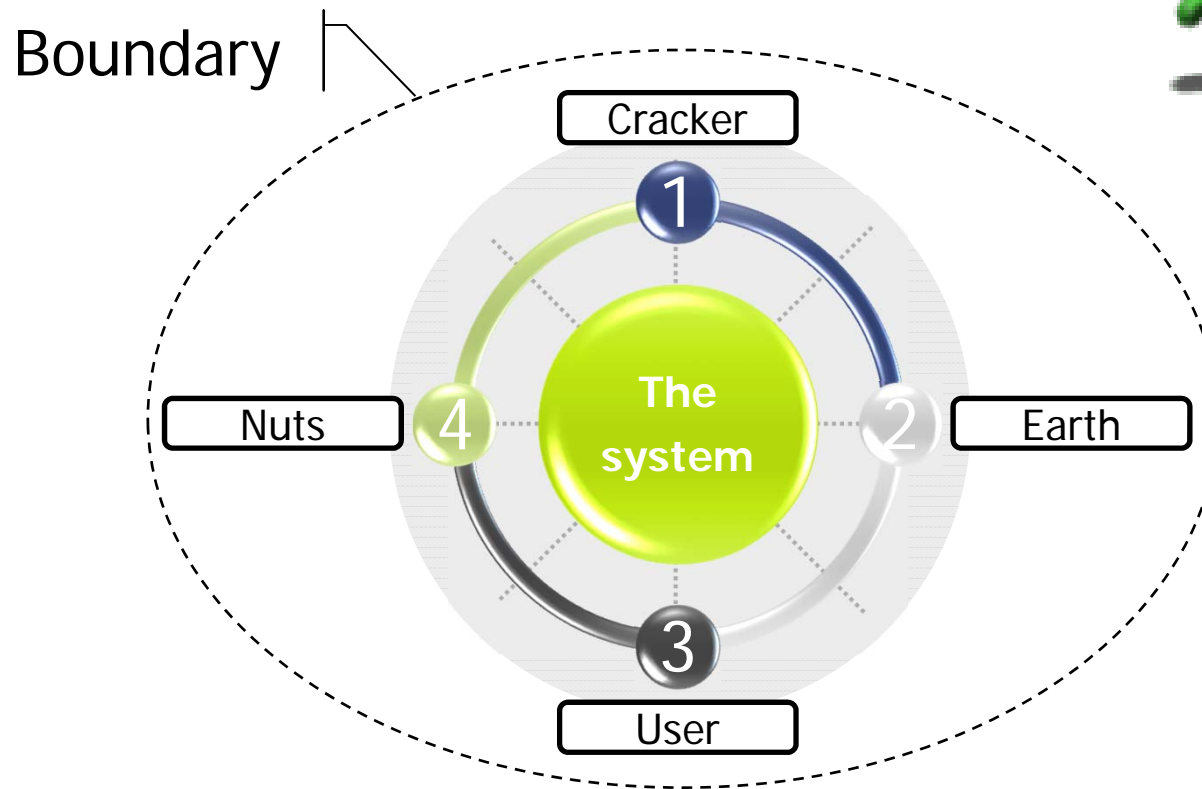
## System

System consists of a set of interacting or interdependent system components (or sub-systems)

- Structure & interconnectivity
- Boundary
- Input & Output
- Surroundings

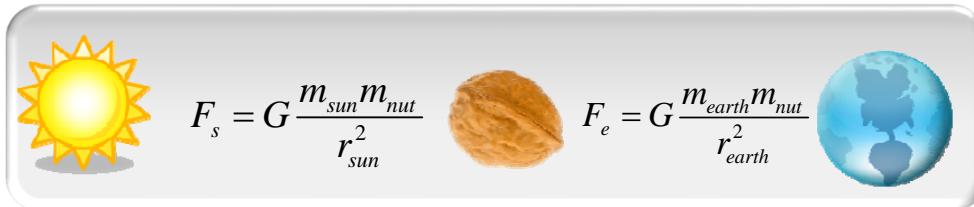


# Components in the system?



**Your experiences?**

# Physics behind: Newton's law of universal gravitation



## Gravity

```
> restart, G := 6.67259·1e-11 : m_nut := 0.02 :
> W_earth := (G·m_earth·m_nut) / r_earth^2 : m_earth := 5.9736·1e24 : r_earth := 6.371
·1e6 :
> evalf(W_earth);
```

0.1964017550 (1.1)

```
> W_sun := (G·m_sun·m_nut) / d^2 : m_sun := 1.9891·1e30 : d := 1.496·1e11 :
> evalf(W_sun);
```

0.0001186090606 (1.2)

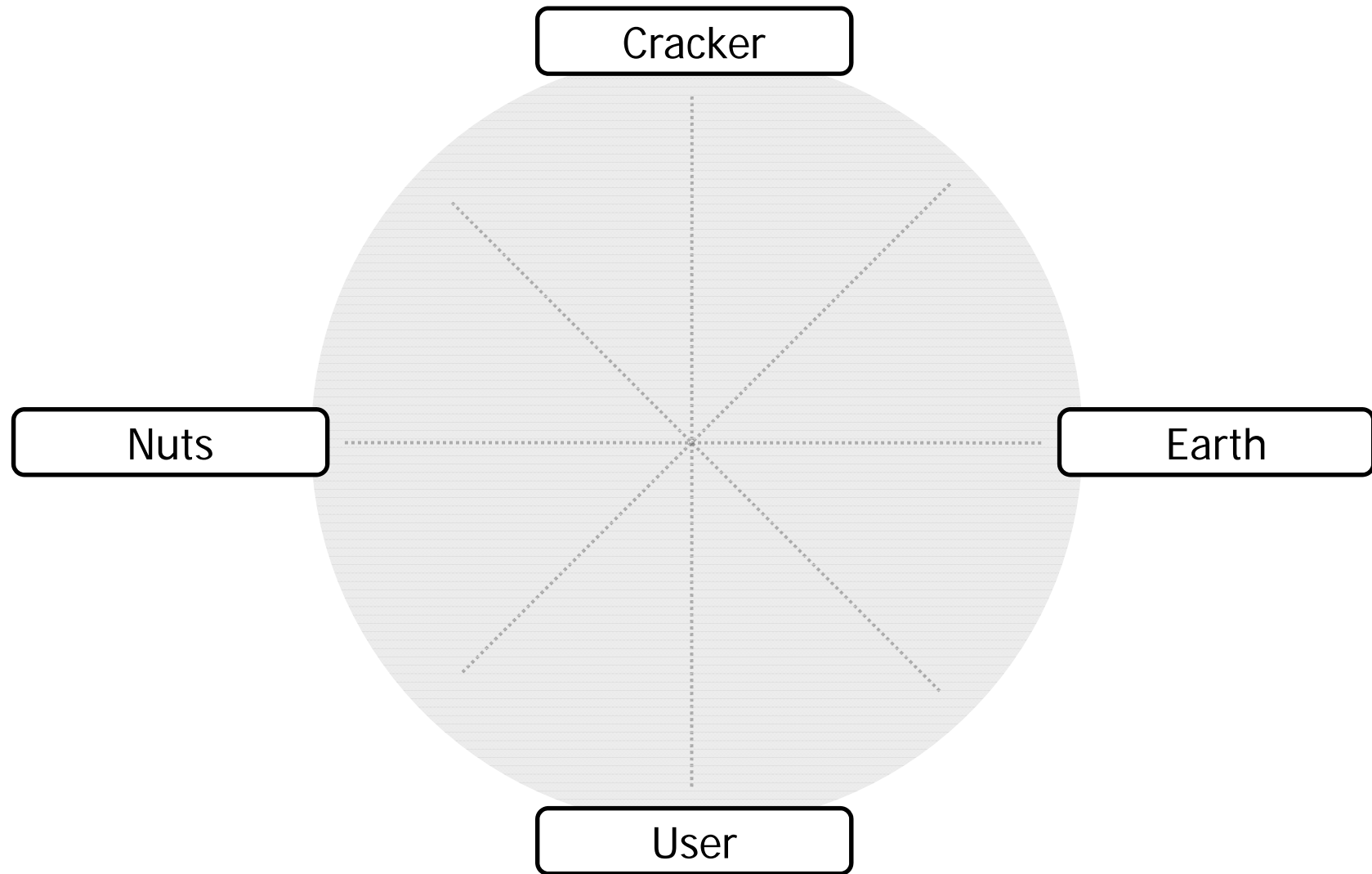
```
> (evalf(W_sun) / evalf(W_earth))
```

0.0006039103907 (1.3)

