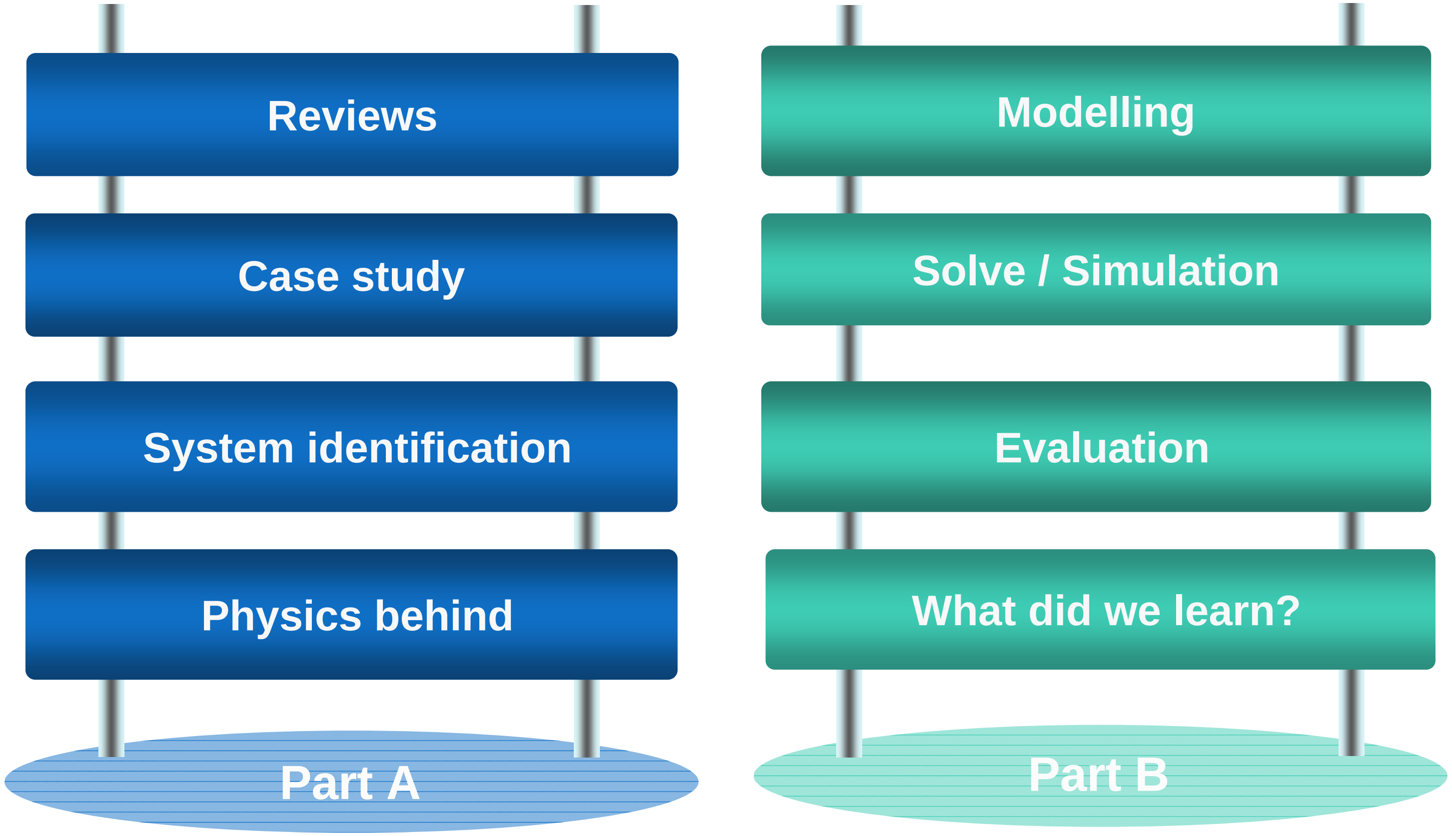
G-L-2

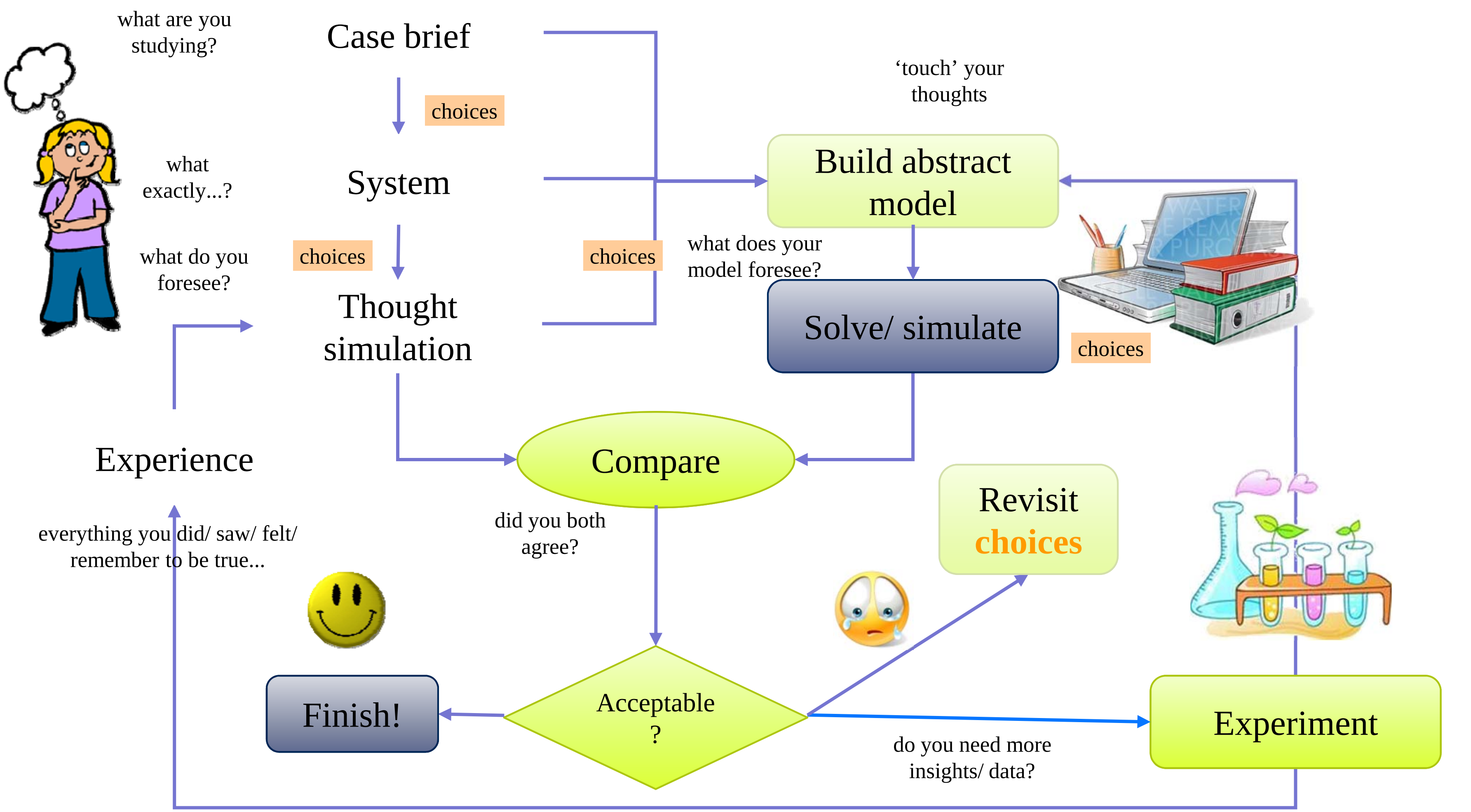
Modelling

Agenda


Reviews
Case study
System identification
Physics behind
Part A
Modelling
Solve / Simulation
Evaluation
What did we learn?
Part B

Reviews

Modelling in design



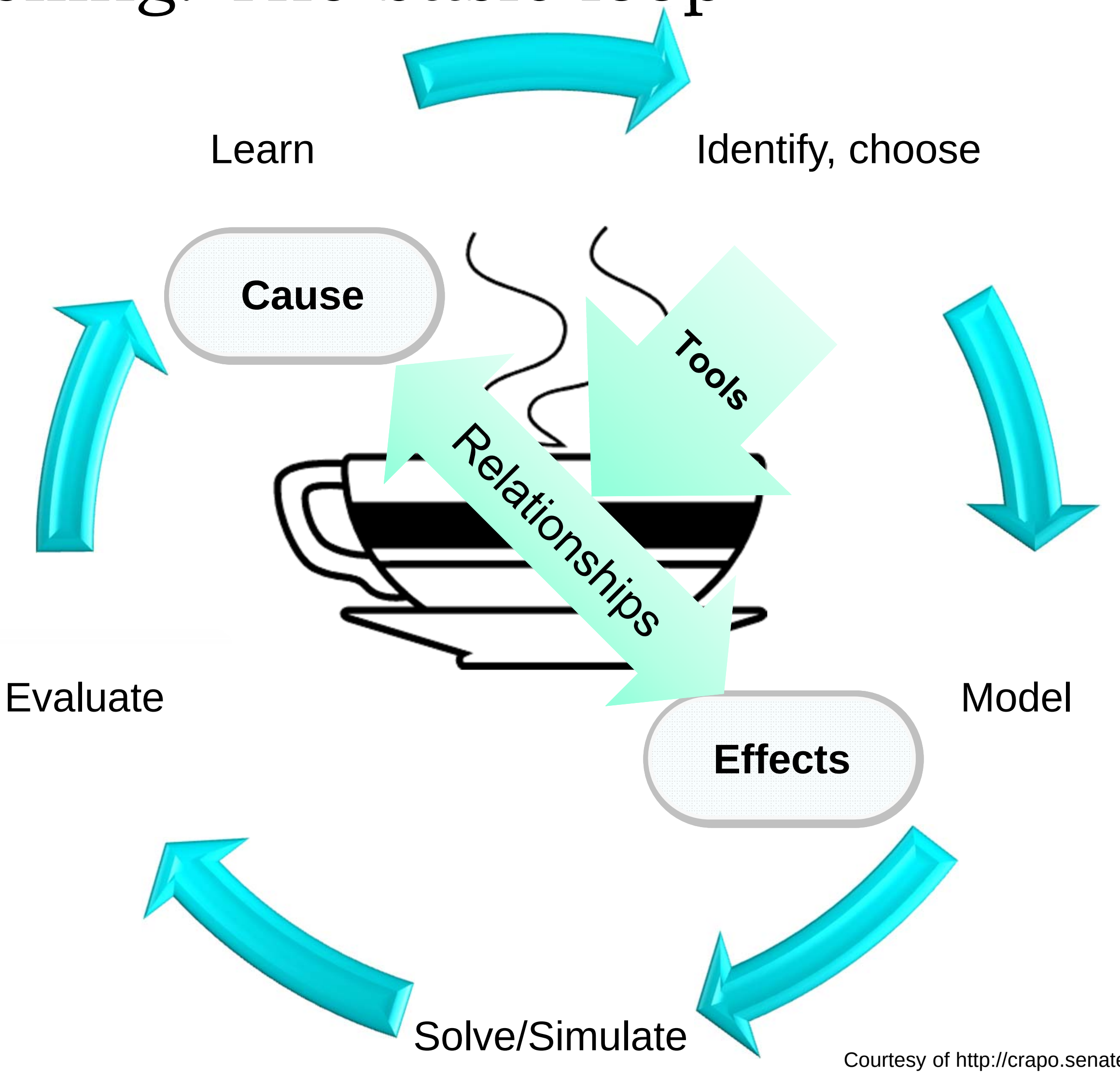
Courtesy of centech.com.pl and http://www.clipsahoy.com/webgraphics4/as5814.htm

Cause & effect – The key to modelling

Cause	Cup falls off a table, hits the floor
Identify cause-effect relationship	What is special about falling off a table? Long distance, high speed, high acceleration at impact? etc
Understand cause-effect relationship	'It is not the *fall* that kills you, but the sudden stop at the end'... Acceleration causes large forces...
Quantify cause-effect relationship	Accelerations during impact
Effect	The cup is damaged