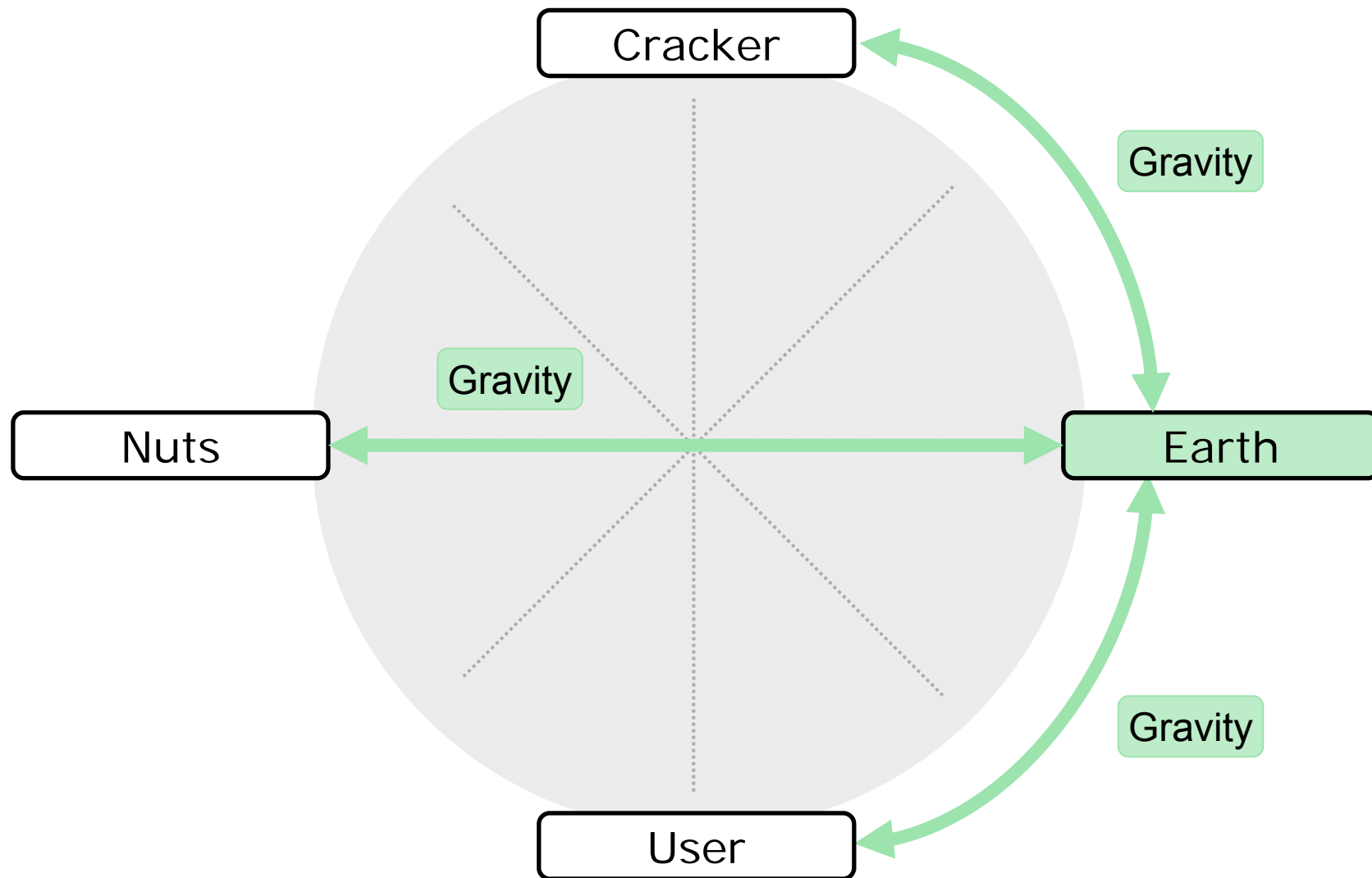


$$Gravity_{sun} = 0.06\% Gravity_{earth}$$

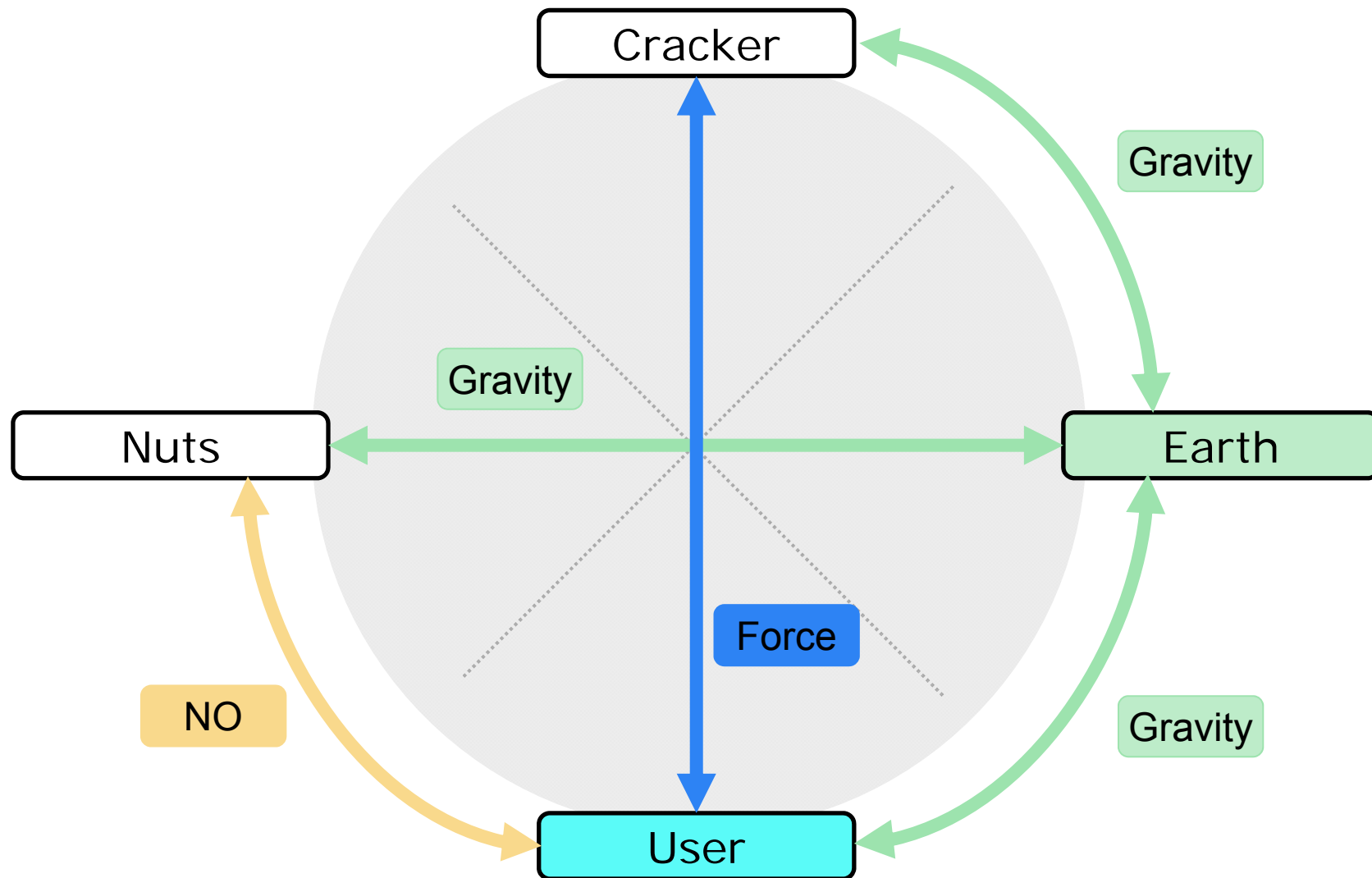
# Explore the relations



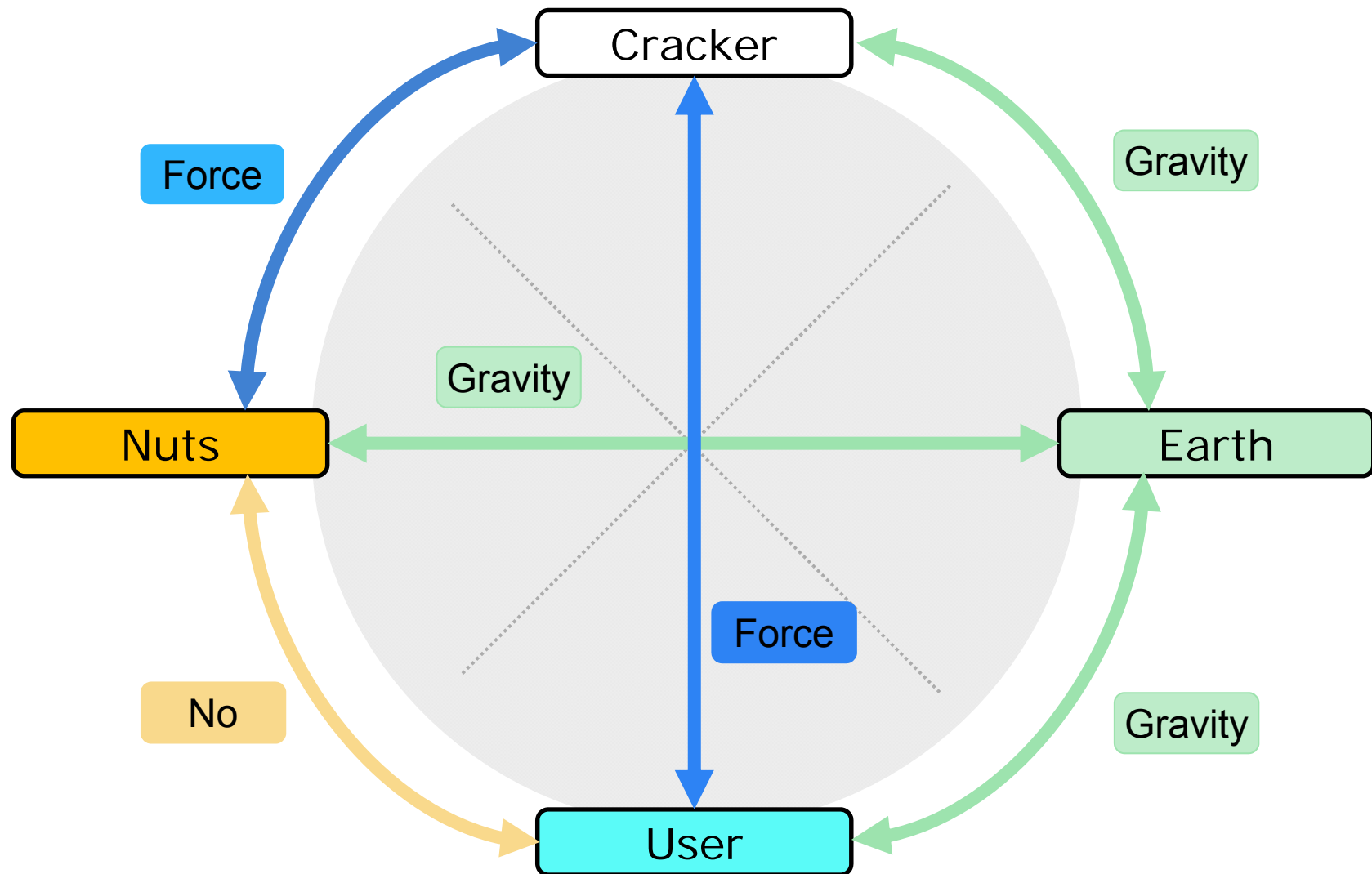
# Explore the relations



# Explore the relations

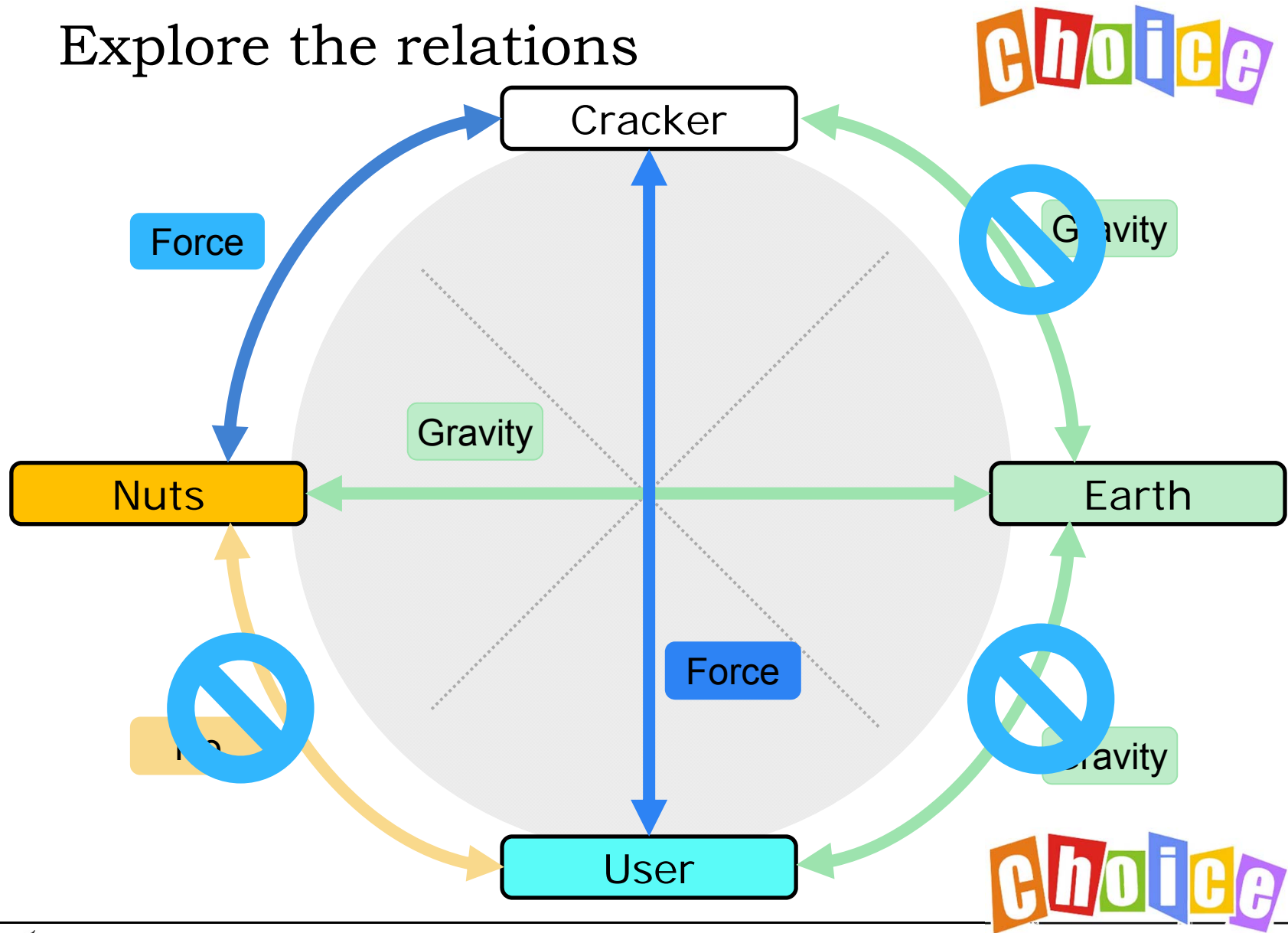


## Explore the relations

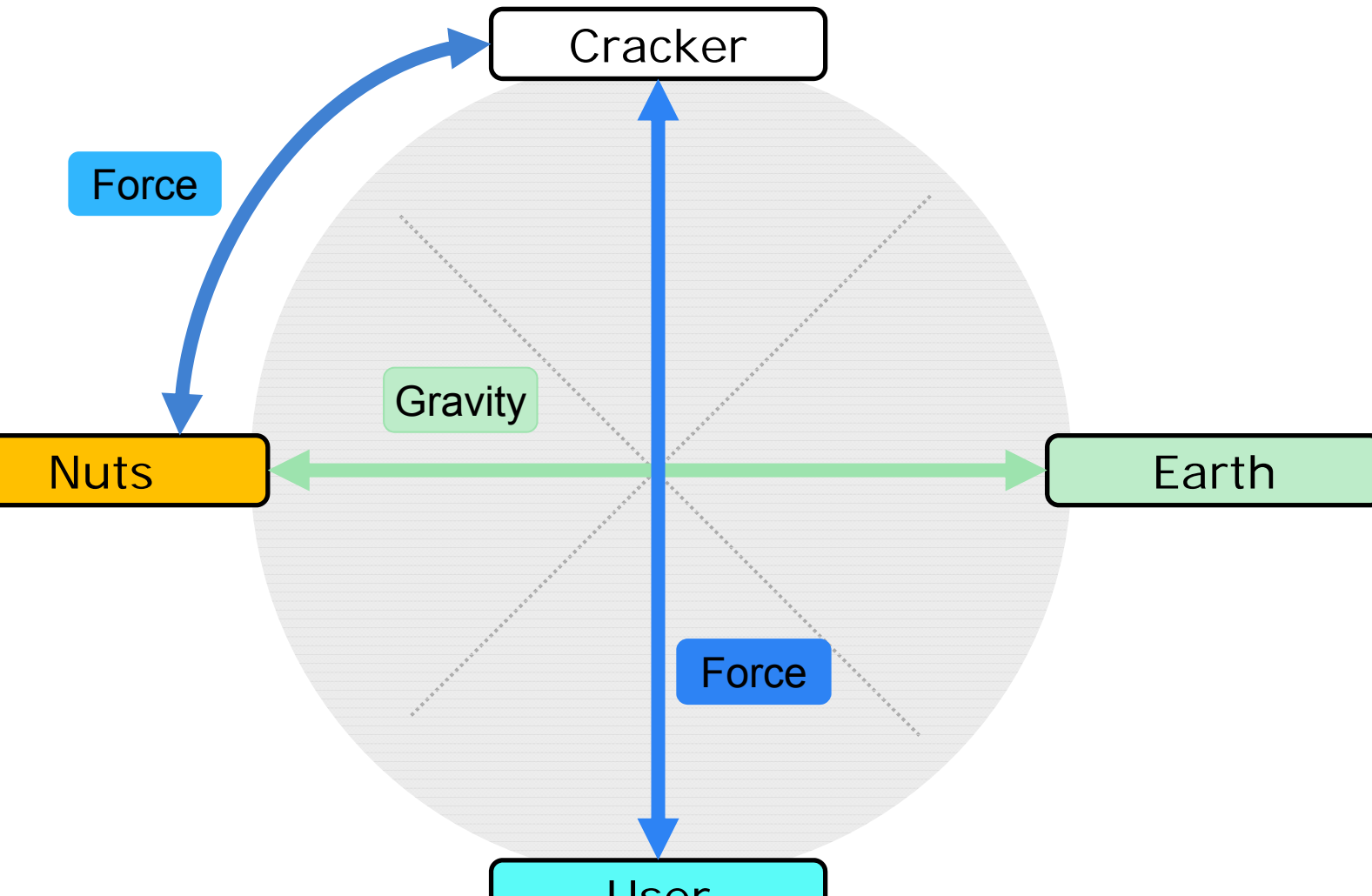




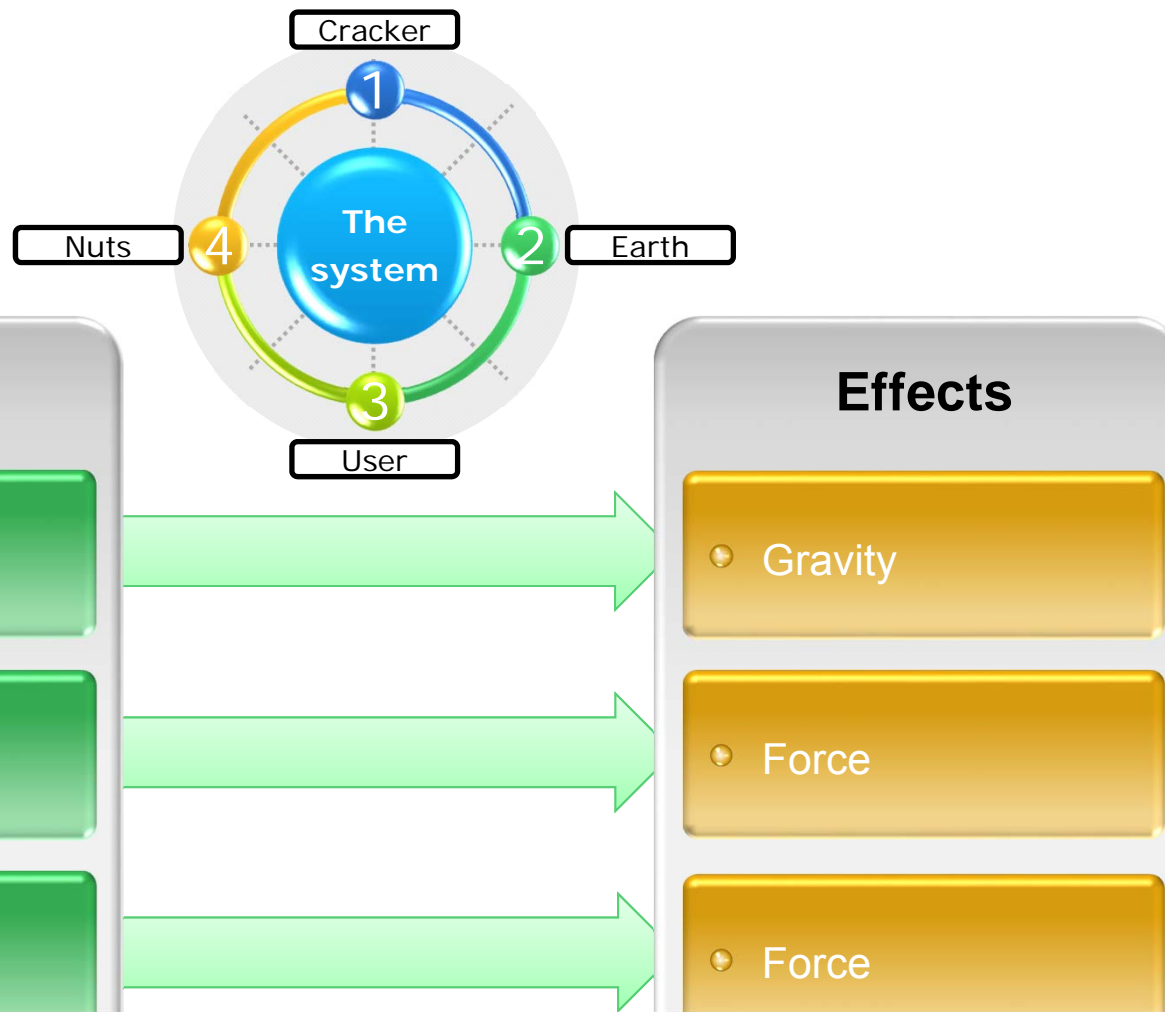
## Explore the relations



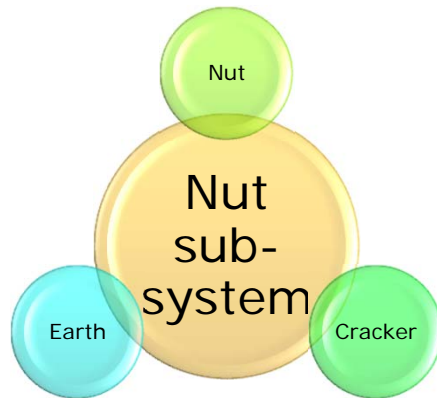
Explore the relations



# The thought simulation



Further focus on the nut



## Cause

• Earth

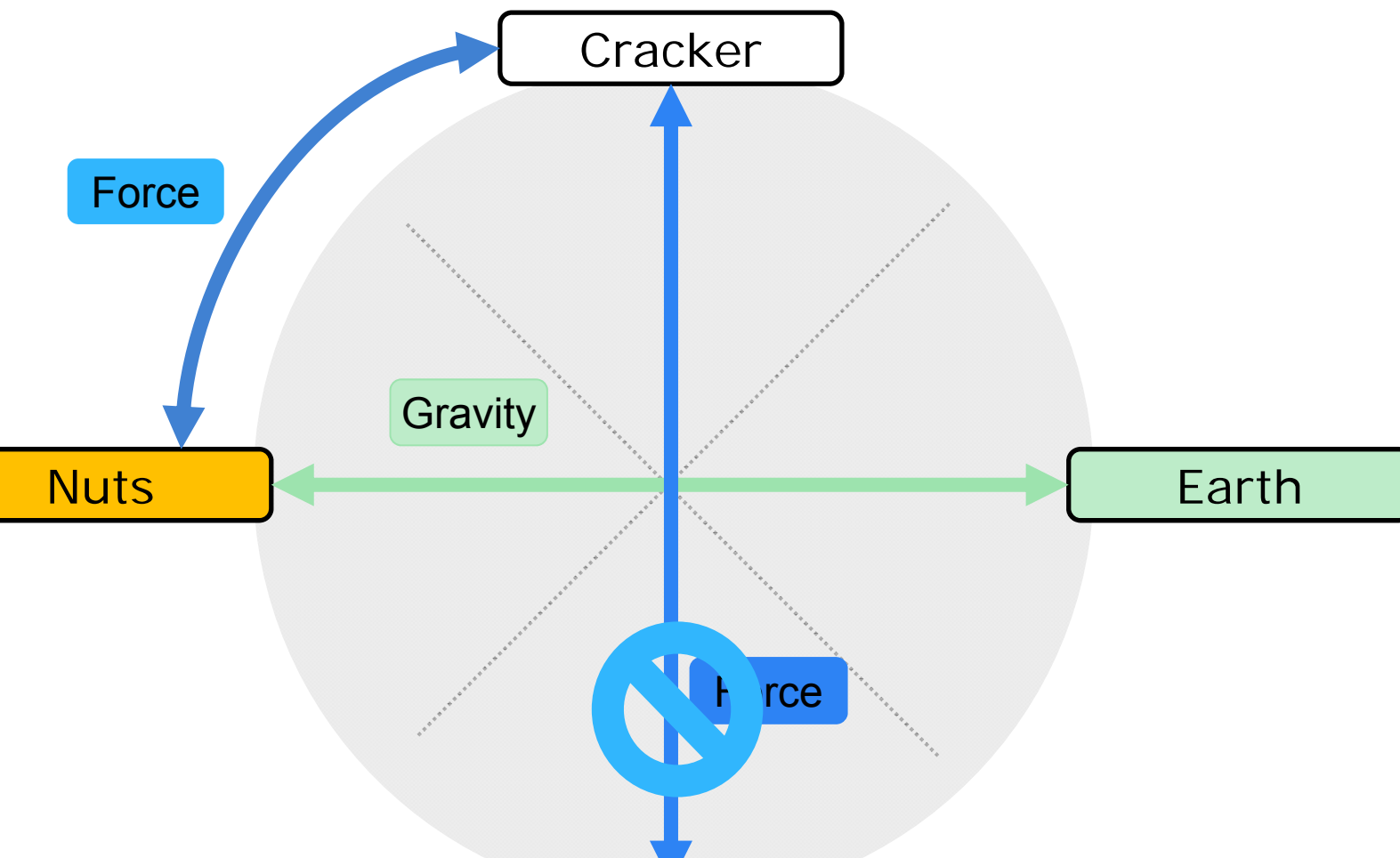
• Cracker ~ Nut

## Effects

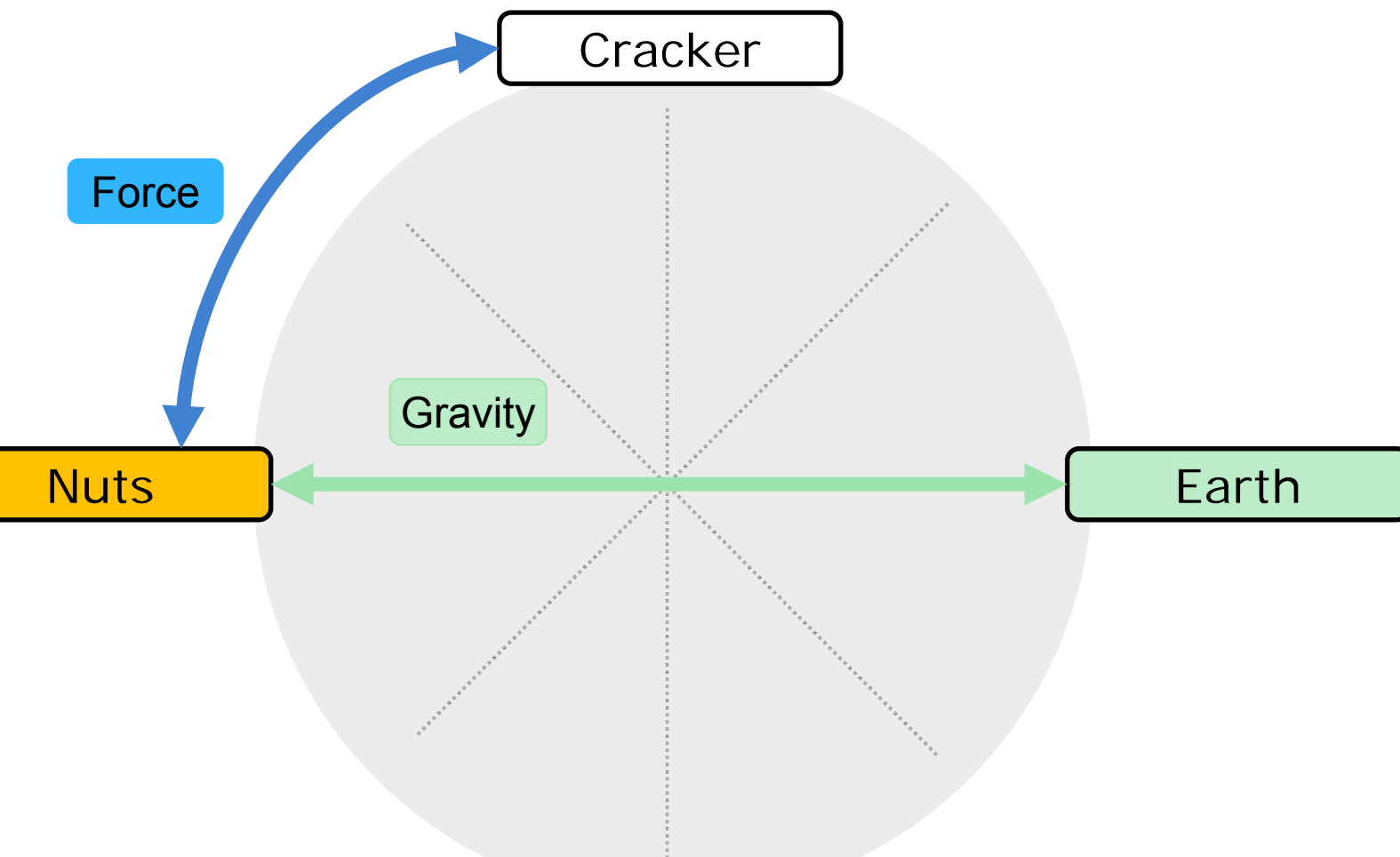
• Gravity

• Force

Explore the relations



Explore the relations

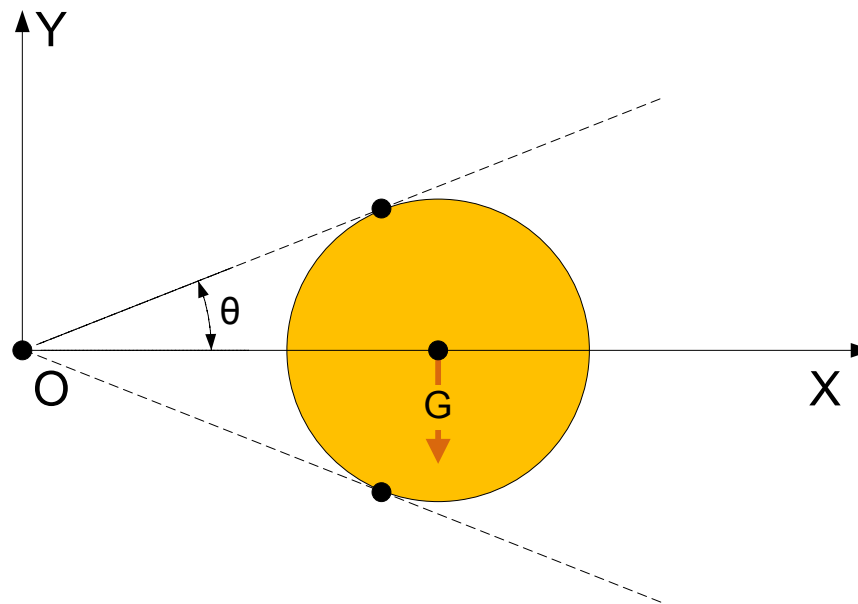


# Modelling - The Nut – Analysis

## Earth - Gravity

avity

•Earth Gravity



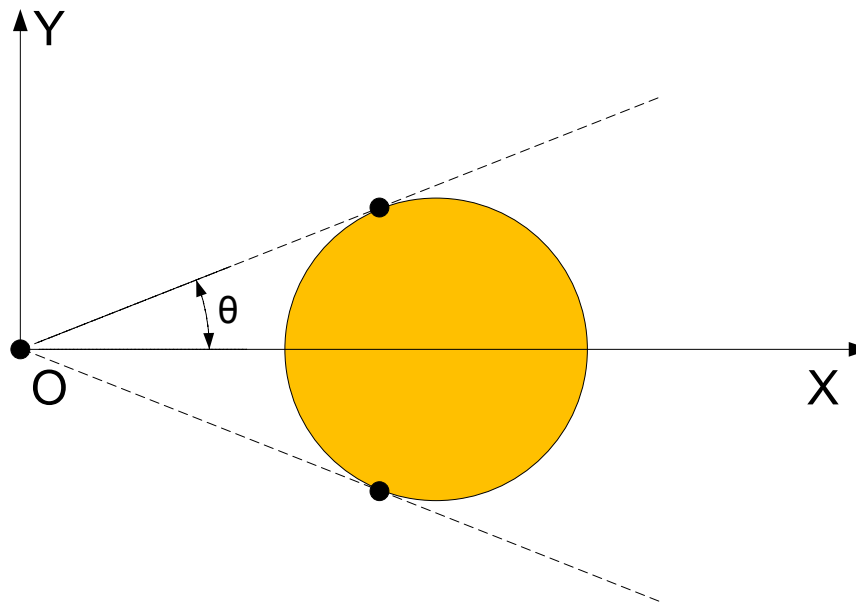
# Modelling - The Nut – Analysis

## Cracker ~ Nuts

Activity

- Earth Gravity

Boundary





# Modelling - The Nut – Analysis

## Cracker ~ Nuts

Activity • Earth Gravity

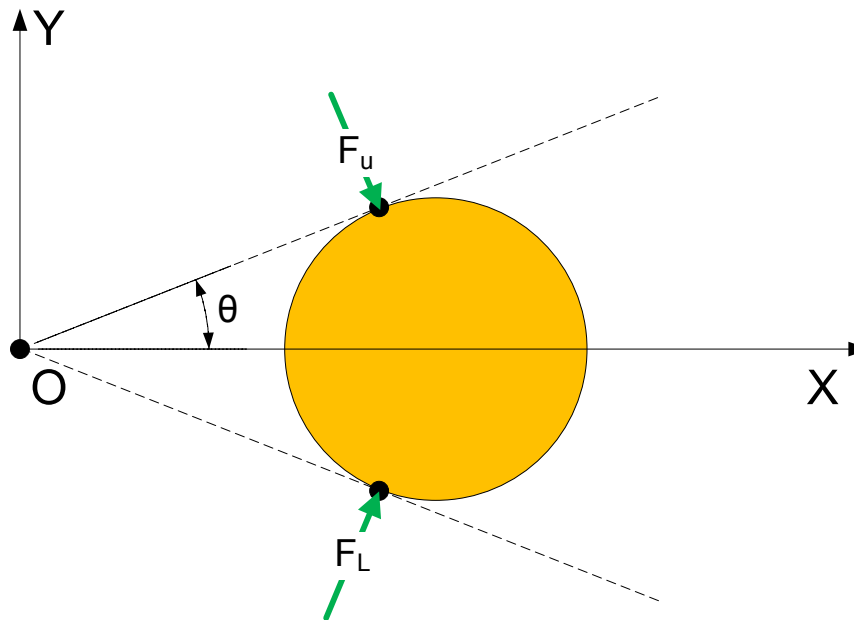
Boundary

Input • From the grips



Reaction

Friction



# Modelling - The Nut – Analysis

## Cracker ~ Nuts

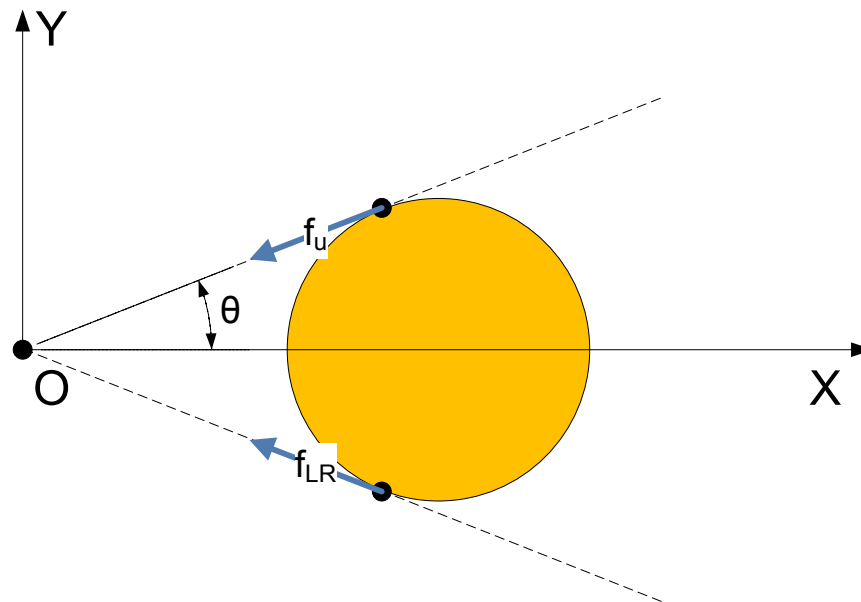
Gravity •Earth Gravity

Boundary

Input •From the grips

Reaction

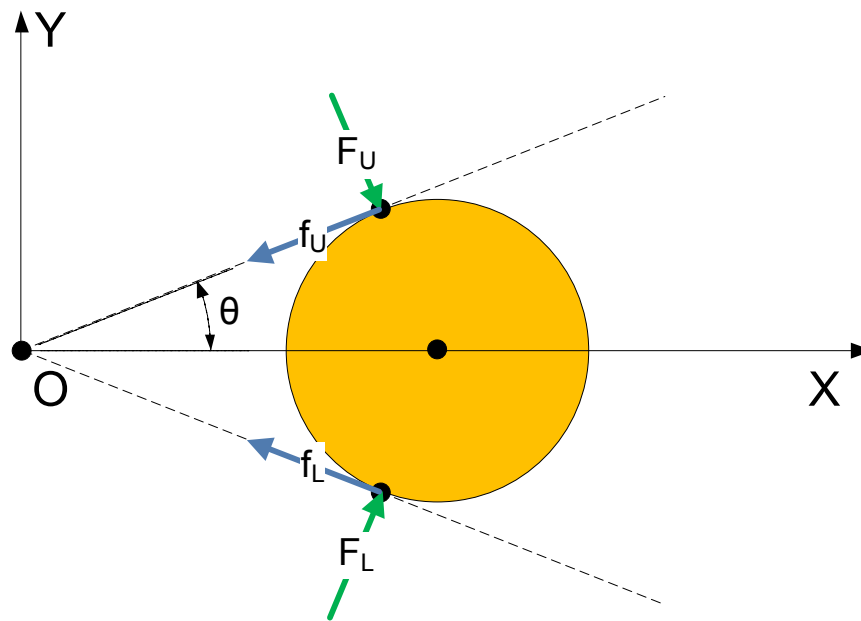
Friction •From the grips



# The attribute of friction force

coefficient  
of friction

$$f_u \leq f F_u$$



# Modelling - The Nut – Equilibrium

Cause ~ effect

Gravity • Earth Gravity

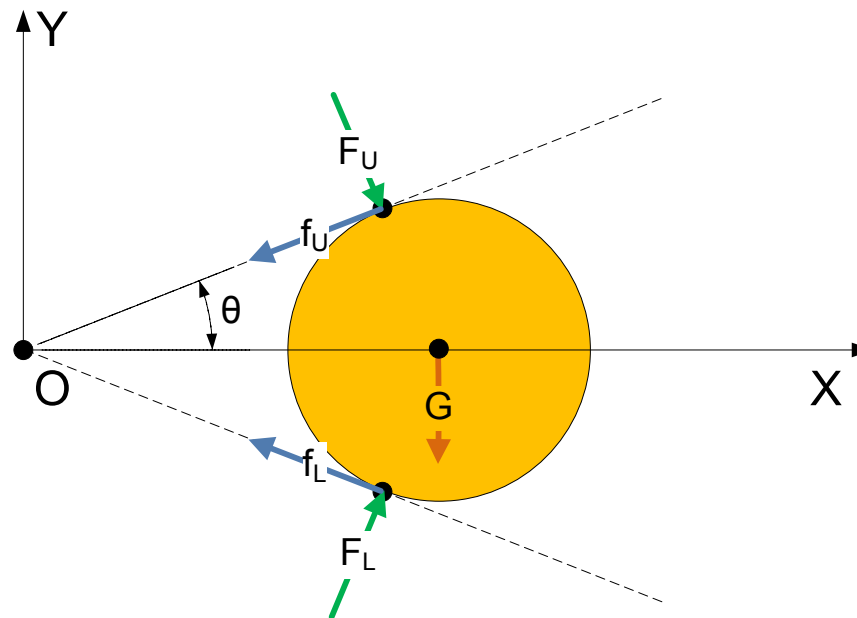
Boundary

Input • From the grips

Reaction

Friction • From the grips

$$\sum \mathbf{F} = 0, \sum \mathbf{M} = 0$$



# Modelling - The Nut – Equilibrium

## Cause ~ effect

Gravity •Earth Gravity

Boundary

Input •From the grips

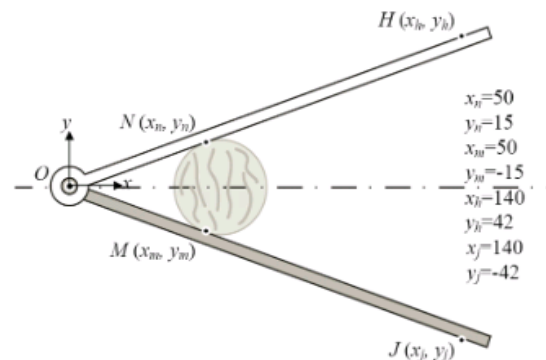
Reaction

Friction •From the grips



•Diameter = 30mm

## Modeling G-L-1



Sum of all force is zero

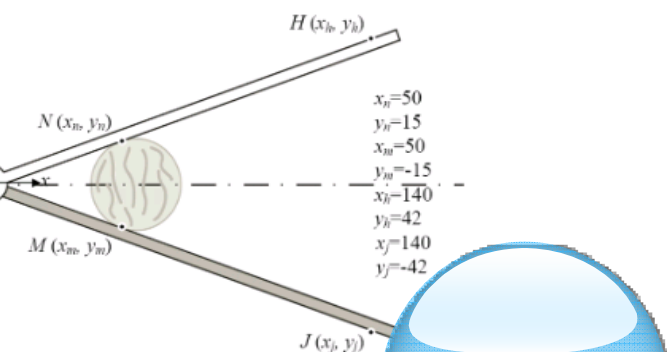
```
> restart : with(VectorCalculus) :
> theta := arctan(15/50) :
> F_U := 300 * (sin(theta), -cos(theta)) :
> F_L := F_LNorm(sin(theta), cos(theta)) :
> f_U := (f.Norm(F_U)) * (-cos(theta), -sin(theta)) :
> f_L := (f.Norm(F_L) - B) * (-cos(theta), sin(theta)) :
> G := 0.2 * 9.8 * (0, -1) :
> SUMF := F_L + F_U + f_U + f_L + G = (0, 0)
> SUMM := (f.Norm(F_U)) - (f.Norm(F_L) - B) = 0 :
> solve({SUMF(1), SUMF(2), SUMM}, {f, F_LNorm, B})
```

# Modelling - The Nut – Large & Small nut



•Diameter = 40mm

## Modeling G-L-1



Sum of all force is zero

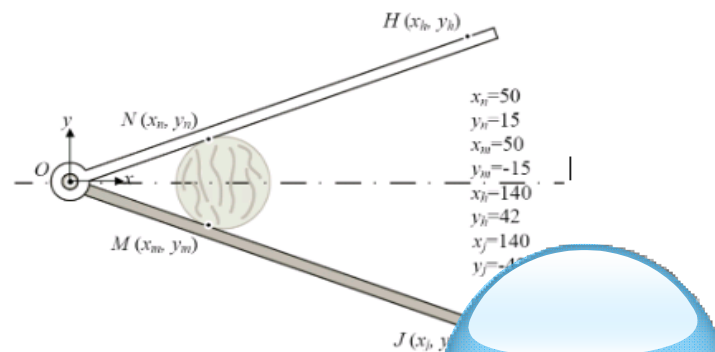
```
restart : with(VectorCalculus) :
theta := arctan(20/50) :
F_U := 300 * (sin(theta), -cos(theta)) :
F_L := F_LNorm * (sin(theta), cos(theta)) :
f_U := (f.Norm(F_U)) * (-cos(theta), -sin(theta)) :
f_L := (f.Norm(F_L) - B) * (-cos(theta), sin(theta)) :
G := 0.2 * 9.8 * (0, -1) :
SUMF := F_L + F_U + f_U + f_L + G = (0, 0) :
SUMM := (f.Norm(F_U)) - (f.Norm(F_L) - B) = 0 :
solve({SUMF(1), SUMF(2), SUMM}, {f, F_LNorm, B})
```

**f = 0.4**



•Diameter = 20mm

## Modeling G-L-1



Sum of all force is zero

```
> restart : with(VectorCalculus) :
> theta := arctan(10/50) :
> F_U := 300 * (sin(theta), -cos(theta)) :
> F_L := F_LNorm * (sin(theta), cos(theta)) :
> f_U := (f.Norm(F_U)) * (-cos(theta), -sin(theta)) :
> f_L := (f.Norm(F_L) - B) * (-cos(theta), sin(theta)) :
> G := 0.2 * 9.8 * (0, -1) :
> SUMF := F_L + F_U + f_U + f_L + G = (0, 0) :
> SUMM := (f.Norm(F_U)) - (f.Norm(F_L) - B) = 0 :
> solve({SUMF(1), SUMF(2), SUMM}, {f, F_LNorm, B})
```

**f = 0.2**

# Evaluation

Approximate coefficients of friction

Materials		Static friction, $\mu_s$	
		Dry & clean	Lubricated
Aluminum	Steel	0.61	
Copper	Steel	0.53	
Brass	Steel	0.51	
Cast iron	Copper	1.05	
Cast iron	Zinc	0.85	
Concrete (wet)	Rubber	0.30	
Concrete (dry)	Rubber	1.0	
Concrete	Wood	0.62 <sup>[10]</sup>	
Copper	Glass	0.68	
Glass	Glass	0.94	
Metal	Wood	0.2-0.6 <sup>[10]</sup>	0.2 (wet) <sup>[10]</sup>
Polythene	Steel	0.2 <sup>[11]</sup>	0.2 <sup>[11]</sup>
Steel	Steel	0.80 <sup>[11]</sup>	0.16 <sup>[11]</sup>
Steel	Teflon	0.04 <sup>[11]</sup>	0.04 <sup>[11]</sup>
Teflon	Teflon	0.04 <sup>[11]</sup>	0.04 <sup>[11]</sup>
Wood	Wood	0.25-0.5 <sup>[10]</sup>	0.2 (wet) <sup>[10]</sup>

cannot guarantee  
that it will work.