Courtesy of http://crapo.senate.gov/services/ConstituentCoffee.cfm

Modelling: The basic loop



Courtesy of http://crapo.senate.gov/services/ConstituentCoffee.cfm

Case studies

Design Brief

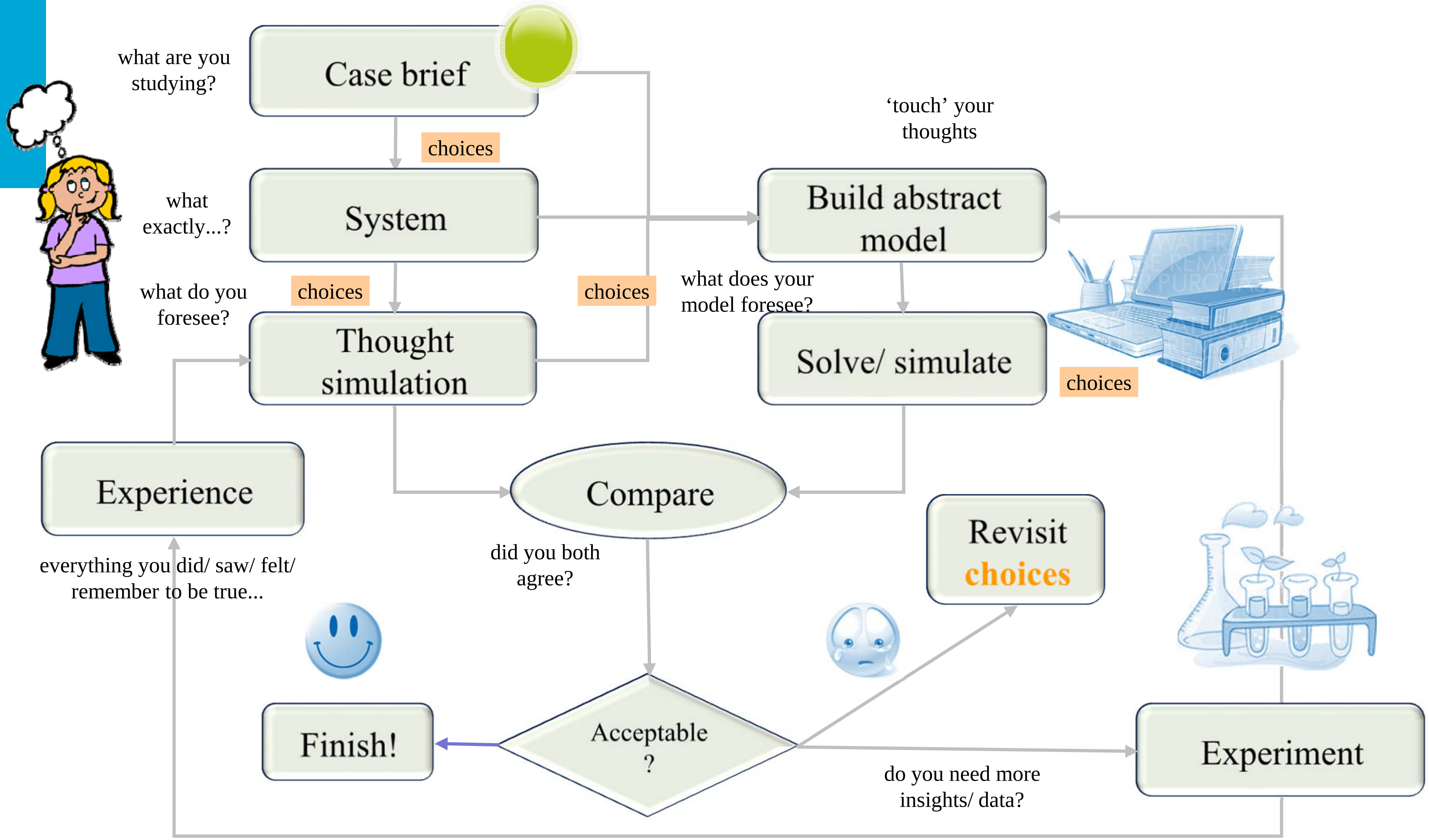
Design the functions of an ice dispenser for McDonald ®

- Cooling the Coke to 0-5 degrees with one touch of the button
- Marketing Worldwide

i'm lovin' it™

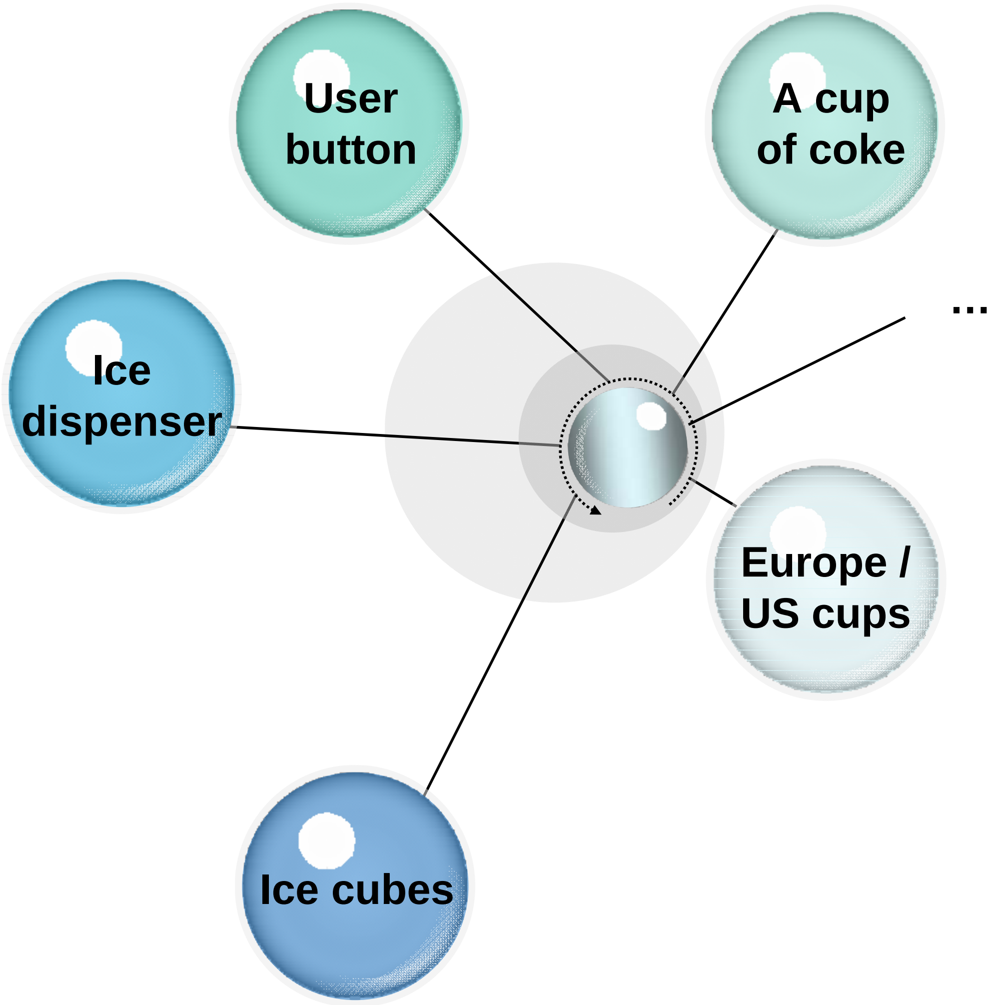
Fiction case study: For education only
Golden Arches, Golden Arches Logo and McDonald's are registered trademarks of the McDonald's Corporation
Ice dispenser image: Courtesy of http://rwmechanicalofsc.com/products.htm