If this is an acceptable design



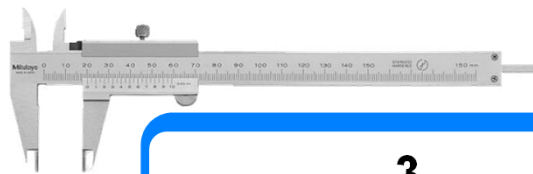
1

We developed a portable nut cracker



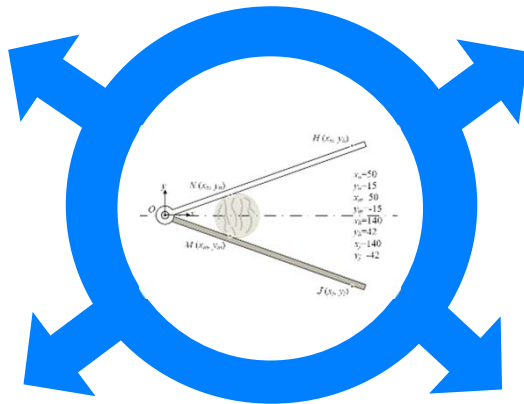
2

In normal conditions, it can crack nuts till 20mm in diameter. (Small nuts)



3

We advise the user to purchase a caliper to make a precise measurement of the nut before usage



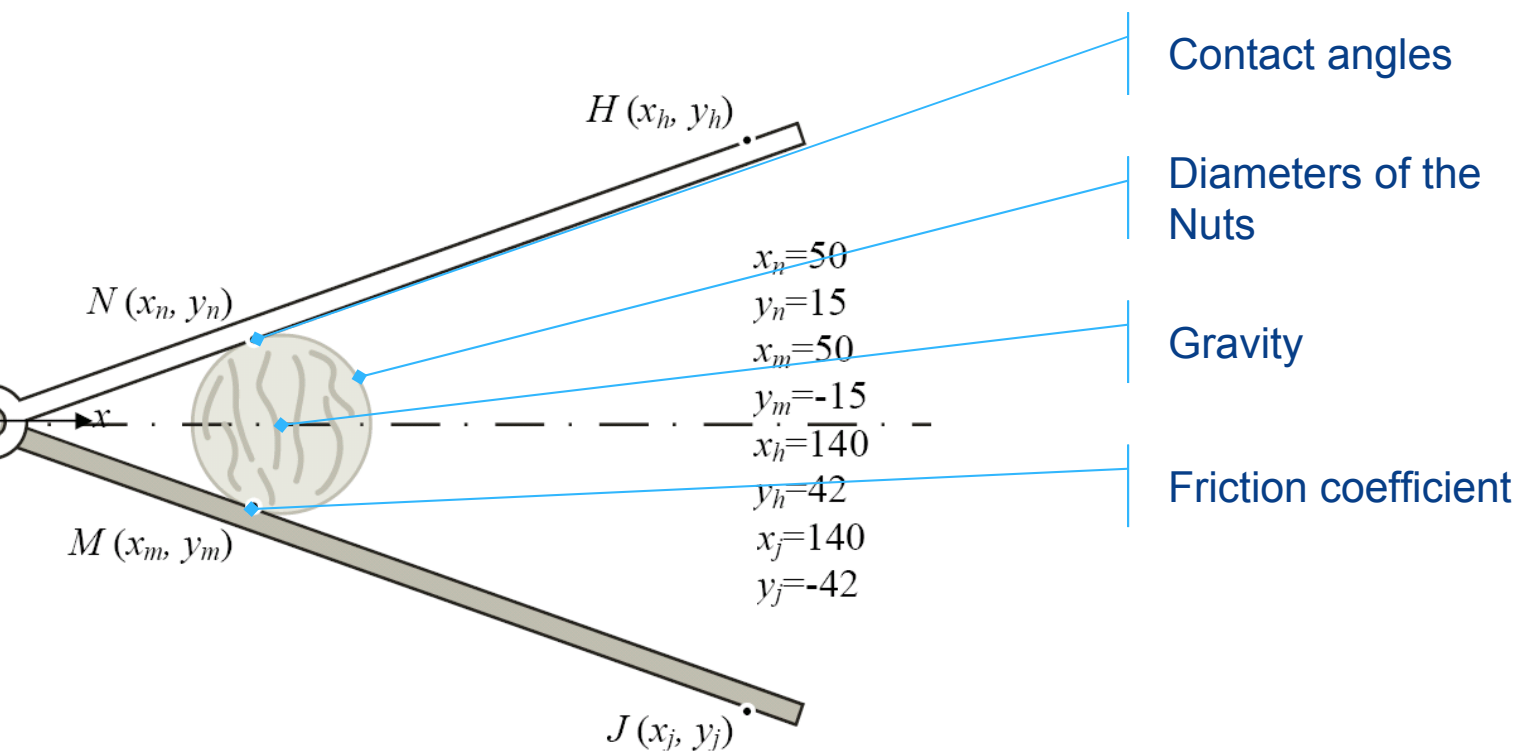
4

We also advise the user to dress a safety glasses in the case of trying to crack large nuts

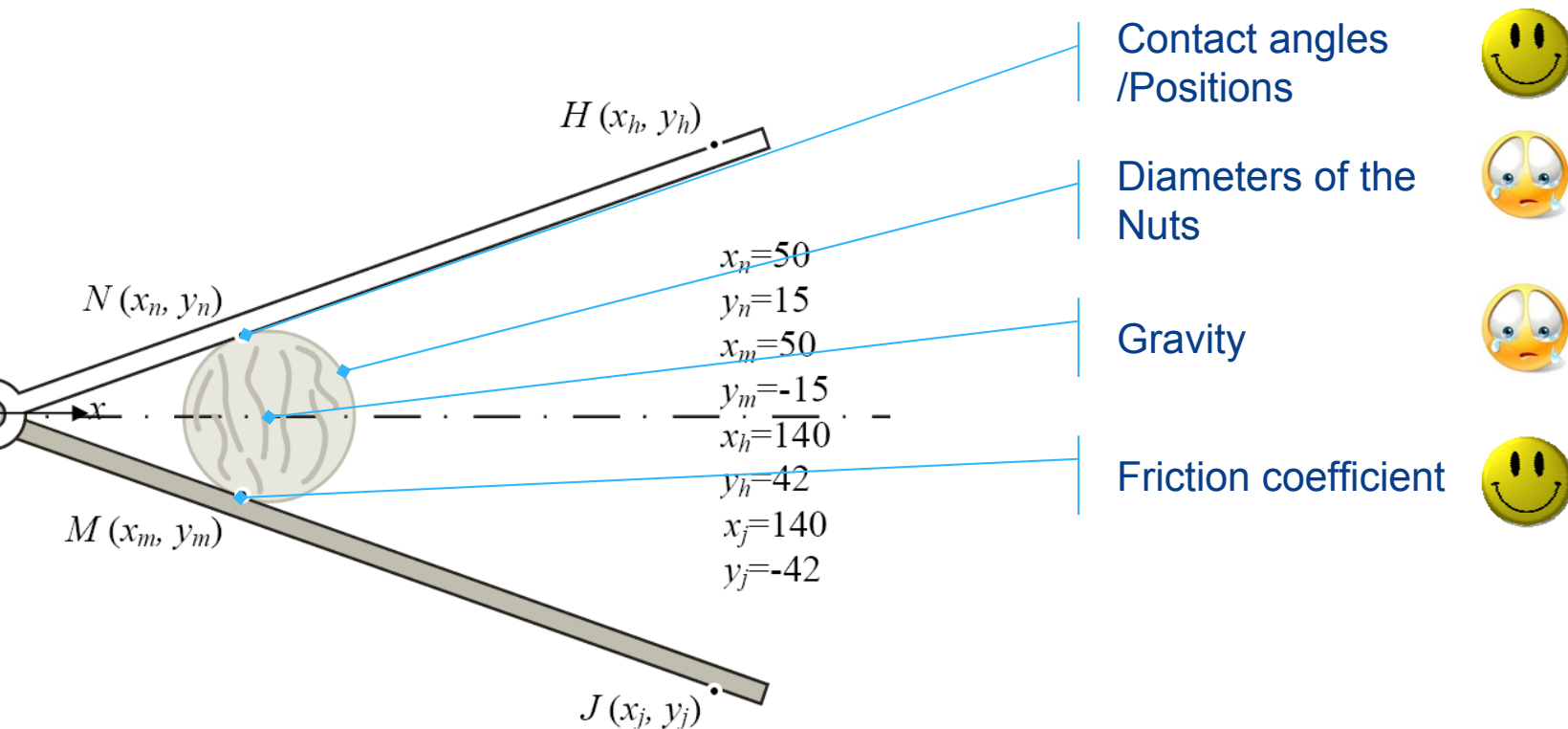




# Evaluation: Revisit choices in the system



# Which parameter can be changed?



# Evaluation: Friction coefficient

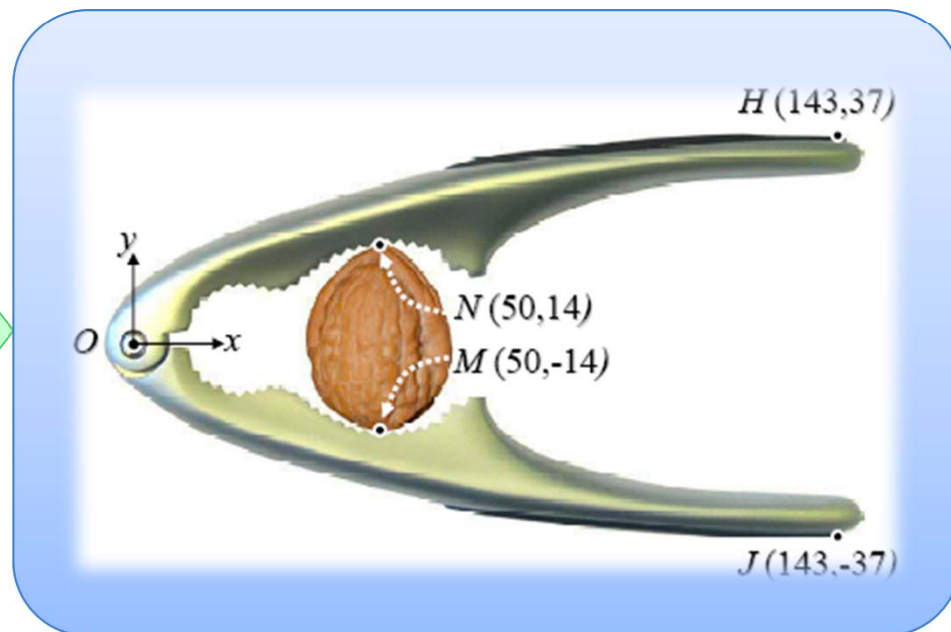
We may try to use rubber in the contact area, but rubber is not durable?

What else can we do?

Approximate coefficients of friction

Materials		Static friction, $\mu_s$	
		Dry & clean	Lubricated
Aluminum	Steel	0.61	
Copper	Steel	0.53	
Brass	Steel	0.51	
Cast iron	Copper	1.05	
Cast iron	Zinc	0.85	
Concrete (wet)	Rubber	0.30	
Concrete (dry)	Rubber	1.0	
Concrete	Wood	0.62 <sup>[10]</sup>	
Copper	Glass	0.68	
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Metal	Wood	0.2-0.6 <sup>[10]</sup>	0.2 (wet) <sup>[10]</sup>
Polythene	Steel	0.2 <sup>[11]</sup>	0.2 <sup>[11]</sup>
Steel	Steel	0.80 <sup>[11]</sup>	0.16 <sup>[11]</sup>
Steel	Teflon	0.04 <sup>[11]</sup>	0.04 <sup>[11]</sup>
Teflon	Teflon	0.04 <sup>[11]</sup>	0.04 <sup>[11]</sup>
Wood	Wood	0.25-0.5 <sup>[10]</sup>	0.2 (wet) <sup>[10]</sup>

A new idea – Change the contact angle

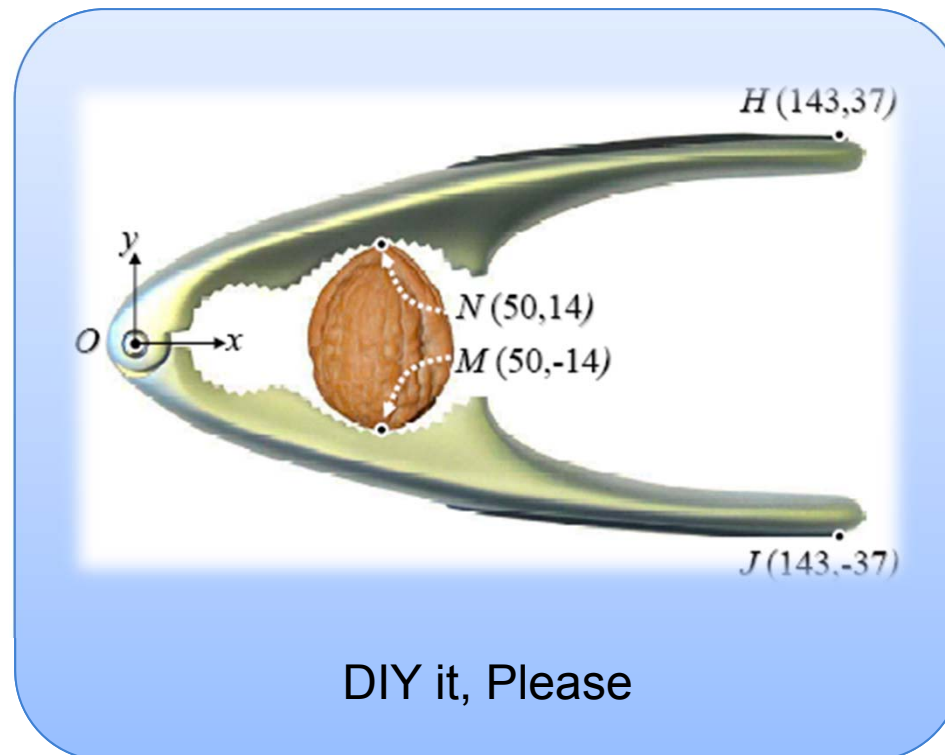


A new idea

# A new cycle of modelling

## Cause ~ effect

- Earth Gravity
- From the grips
- From the grips





# Free your imagination in designing



Courtesy of  
<http://www.signetemporium.com/promo.php>  
<http://www.craftsmanspace.com/free-projects/rounded-wooden-nut-cracker-plan.html>  
<http://www.ohgizmo.com/2009/12/02/northern-industrial-tools-nut-cracker-is-one-serious-piece-of-holiday-kit/>  
<http://www.legendarykitchen.co.uk/retro-nut-cracker-2789-0.html>

# rigid body dynamics

Special Thanks to Dr. Bas Flipsen



Everything starts from here



Isaac Newton

Law of Motion

Observed from an inertial reference frame, the net force on a particle is equal to the time rate of change of its linear momentum:

$$\mathbf{F} = \frac{d}{dt} m\mathbf{v}$$

He never wrote

$$\mathbf{F} = m\mathbf{a}$$

# Dynamics – Equilibrium

Forces related to Cause ~ effect

Inertia force  $-\frac{d}{dt}m\mathbf{v}$

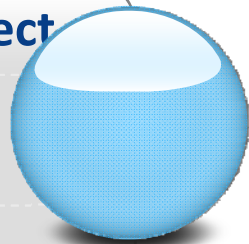
Gravity •Earth Gravity

Boundary

Input

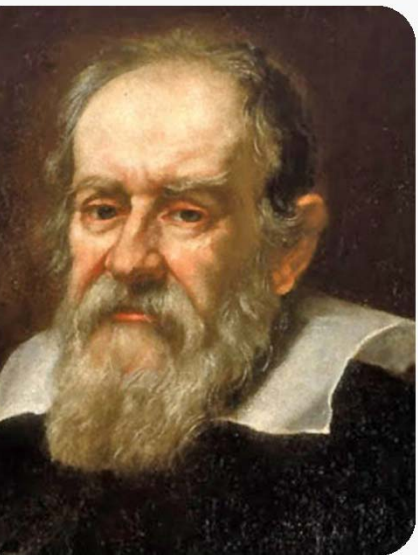
Reaction

Friction



$$\sum \mathbf{F} = 0, \sum \mathbf{M} = 0$$

# Case study: Galileo's Leaning Tower of Pisa experiment

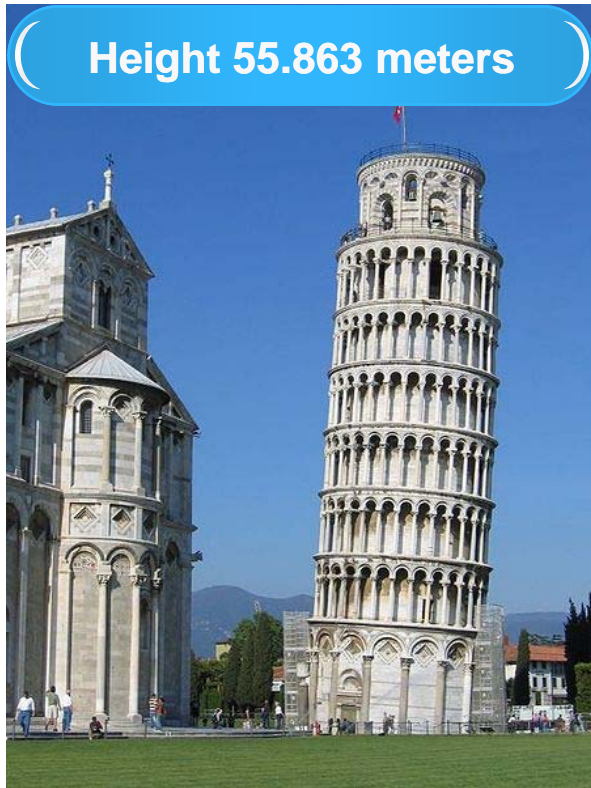


*Galileo Galilei*

Physicist  
Mathematician  
Astronomer



Is it really true?



# Hammer & Feather drop: Apollo 15

ref. [http://nssdc.gsfc.nasa.gov/planetary/lunar/apollo\\_15\\_feather\\_drop.html](http://nssdc.gsfc.nasa.gov/planetary/lunar/apollo_15_feather_drop.html)



The Modelling TV

It seems true on the moon, how about earth?



# NASA “metric mixup” -Mars Climate Orbiter



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## exploringmars

in-depth specials

### NASA's metric confusion caused Mars orbiter loss

September 30, 1999  
Web posted at: 1:46 p.m. EDT (1746 GMT)

(CNN) -- NASA lost a \$125 million Mars orbiter because one engineering team used metric units while another used English units for a key spacecraft operation, according to a review finding released Thursday.

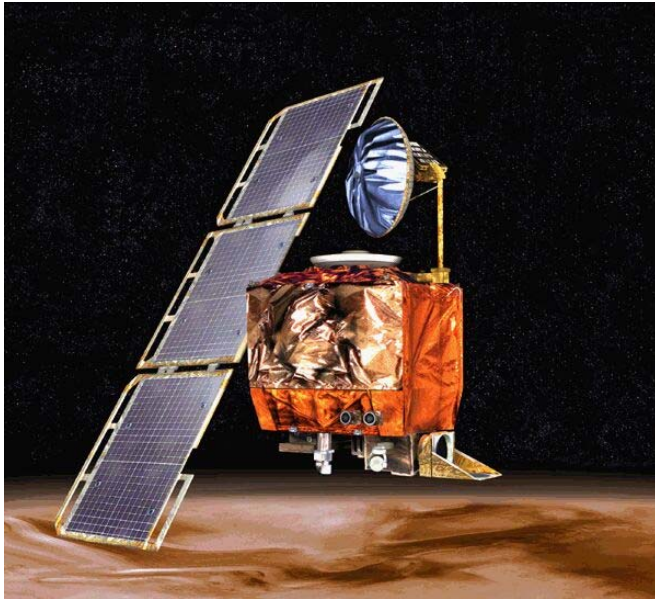
For that reason, information failed to transfer between the Mars Climate Orbiter spacecraft team at Lockheed Martin in Colorado and the mission navigation team in California. Lockheed Martin built the



NASA's Climate Orbiter was lost  
September 23, 1999

Courtesy of [http://en.wikipedia.org/wiki/Mars\\_Climate\\_Orbiter](http://en.wikipedia.org/wiki/Mars_Climate_Orbiter), [www.cnn.com](http://www.cnn.com)

# NASA “*metric mixup*” - The tuition fees



\$327.6 million: Orbiter and Lander  
\$193.1 million: Spacecraft development  
\$91.7 million: Launch  
\$42.8 million: Mission operations



\$655.3 Million

Ref. [http://en.wikipedia.org/wiki/Mars\\_Climate\\_Orbiter](http://en.wikipedia.org/wiki/Mars_Climate_Orbiter)