The next step: System identification

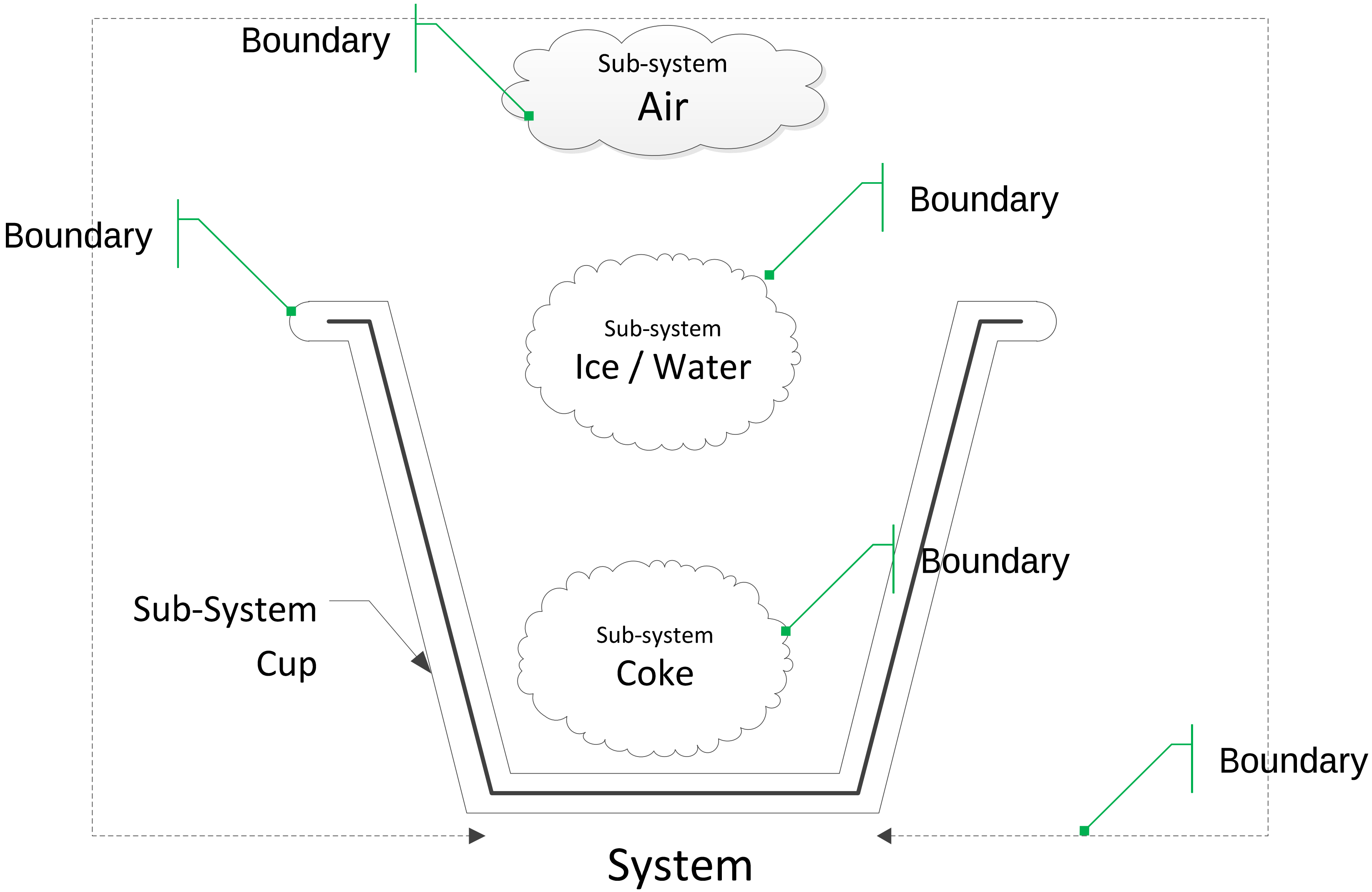


Courtesy of centech.com.pl and http://www.clipsahoy.com/webgraphics4/as5814.htm

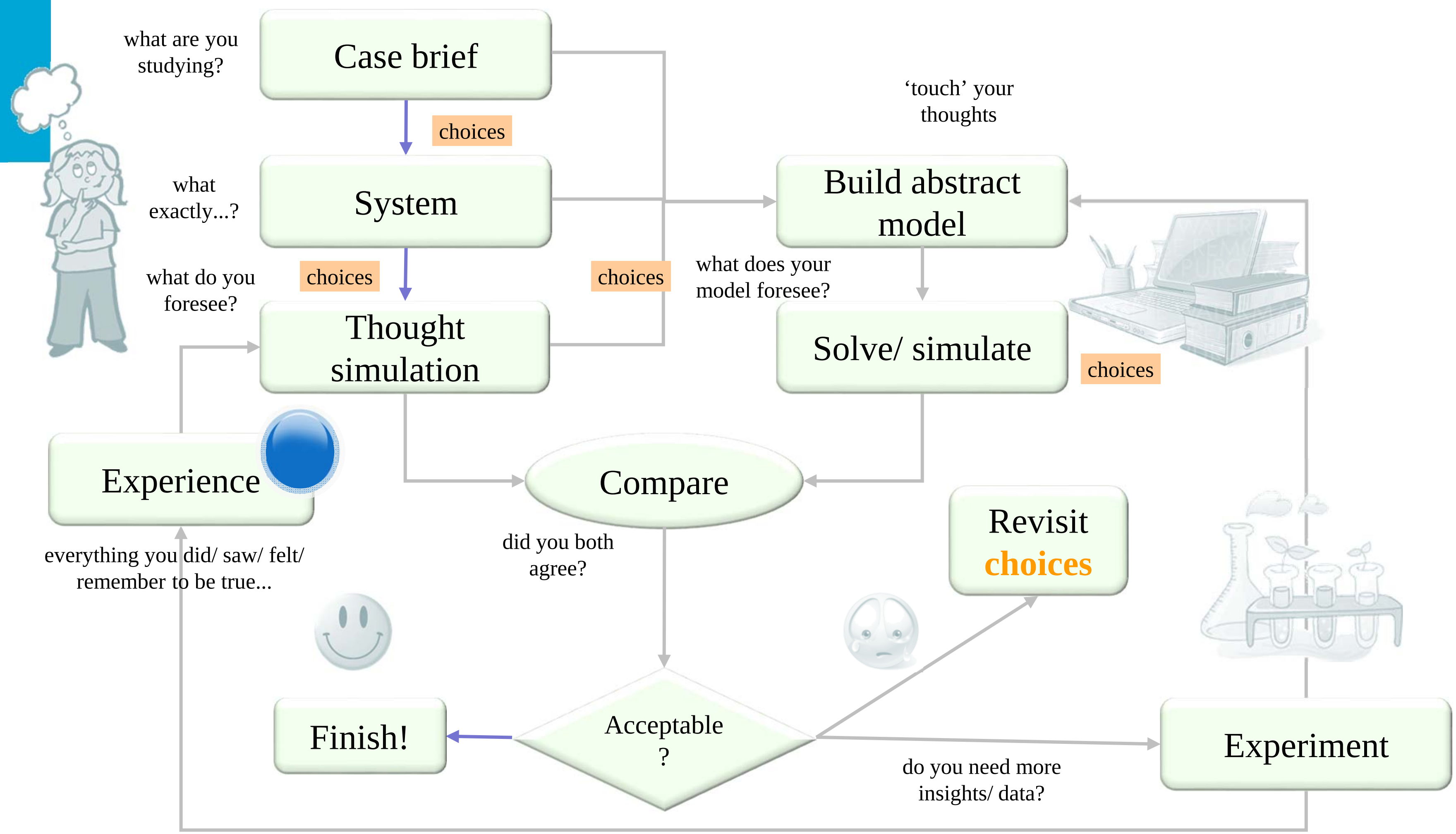
What is inside the system?