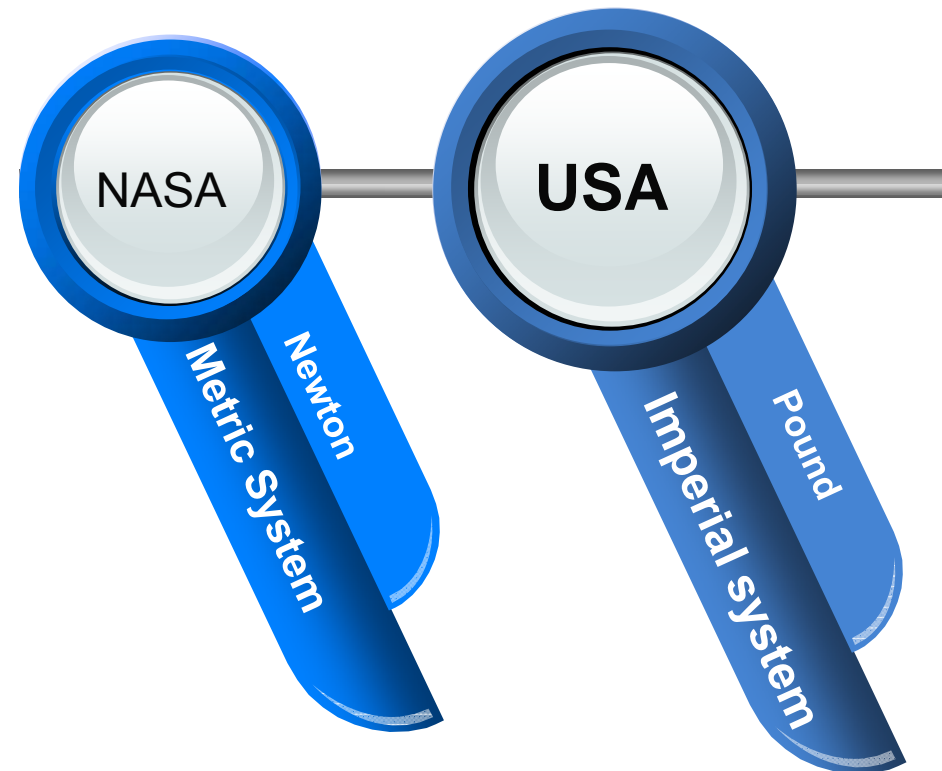
# Dimension problems



\$655.3 Million  
+  
Time

Metric system  
+  
Imperial Units

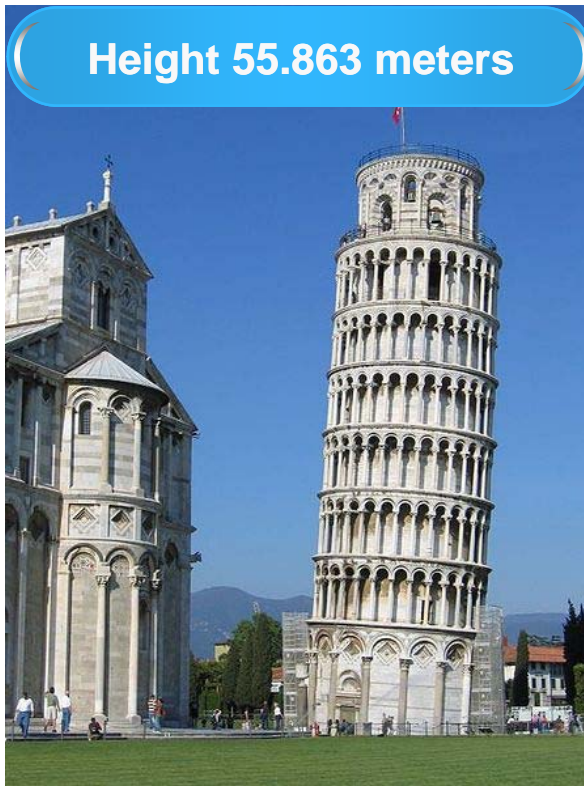
Imperial units



Ref. [http://en.wikipedia.org/wiki/Mars\\_Climate\\_Orbiter](http://en.wikipedia.org/wiki/Mars_Climate_Orbiter)

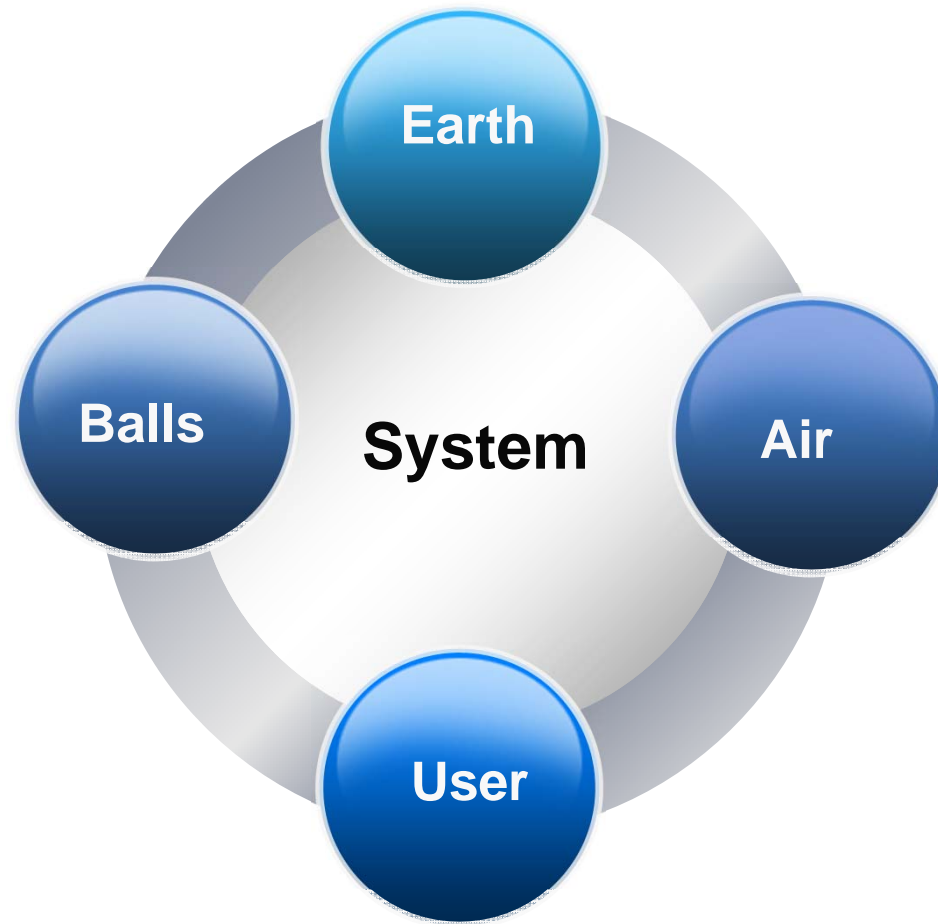


# Is it really true?

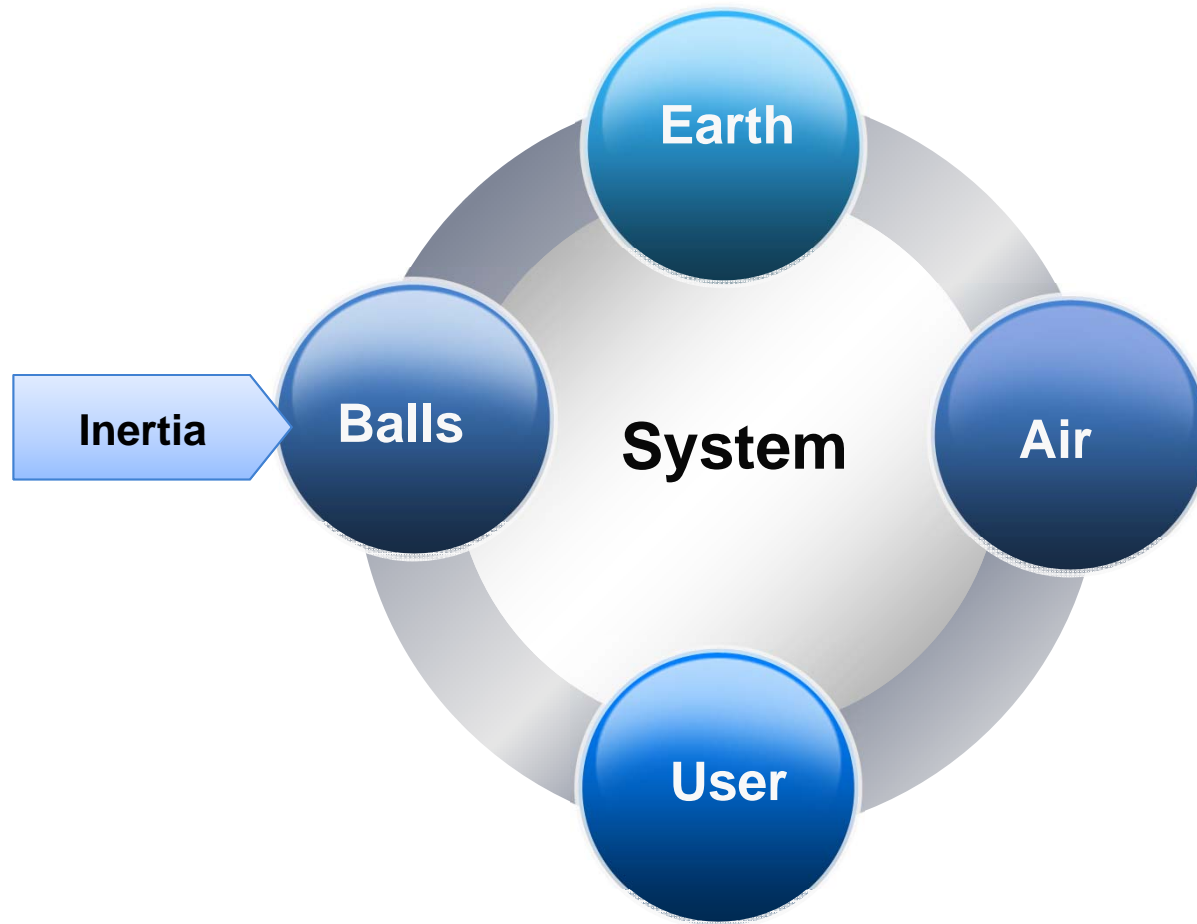


Courtesy of [http://en.wikipedia.org/wiki/Leaning\\_Tower\\_of\\_Pisa](http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa), <http://www.bluffton.edu/~bergerd/classes/NSC109/Handouts/answers2-3.html>

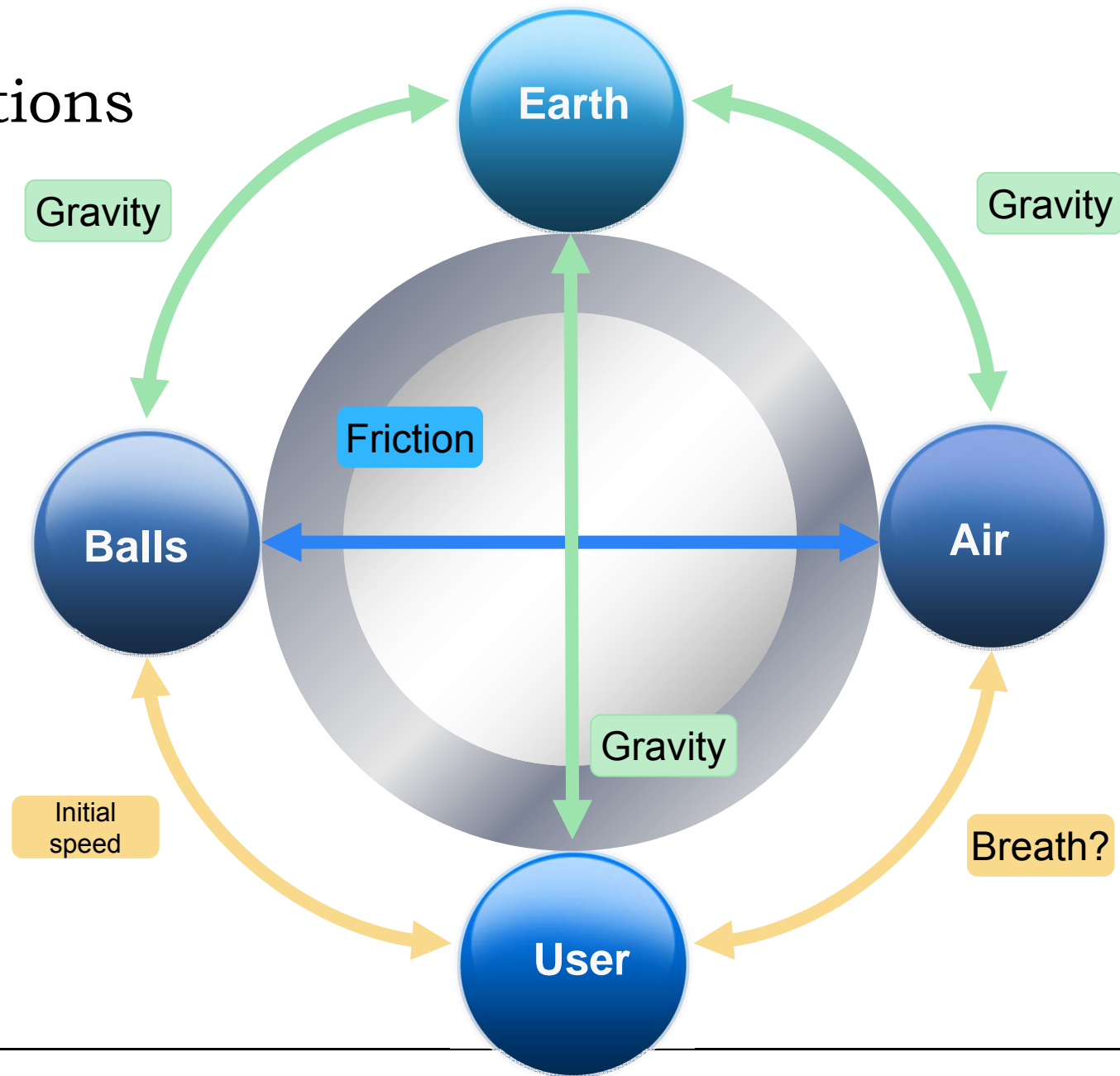
# Inside the system?



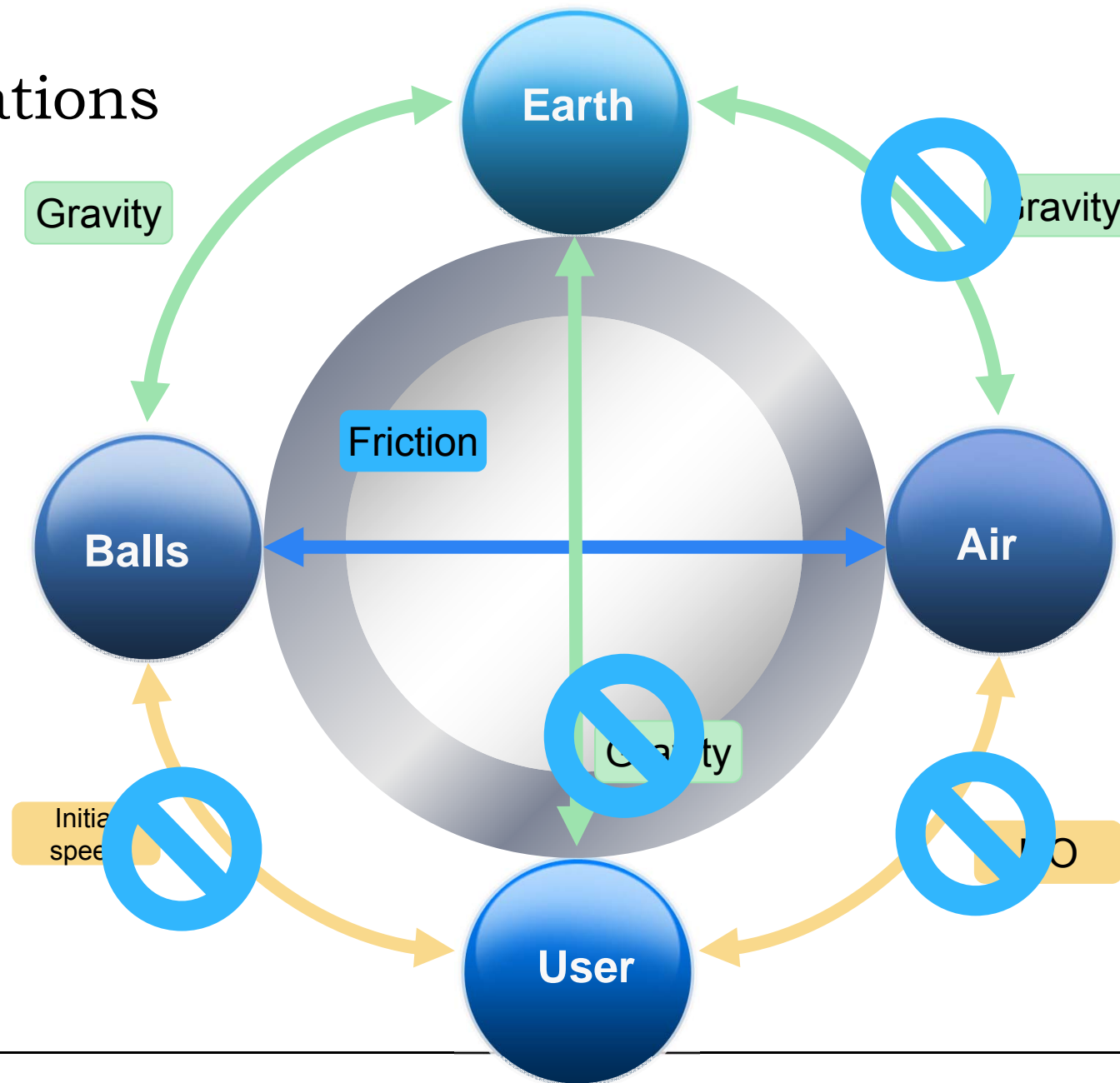
# Reasoning about components: Which one is moving?



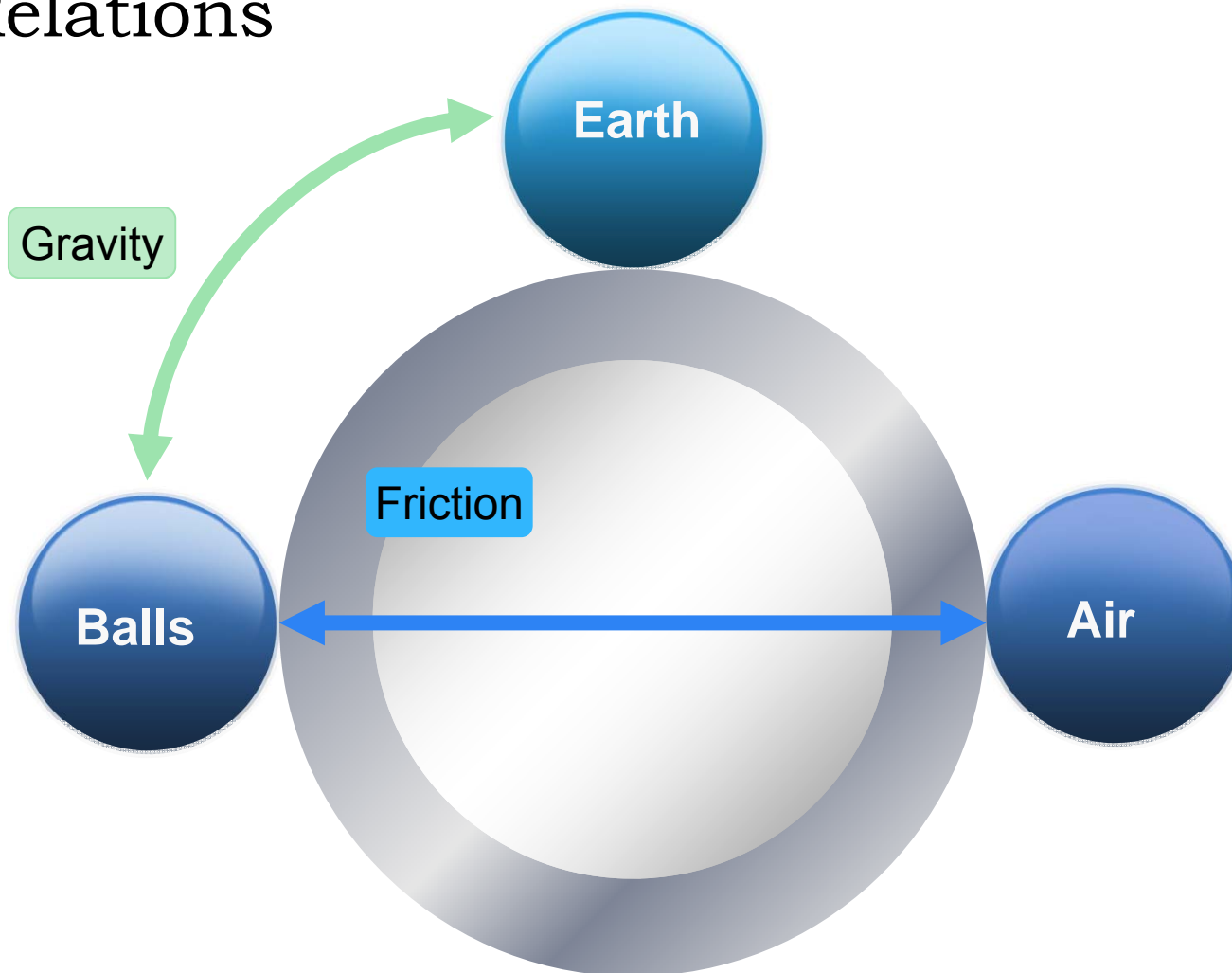
# Relations



## Relations



# Relations



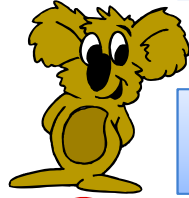
# Cause-Effect

Cause



**Inertia**

- Resist to change



**Gravity**

- Ball drops



**Air drag**

- Ball slows down

Effect

# Physics behind – The air drag

The Air drag (scalar form)

$$F_D = \frac{1}{2} \rho A C_d V^2$$

Ref. [http://en.wikipedia.org/wiki/Drag\\_\(physics\)](http://en.wikipedia.org/wiki/Drag_(physics))



# Physics behind – The air drag

Density  
Unit(kg/m<sup>3</sup>)

The density of air

1.204 kg/m<sup>3</sup>

The Air drag (scalar form)

$$F_D = \frac{1}{2} \rho A C_d V^2$$

Ref. [http://en.wikipedia.org/wiki/Drag\\_\(physics\)](http://en.wikipedia.org/wiki/Drag_(physics))

# Physics behind – The air drag

Reference Area  
Unit(m<sup>2</sup>)

The Air drag (scalar form)

Cross section area  
Perpendicular to  
the velocity

$$F_D = \frac{1}{2} \rho A C_d V^2$$



The velocity

Ref. [http://en.wikipedia.org/wiki/Drag\\_\(physics\)](http://en.wikipedia.org/wiki/Drag_(physics)), Courtesy of <http://www.redlineblog.com/reventon-lambos-ruthless-new-bull/>

# Physics behind – The air drag

## The Air drag (scalar form)

$$F_D = \frac{1}{2} \rho A C_d V^2$$

Shape	Drag Coefficient
Sphere	0.11
1/2 sphere	0.12
Cone	0.50
Cube	0.80
Angled Cube	0.80
Long Cylinder	0.82
Short Cylinder	1.0
Streamlined body	0.04
Streamlined 1/2 body	0.09
Measured Drag Coefficients	

Drag coefficient  
(dimensionless)



Courtesy of [http://en.wikipedia.org/wiki/Drag\\_\(physics\)](http://en.wikipedia.org/wiki/Drag_(physics)), <http://www.cyclingweekly.co.uk/archive/348018/how-to-improve-your-aero-position.html>

# Why?

Shape	Drag Coefficient
Sphere	0.11
Half sphere	0.12
Cone	0.30
Cube	0.5
Angled Cube	0.80
Square Cylinder	0.82
Square Cylinder	1
Streamlined body	0.01
Streamlined body	0.09
Measured Drag Coefficients	



Courtesy of [http://en.wikipedia.org/wiki/Drag\\_\(physics\)](http://en.wikipedia.org/wiki/Drag_(physics)), <http://www.cyclingweekly.co.uk/archive/348018/how-to-improve-your-aero-position.html>

# Physics behind – The air drag

## The Air drag (scalar form)

$$F_D = \frac{1}{2} \rho A C_d V^2$$

Density  
Unit(kg/m<sup>3</sup>)  
Reference Area  
Unit(m<sup>2</sup>)

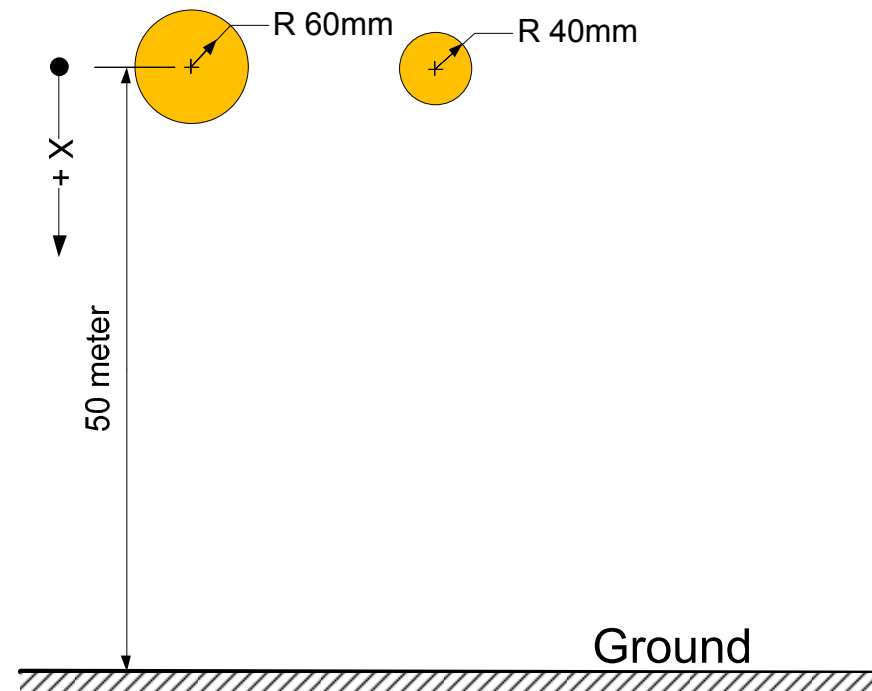
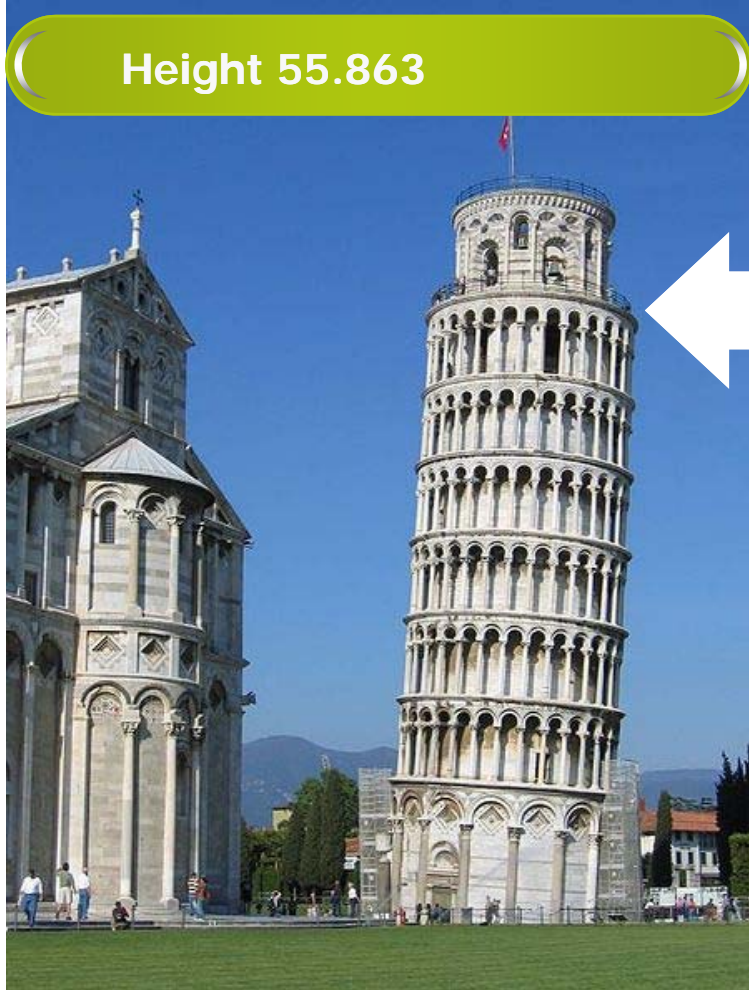
Velocity  
Unit(m/s)

Drag coefficient  
(dimensionless)

Ref. [http://en.wikipedia.org/wiki/Drag\\_\(physics\)](http://en.wikipedia.org/wiki/Drag_(physics))

# Modeling

Height 55.863



Courtesy of [http://en.wikipedia.org/wiki/Leaning\\_Tower\\_of\\_Pisa](http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa)

# Dynamics – Equilibrium (Scalar)

## Forces related to Cause ~ effect

Inertia force  $-\frac{d}{dt}m\mathbf{v}$

$$-m\frac{d^2x(t)}{dt^2}$$

Gravity •Earth Gravity

$$mg$$

Boundary

Input

Reaction

Friction •From the air

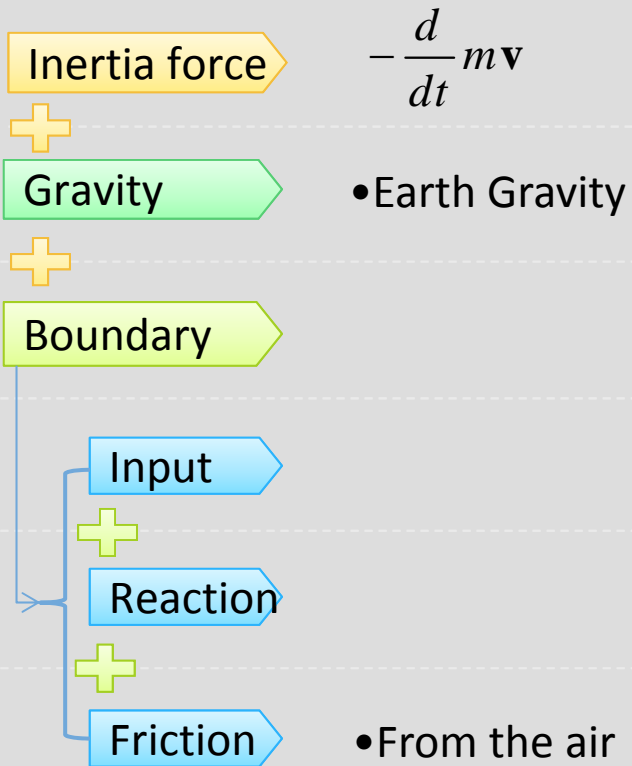
$$-\frac{1}{2}\rho AC_d\left(\frac{dx(t)}{dt}\right)^2$$

$$\sum F_x = 0$$

$x(t)$  - travel along X-axis

# Dynamics – The model

## Forces related to Cause ~ effect



$$\sum F_x = 0$$

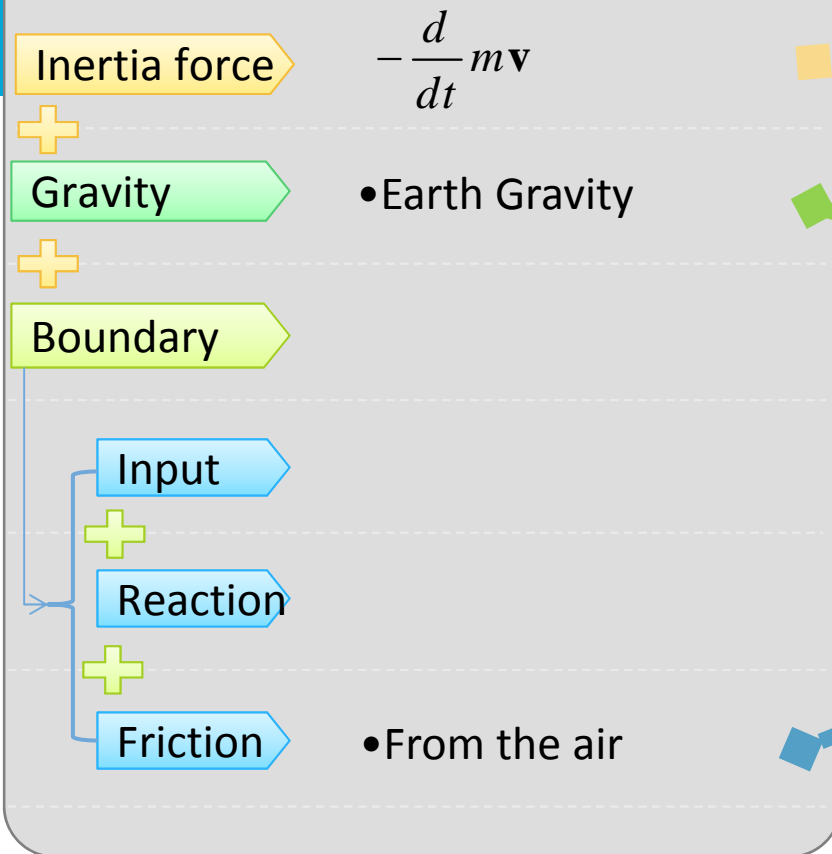
$$-m \frac{d^2 x(t)}{dt^2} + mg - \frac{1}{2} \rho A C_d \left( \frac{dx(t)}{dt} \right)^2 = 0$$

$x(t)$  - travel along X-axis



# Dynamics – The model

## Forces related to Cause ~ effect



$$\sum F_x = 0$$

$$-m \frac{d^2 x(t)}{dt^2} + mg - \frac{1}{2} \rho A C_d \left( \frac{dx(t)}{dt} \right)^2 = 0$$

$x(t)$  - travel along X-axis

# Dynamics – Equilibrium

$$-m \frac{d^2 x(t)}{dt^2} + mg - \frac{1}{2} \rho A C_d \left( \frac{dx(t)}{dt} \right)^2 = 0$$

## We choose

- The ball radiuses are 60mm and 40mm, respectively
- The balls are made from Iron, density is 7800kg/m<sup>3</sup>
- The travel distances is 50 meters
- The density of air is 1.204kg/m<sup>3</sup>
- The reference area is  $\pi r^2$
- The drag coefficient is 0.47

# Dynamics – Equilibrium

$$-m \frac{d^2 x(t)}{dt^2} + mg - \frac{1}{2} \rho A C_d \left( \frac{dx(t)}{dt} \right)^2 = 0$$

$$\rho_{\text{Iron}} V$$
$$= \rho_{\text{Iron}} \frac{4}{3} \pi r^3$$

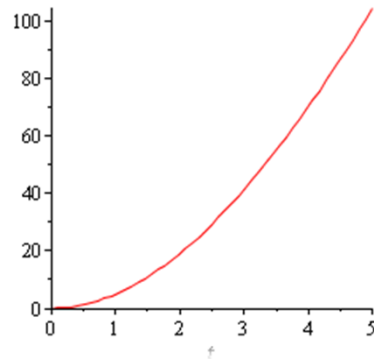
## We choose

- The ball radiuses are 60mm and 40mm, respectively
- The balls are made from Iron, density is 7874kg/m<sup>3</sup>
- The travel distances is 50 meters
- The density of air is 1.204kg/m<sup>3</sup>
- The reference area is  $\pi r^2$
- The drag coefficient is 0.47

# Dynamics – Solve

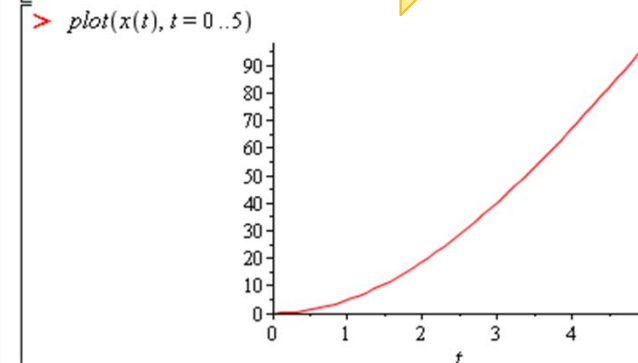
Big

```
> restart;
> equ := -m·diff(x(t), t$2) + m·g - 0.5·pair·A·Cd·(diff(x(t), t))^2 = 0 :
> m :=  $\frac{4}{3}$ ·pIron·3.1415926·r3 : pIron := 7684 : r := 0.006 : g := 9.8 : pair
:= 1.204 : A := 3.1415926·r2 : Cd := 0.47 :
> ics := x(0) = 0, D(x)(0) = 0 :
> sol := dsolve({equ, ics}, x(t)) :
> x := unapply(op(2, sol), t) :
> evalf(solve(x(t) = 50, t));
-3.318200010, 3.318200013
```



Small

```
> restart,
> equ := -m·diff(x(t), t$2) + m·g - 0.5·pair·A·Cd·(diff(x(t), t))^2 = 0 :
> m :=  $\frac{4}{3}$ ·pIron·3.1415926·r3 : pIron := 7684 : r := 0.004 : g := 9.8 : pair
:= 1.204 : A := 3.1415926·r2 : Cd := 0.47 :
> ics := x(0) = 0, D(x)(0) = 0 :
> sol := dsolve({equ, ics}, x(t)) :
> x := unapply(op(2, sol), t) :
> evalf(solve(x(t) = 50, t));
3.380945089
```



The big ball is 0.062 second faster

Indeed, our experience is right, the big ball is faster.  
It is just hardly noticeable in this case (Human average reaction time: 0.215 second)

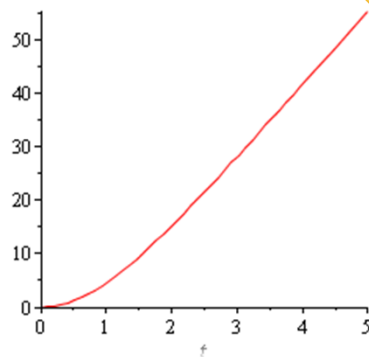
Ref. <http://www.humanbenchmark.com/tests/reactiontime/index.php>

# Evaluations – Wooden ball

Big

Galileo - Wood

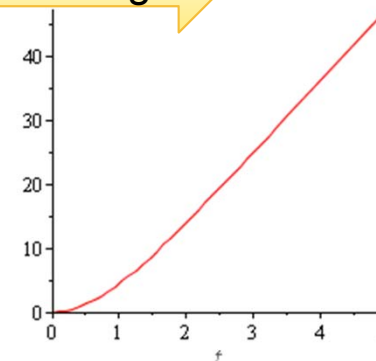
```
> restart;
> equ := -m·diff(x(t), t$2) + m·g - 0.5·C_d·(diff(x(t), t))^2 = 0 :
> m :=  $\frac{4}{3} \cdot \rho_{Wood} \cdot 3.1415926 \cdot r^3$  :  $\rho_{Wood} := 684$  :  $r := 0.006$  :  $g := 9.8$  :  $\rho_{air} := 1.204$  :  $A := 3.1415926 \cdot r^2$  :  $C_d := 0.47$  :
> ics := x(0) = 0, D(x)(0) = 0 :
> sol := dsolve({equ, ics}, x(t)) :
> x := unapply(op(2, sol), t) :
> evalf(solve(x(t) = 50, t));
```



Small

Galileo - Wood

```
> restart;
> equ := -m·diff(x(t), t$2) + m·g - 0.5·C_d·(diff(x(t), t))^2 = 0 :
> m :=  $\frac{4}{3} \cdot \rho_{Wood} \cdot 3.1415926 \cdot r^3$  :  $\rho_{Wood} := 684$  :  $r := 0.004$  :  $g := 9.8$  :  $\rho_{air} := 1.204$  :  $A := 3.1415926 \cdot r^2$  :  $C_d := 0.47$  :
> ics := x(0) = 0, D(x)(0) = 0 :
> sol := dsolve({equ, ics}, x(t)) :
> x := unapply(op(2, sol), t) :
> evalf(solve(x(t) = 50, t));
```