System(s) and system boundaries



The next step



Courtesy of centech.com.pl and http://www.clipsahoy.com/webgraphics4/as5814.htm

Our experience

Ice will melt | **Water cools down** | **Different volumes**

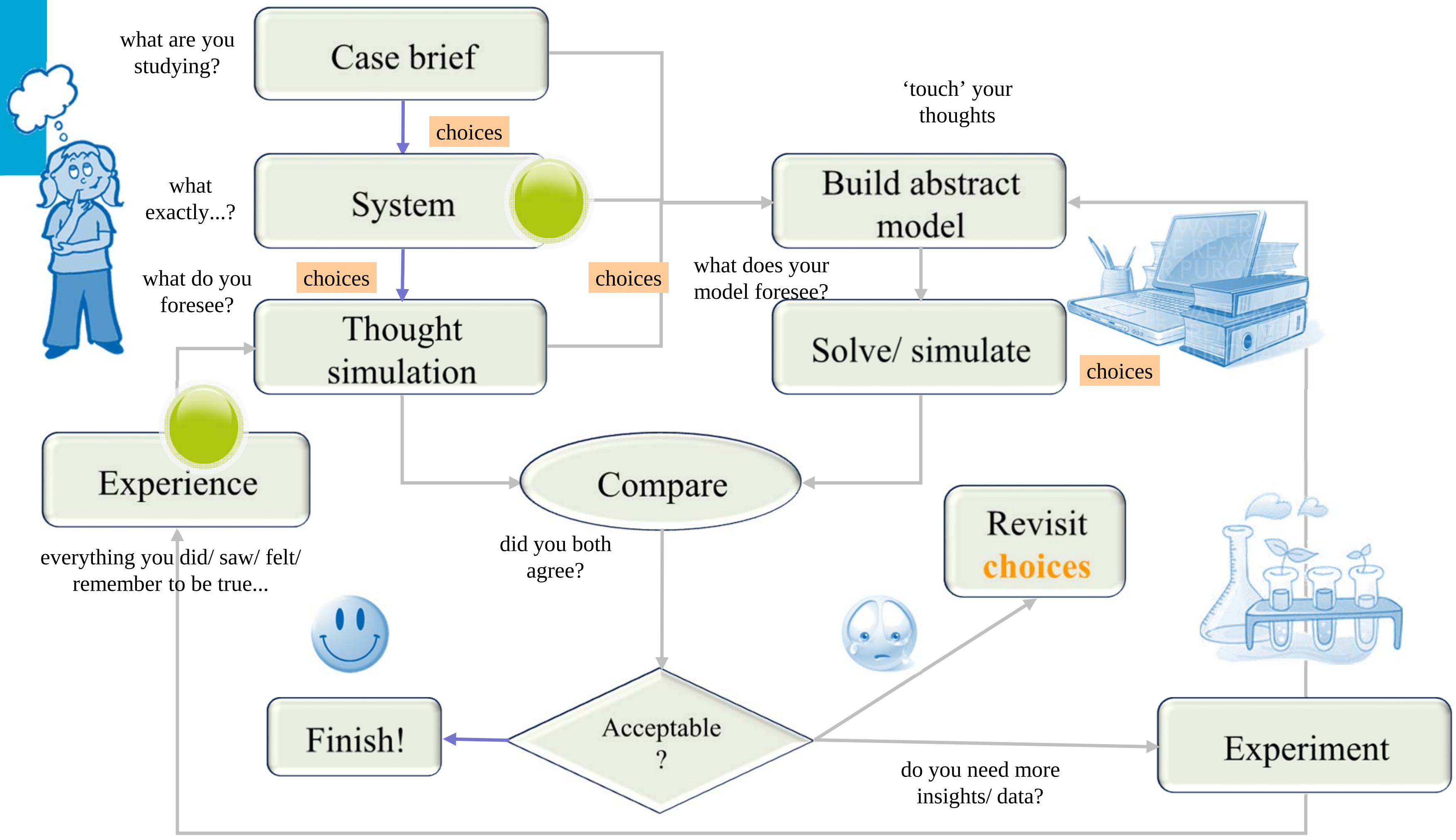
Absorb heat

Release heat

Different cups

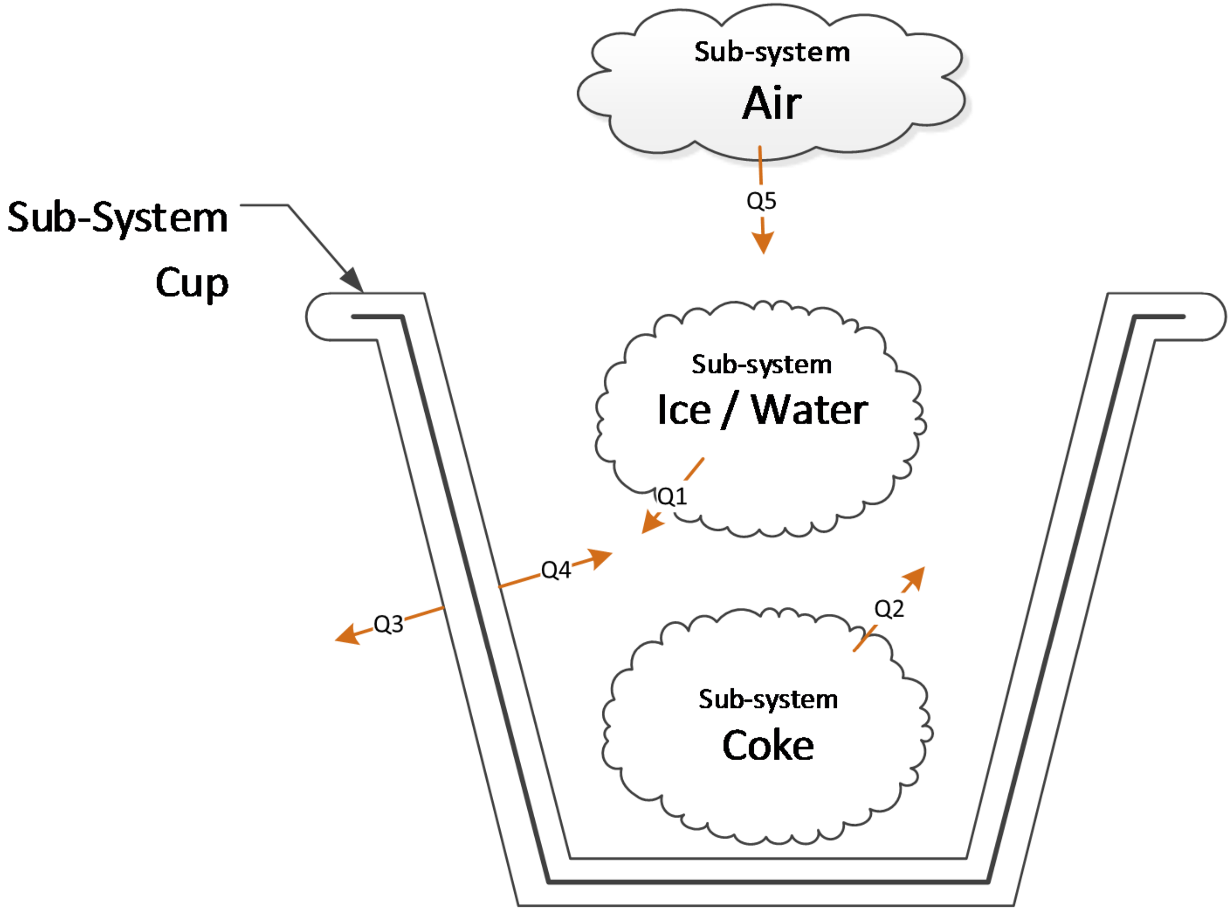
Courtesy of http://www.eriding.net/media/states.shtml, http://www.coffeee.net/Kettles-And-Toasters/Kettles/Red-Kettles, http://cater4you.blogspot.com/2009/01/new-products-paper-coke-cups.html

The next step

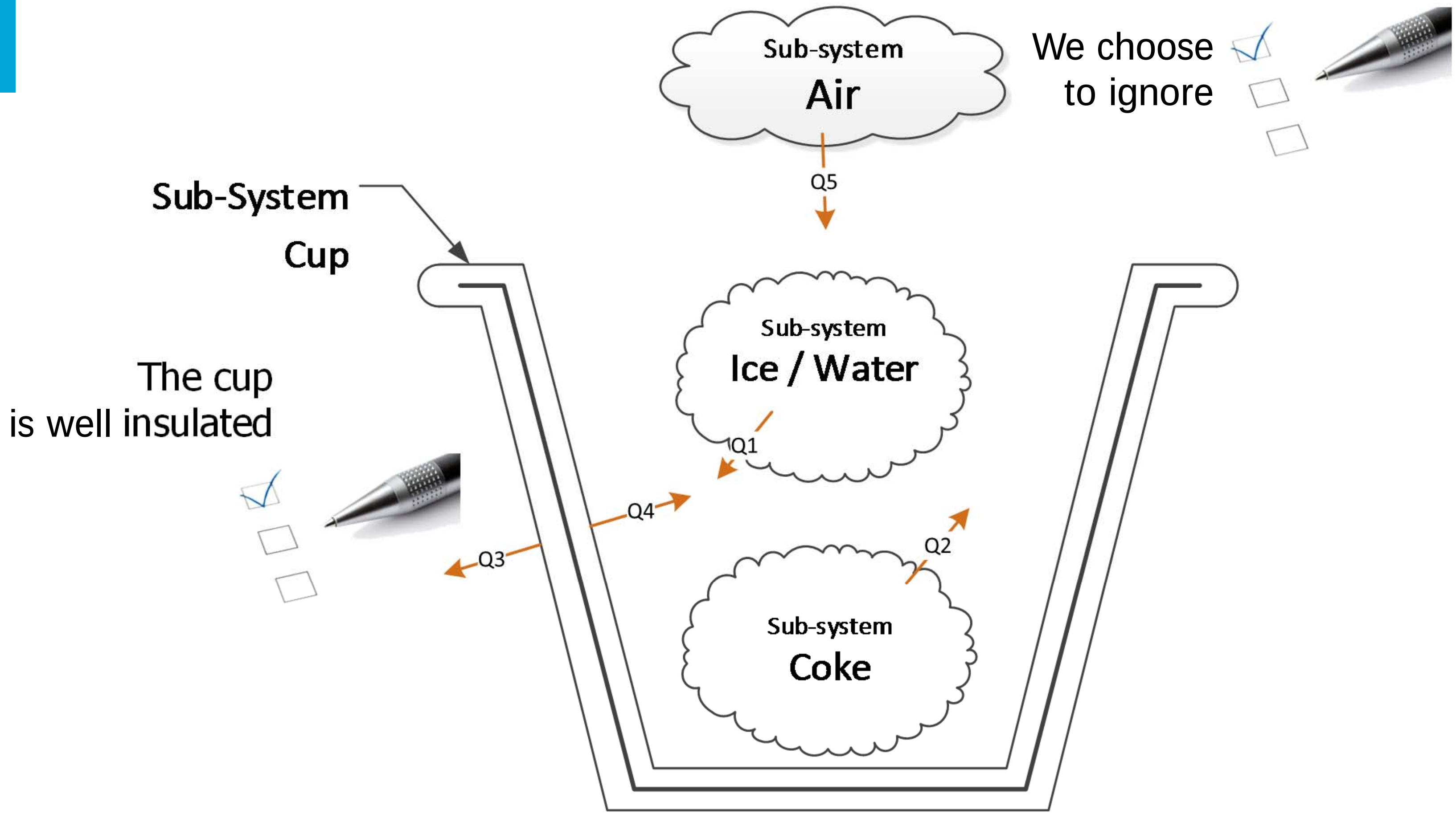


Courtesy of centech.com.pl and http://www.clipsahoy.com/webgraphics4/as5814.htm

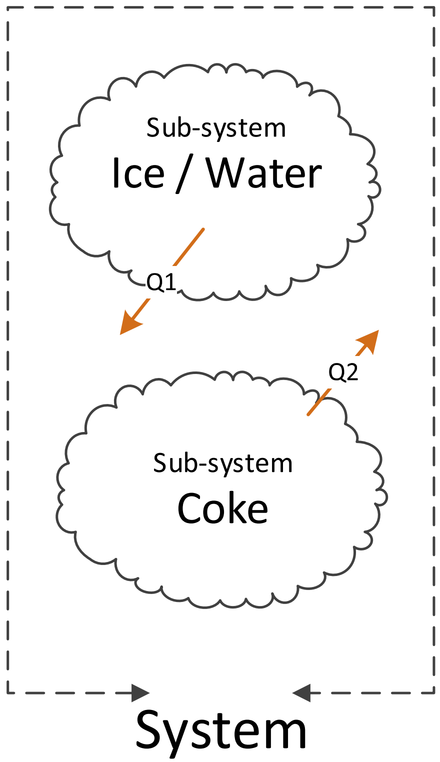
Heat exchange in system(s)