0.64 second, which is 10 times larger

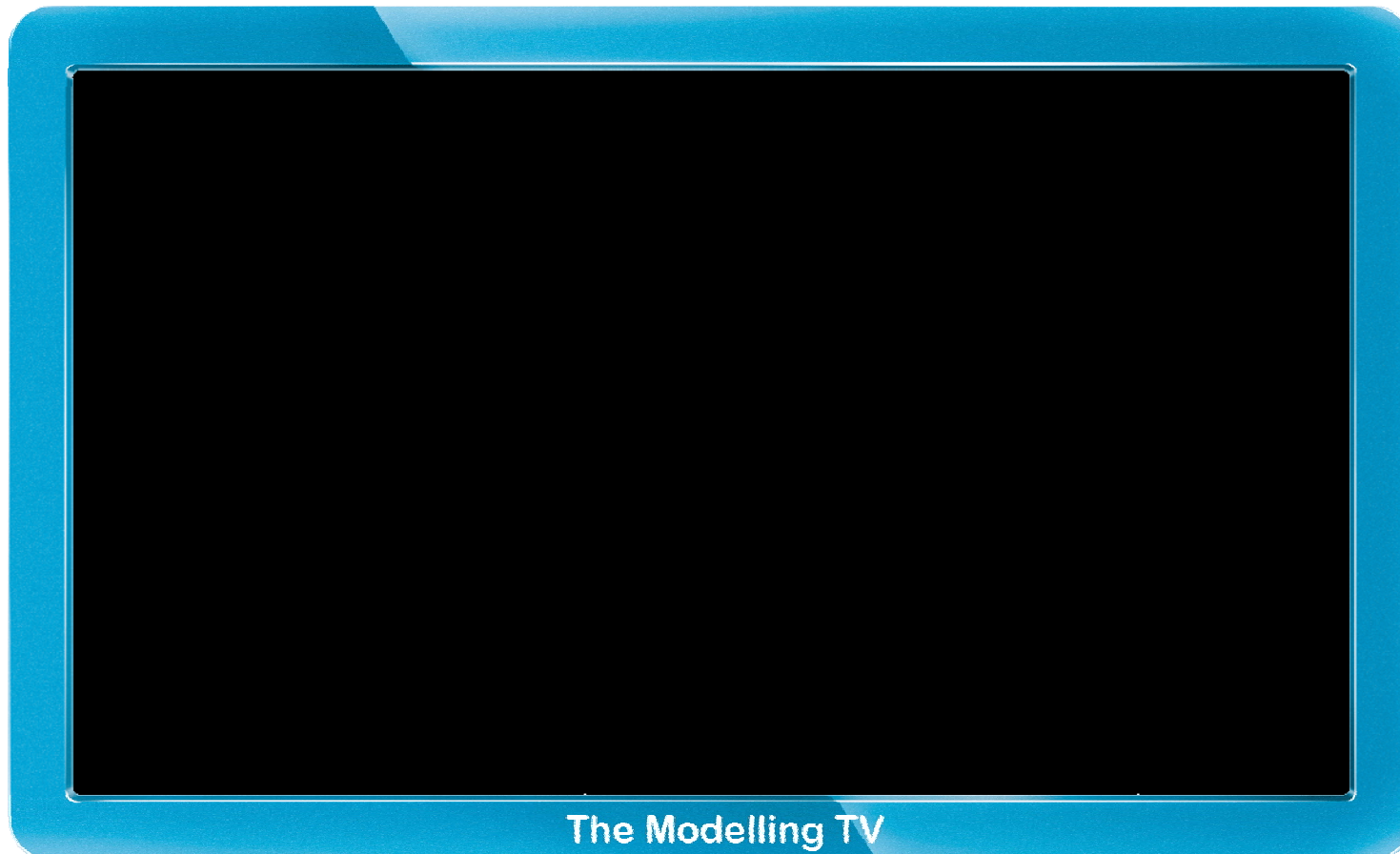
Now we know how smart Galileo Galilei is

# Safety regarding Wooden ball



Courtesy of <http://www.theage.com.au/news/world/deadly-storms-lash-europe/2007/01/12/1168105153228.html>, <http://www.bluffton.edu/~bergerd/classes/NSC109/Handouts/answers2-3.html>

# The power of wind



April 18, 2013





# Dynamics – Equilibrium

es related to Cause ~ effect

force  $-\frac{d}{dt}m\mathbf{v}$

•Earth Gravity

ary

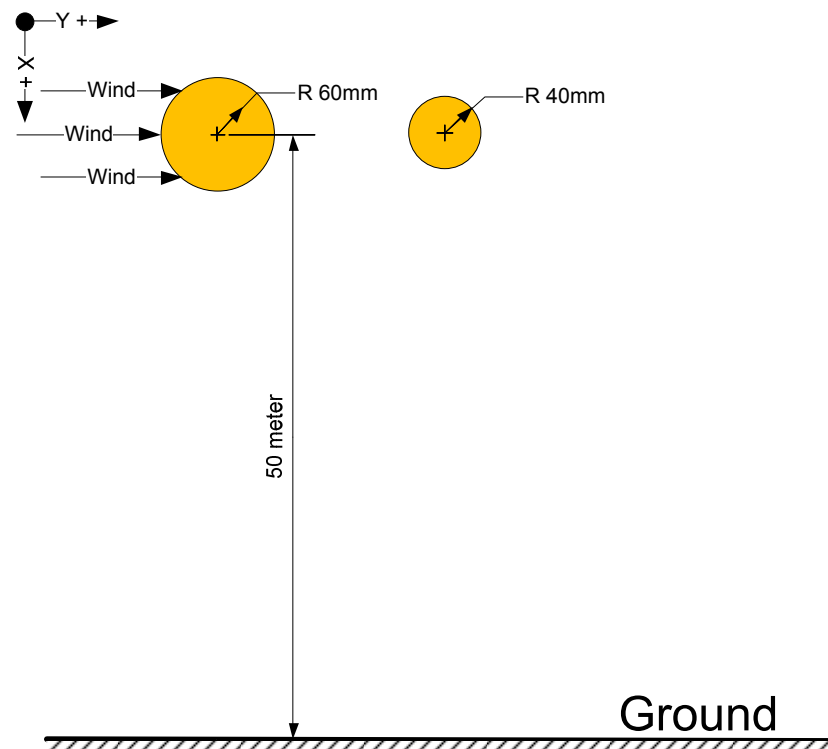
out

action

iction

•From the air

$$\sum F_y = 0$$



# Dynamics – Equilibrium along Y

Forces related to Cause ~ effect

Force  $-\frac{d}{dt}m\mathbf{v}$

•Earth Gravity

ry

ut •From the air

action

ction

$$-m \frac{d^2 y(t)}{dt^2}$$

$$\frac{1}{2} \rho A C_d \left( V_{wind} - \frac{dy(t)}{dt} \right)^2$$

$$\sum F_y = 0$$

# Dynamics – Equilibrium along Y

Forces related to Cause ~ effect

Force  $-\frac{d}{dt}m\mathbf{v}$

Gravity •Earth Gravity

Drag

Input •From the air

Reaction

Friction

$$\sum F_y = 0$$

$$-m \frac{d^2 y(t)}{dt^2} + \frac{1}{2} \rho A C_d \left( V_{wind} - \frac{dy(t)}{dt} \right)^2 = 0$$

# Dynamics – Equilibrium

$$-m \frac{d^2 y(t)}{dt} + \frac{1}{2} \rho A C_d \left( V_{wind} - \frac{dy(t)}{dt} \right)^2 = 0$$

## We choose

- ☐ The ball radiuses are 60mm and 40mm, respectively
- ☐ The balls are made from Wood, the density is 684kg/m<sup>3</sup>
- ☐ The wind speed is 5.66 m/s
- ☐ The density of air is 1.204kg/m<sup>3</sup>
- ☐ The reference area is  $\pi r^2$
- ☐ The drag coefficient is 0.47

# Dynamics – Equilibrium

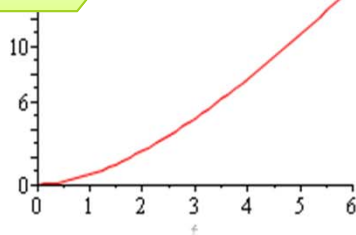
Small Fly - Wood

```

> restart;
> equ := -m·diff(y(t), t$2) + 0.5·pair·A·Cd·(diff(y(t), t))2 = 0 :
> m := 4/3·pWood·3.1415926·r3 : pWood := 684 : r := 0.006 : g := 9.8 : pair
:= 1.204 : A := 3.1415926·r2 : Cd := 0.47 :
> ics := y(0) = 0, D(y)(0) = 0 :
> sol := dsolve({equ, ics}, y(t), numeric, output = listprocedure) :
> y := rhs(sol[2]);
end proc
y(5.243)
9.54275497985485721

```

5 meters



Small

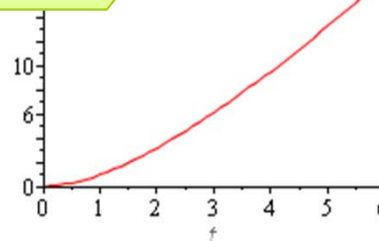
Small Fly - Wood

```

> restart;
> equ := -m·diff(y(t), t$2) + 0.5·pair·A·Cd·(diff(y(t), t))2 = 0 :
> m := 4/3·pWood·3.1415926·r3 : pWood := 684 : r := 0.004 : g := 9.8 : pair
:= 1.204 : A := 3.1415926·r2 : Cd := 0.47 :
> ics := y(0) = 0, D(y)(0) = 0 :
> sol := dsolve({equ, ics}, y(t), numeric, output = listprocedure) :
> y := rhs(sol[2]);
end proc
y(5.243)
14.2755336349381388

```

14.28 meters



## Safety region – Wooden ball



## Safety region – Wooden ball



Lean angle 5.5 degree



Height 50 meters



$D = \text{Height} * \tan(\text{Lean angle}) = 4.2 \text{ meters}$



Small wooden ball 14.28 meters

# How about an iron ball?

Big

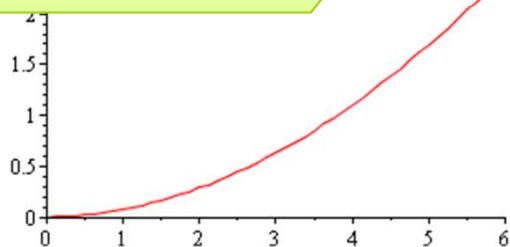
Gameo - Fly - Iron

```
> restart;
> equ := -m·diff(y(t), t$2) + 0.5·pair·A·Cd·(diff(y(t), t))2 = 0 :
> m :=  $\frac{4}{3} \cdot \rho_{\text{Iron}} \cdot 3.1415926 \cdot r^3$  :  $\rho_{\text{Iron}} := 7684$  :  $r := 0.006$  :  $g := 9.8$  :  $p_{\text{air}}$ 
:= 1.204 :  $A := 3.1415926 \cdot r^2$  :  $C_d := 0.47$  :
> ics := y(0) = 0, D(y)(0) = 0 :
> sol := dsolve({equ, ics}, y(t), numeric, output = listprocedure) :
> y := rhs(sol[2]);
```

y := proc and proc

0.767721228050735838

0.77 meter



Small

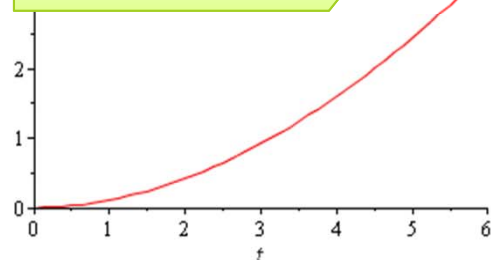
Fly - Iron

```
> restart;
> equ := -m·diff(y(t), t$2) + 0.5·pair·A·Cd·(diff(y(t), t))2 = 0 :
> m :=  $\frac{4}{3} \cdot \rho_{\text{Iron}} \cdot 3.1415926 \cdot r^3$  :  $\rho_{\text{Iron}} := 7684$  :  $r := 0.004$  :  $g := 9.8$  :  $p_{\text{air}}$ 
:= 1.204 :  $A := 3.1415926 \cdot r^2$  :  $C_d := 0.47$  :
> ics := y(0) = 0, D(y)(0) = 0 :
> sol := dsolve({equ, ics}, y(t), numeric, output = listprocedure) :
> y := rhs(sol[2]);
```

y := proc and proc

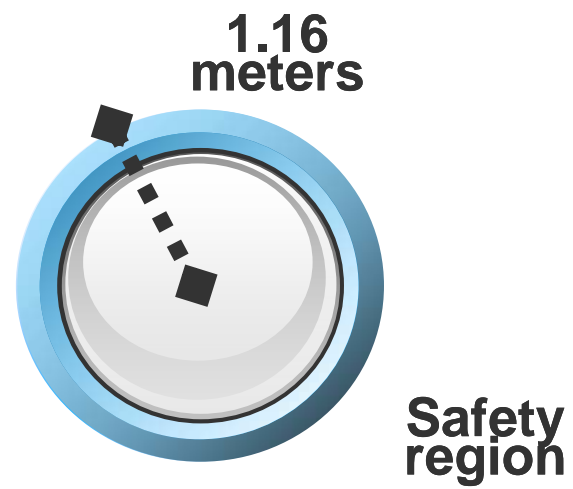
1.16213310612100962

1.16 meter

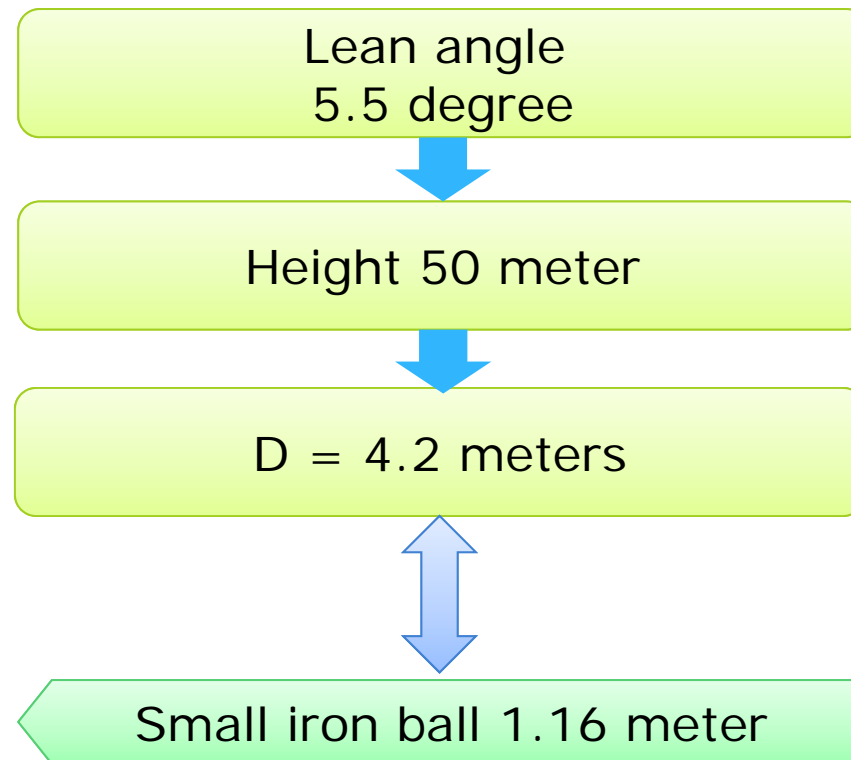




## Safety region – Iron ball



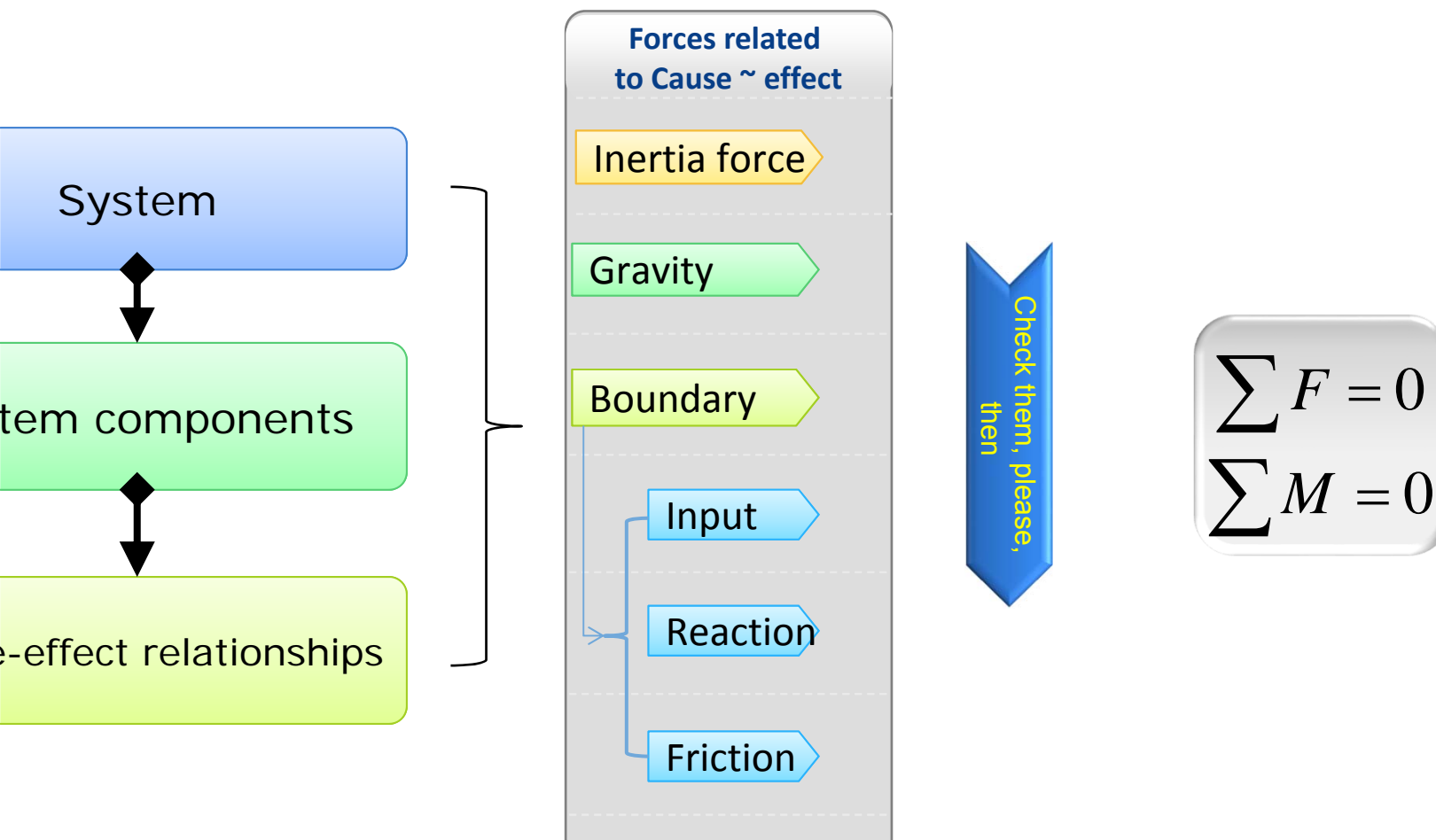
## Safety region – Iron ball



Now we know more about Galileo Galilei.  
He is sure that the ball will reach the ground.