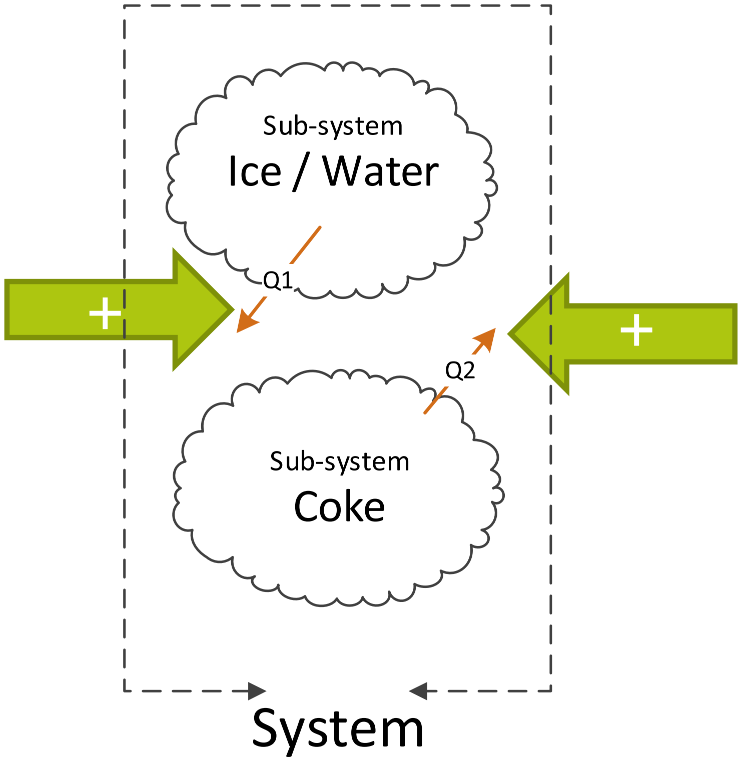
Heat exchange in system(s) -Choices



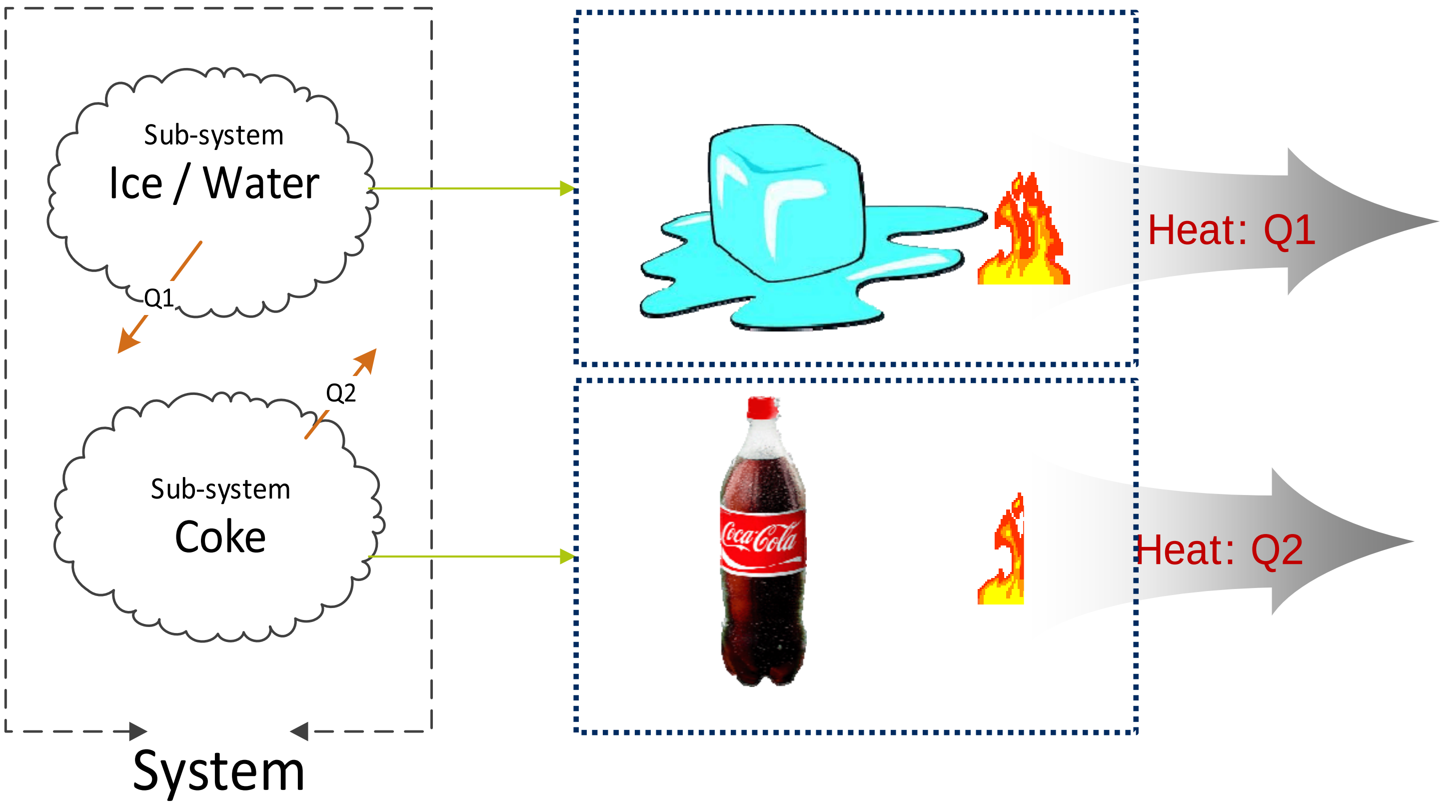
Heat exchange in system(s) -Choices



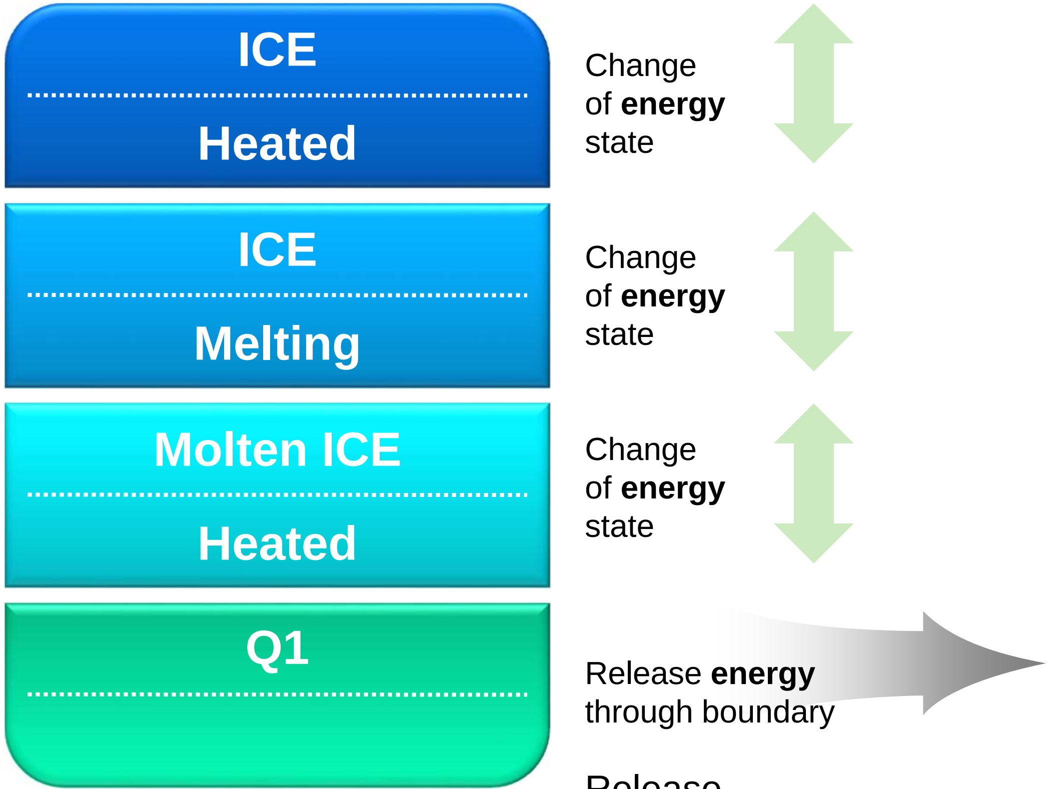
Heat exchange in system(s) -The conventions