Summary

What did we learn



# Workshop

Design brief

A seesaw (also known as a teeter-totter or teeter board) is a long, narrow board pivoted in the middle so that, as one end goes up, the other goes down. (ref. <http://en.wikipedia.org/wiki/Seesaw>)

Question:  
Ergonomics study indicates that when child starts to swing by kicking the ground, the force is about 100N force, the time is about 0.2 second. Specify the constant of the torsion spring to make sure the children can swing more than 5 times.

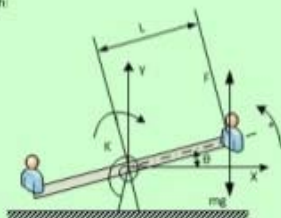
We choose

- Two children (20kg) sit on each side, respectively;
- We suppose the child is sit at the end of the board and the gravity center is on the board;
- To simplify the question, we choose all rotation is around the center of the board;
- The board is made by pinewood, the density is  $700\text{ kg/m}^3$ ;
- The total length of the board is 4 meters, the width is 0.2 m and the thickness is 0.1 m;
- At the center of the seesaw, there is friction torque 15Nm, and it always applied in the direction against the rotation;
- Neglect the weight of the handle
- Neglect the air friction

System



Sketch:



Courtesy of <http://www.bkn.nl/products/spel-spuit/wigwagtoetsketen/wigwag-de-waaker/taftankings-1243>

EM=0					
Cause	Mass of child	Mass of Board	Spring	Friction force	Input
Effect	Inertia force	Child goes down	Drags the board back	Slows the motion	Slows the motion

$$-2I_{khd} \frac{d^2 \theta(t)}{dt^2} - 2I_{bound} \frac{d^2 \theta(t)}{dt^2} - k\theta(t) - \begin{cases} f \frac{d\theta(t)}{dt} > 0 \\ -f \frac{d\theta(t)}{dt} \leq 0 \end{cases} + F(t)L = 0$$

Green line

Your choices?

Solution  
See Maple file P-W-2

# Workshops: Questions & Coaching

## Workshops: A sample question

### Workshop G-W-0: The slide - Self-study



Courtesy of <http://www.kidsplanet.net>

A child slides down from the top of a slide:

1. What is his speed at the bottom of the slide?
2. To guarantee the safety of the child, the maximum velocity of the child should be (degree)

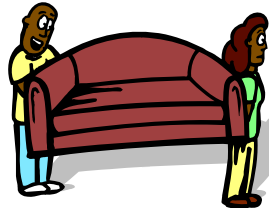
#### ▼ We Choose

Slide:

The slide is straight, 5 meters long, leaning angle 30 degrees.  
The slide is fixed on the ground, its surface friction coefficient 0.2.

Child:

Weight 20kg, He keeps still in the process.



1 coach

1 ~ 2 TA(s)

### Grading

0

•Not presented

0.5

•No progress

1

•Progressing

# How to do it?

Problem Brief

Choices

System

Sketch

IO2081 Modelling

Workshop G-W-0: The slide

Attention: Fiction case study, for education only

Design a slide for children.

Consider a child slides down from the top of a slide:  
1. What is his speed at the bottom of the slide?  
2. To guarantee the safety, the maximum velocity of the child should be less than 3.5m/s. What is the maximum leaning angle of the slide in your design?

Slide:  
The slide is straight, 5 meters long. In the initial design, the leaning angle is 30 degrees.  
The slide is fixed on the ground, its surface friction coefficient is 0.1.  
The accuracy of the leaning angle specified by you should be within 1 degree.

Child:  
Weight 20kg, he keeps still in the process.

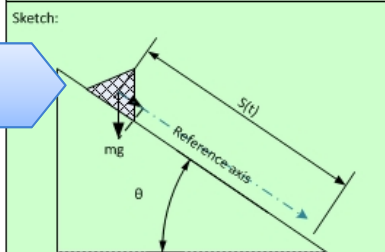
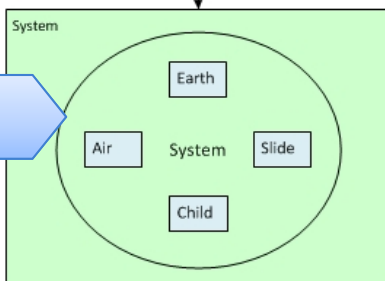
If we consider air drag: Air drag coefficient 0.5, air density 1.2 kg/m<sup>3</sup>, the frontal area of the child is 0.09 m<sup>2</sup>.

Cause-Effects

Advanced



Courtesy of <http://www.kidplanetusa.com/Schedules.htm>



Neglect air drag			
Cause	Mass of child	Gravity	Slide friction
Effect	Inertia force	Child goes down	Childs slows down

Consider all forces along reference axis

$$-m \frac{d^2 S(t)}{dt^2} + mg \sin(\theta) - fmg \cos(\theta) = 0$$

Consider air drag				
Cause	Mass of child	Gravity	Slide friction	Air drag
Effect	Inertia force	Child goes down	Child slows down	Child slows down

Consider all forces along reference axis

$$-m \frac{d^2 S(t)}{dt^2} + mg \sin(\theta) - fmg \cos(\theta) - \frac{1}{2} \rho A c \left( \frac{dS(t)}{dt} \right)^2 = 0$$

Green Line

See M -W-0

G-W-0



Courtesy of <http://www.kidsplanetusa.com/Schedules.htm>

Design a slide for children.

Consider a child slides down from the top of a slide:

1. What is his speed at the bottom of the slide?
2. To guarantee the safety, the maximum velocity of the child should be less than 3.5m/s. What is the maximum leaning angle of the slide in your design?

Slide:

The slide is straight, 5 meters long. In the initial design, the leaning angle 30 degrees.

The slide is fixed on the ground, its surface friction coefficient 0.2.

The accuracy of the leaning angle specified by you should be within  $\pm 1$  degree.

Child:

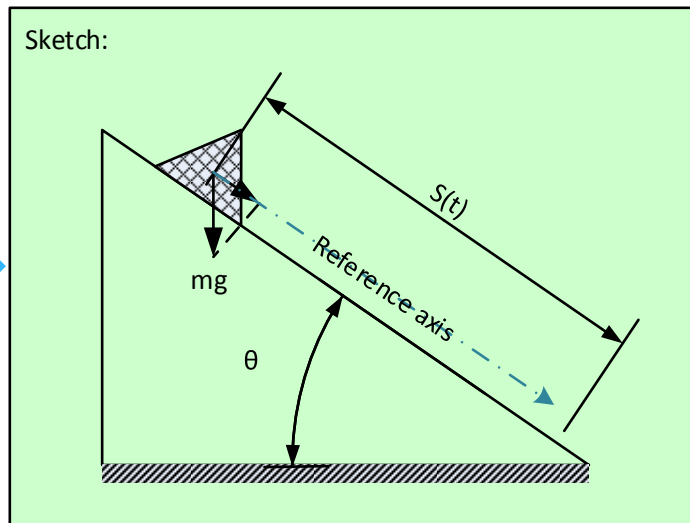
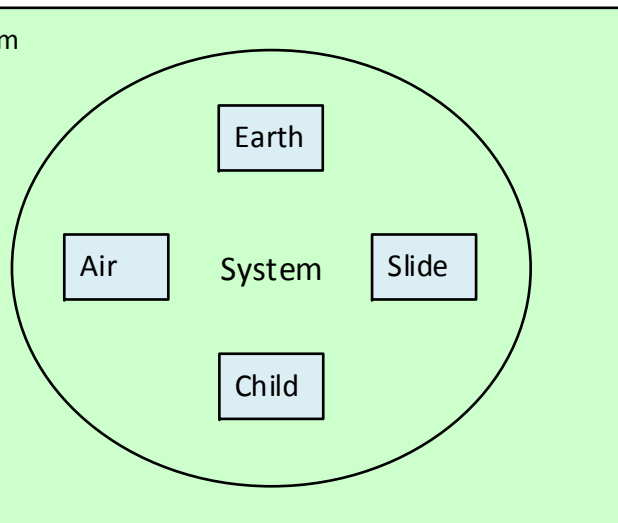
Weight 20kg, he keeps still in the process.

If we consider air drag: Air drag coefficient 0.5, air density  $1.2 \text{ kg/m}^3$ , front area of the child is  $0.09 \text{ m}^2$ .



# G-W-0

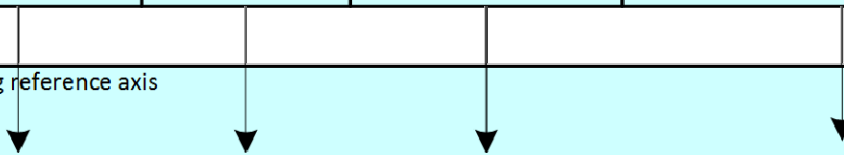
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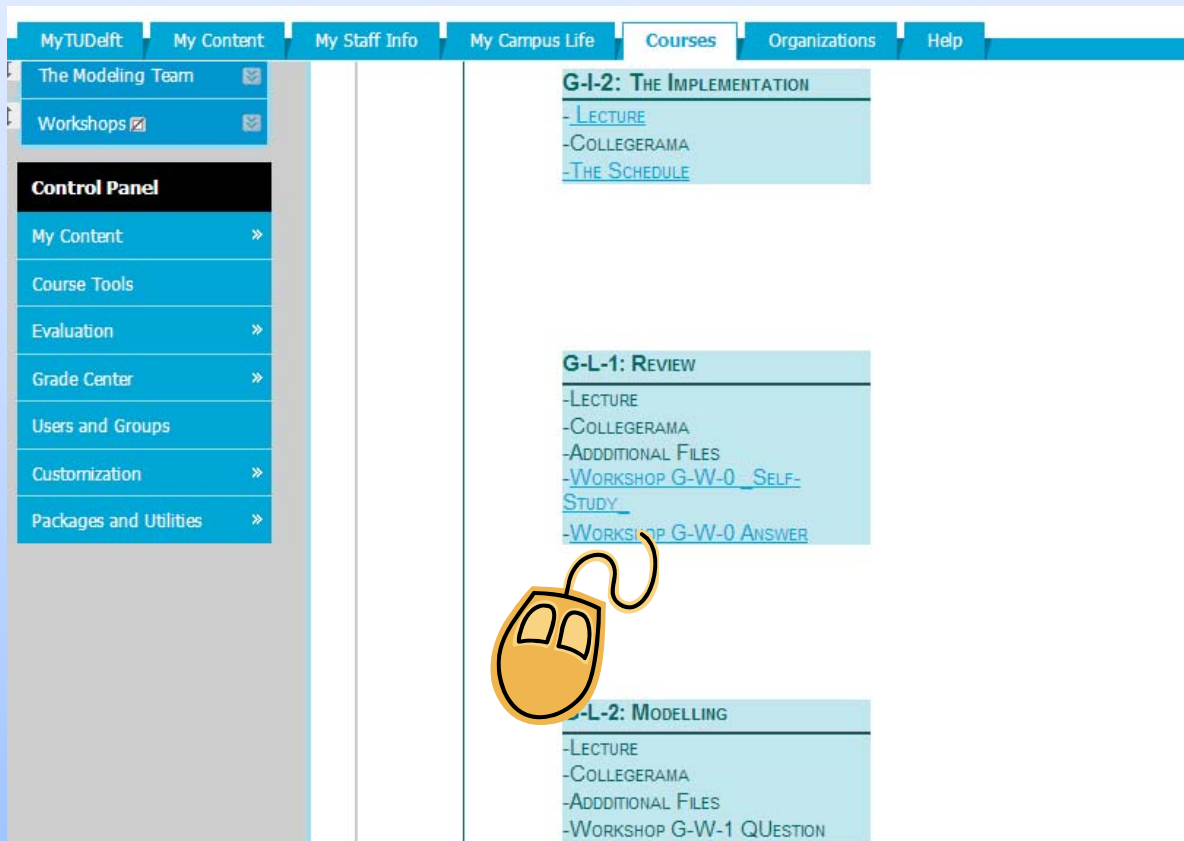
Consider air drag

Cause	Mass of child	Gravity	Slide friction	Air drag
Effect	Inertia force	Child goes down	Child slows down	Child slows down

Consider all forces along reference axis

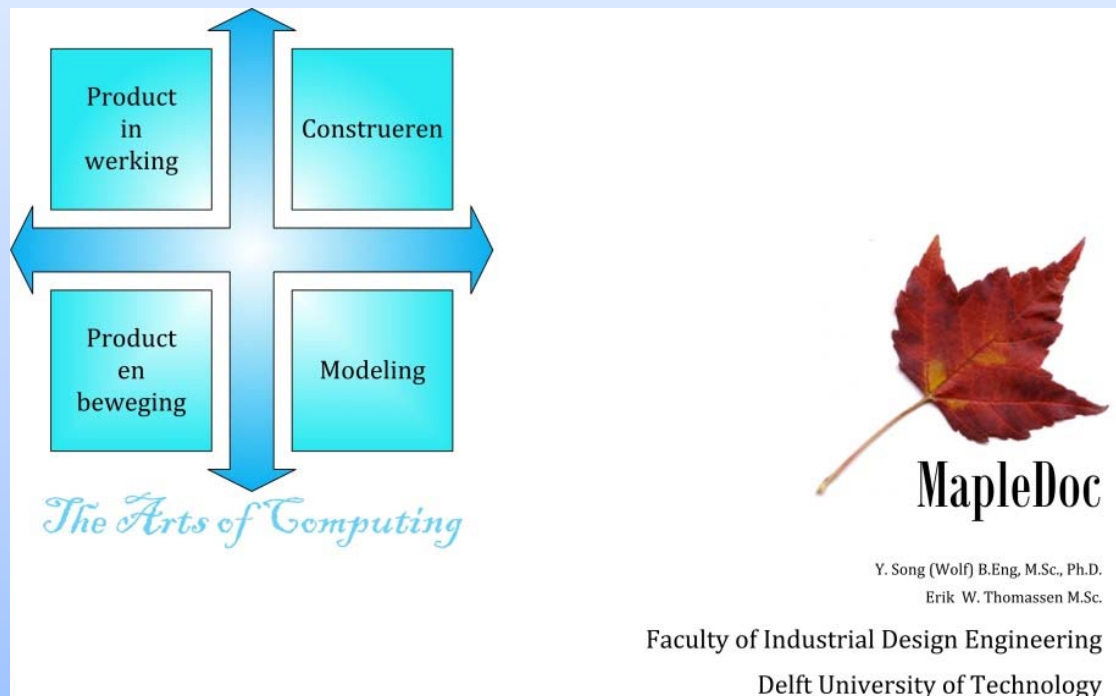


The Maple solution will be online after the workshop

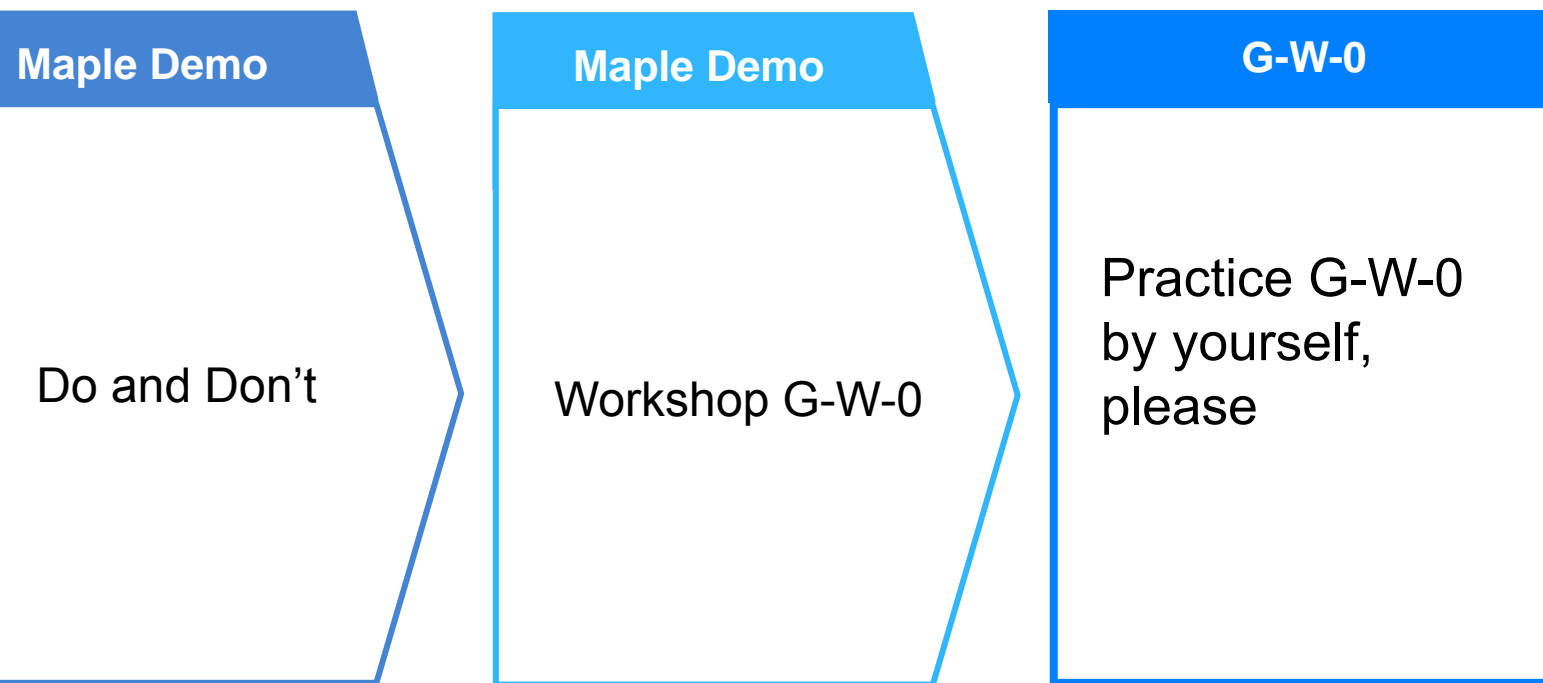


Please check MapleDoc

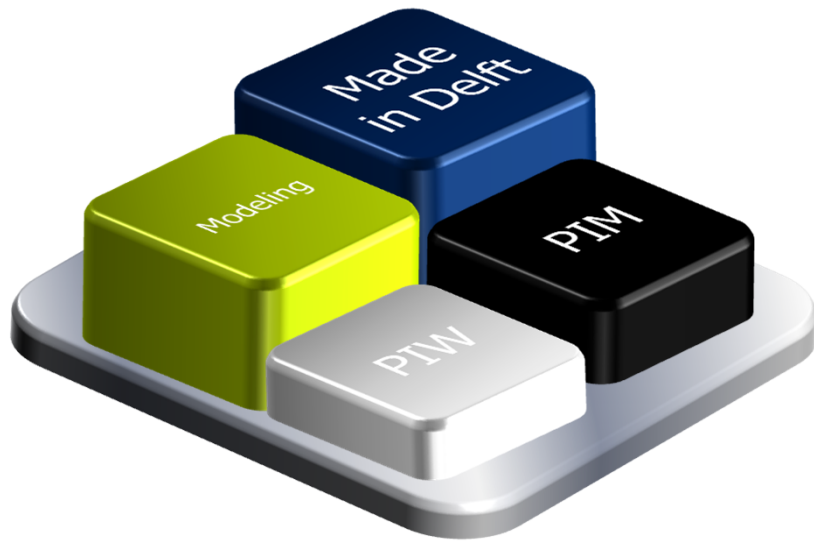
## The MapleDoc



# Demo



Thank You!



The Modelling Team  
Department of Design Engineering  
Faculty of Industrial Design Engineering

Enjoy Sunshine, Enjoy Modelling

When the spring is coming, Modelling is coming