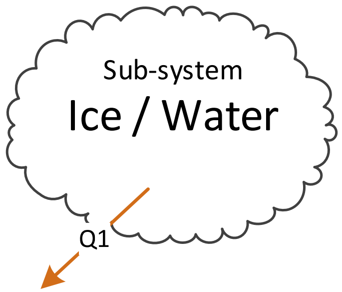


Thought simulation
System



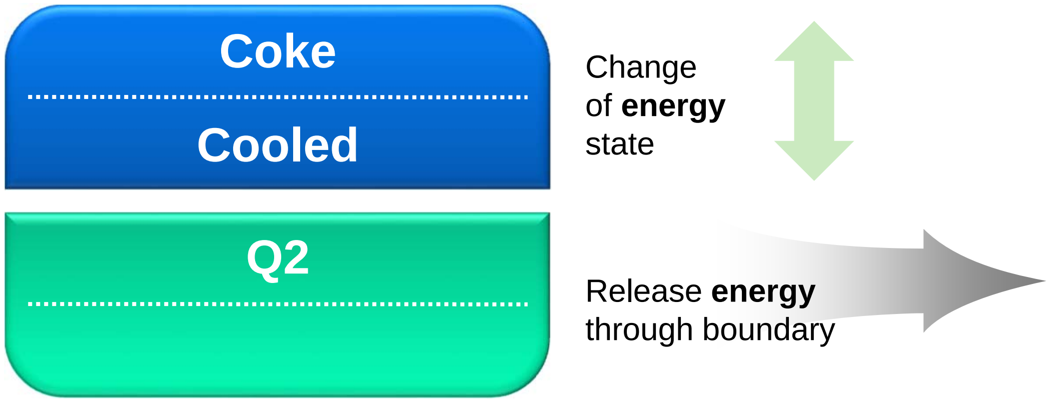
Thought simulation
-Sub-system Ice

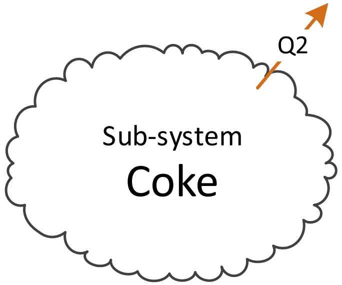




Release
= give **energy** 'back to circulation'

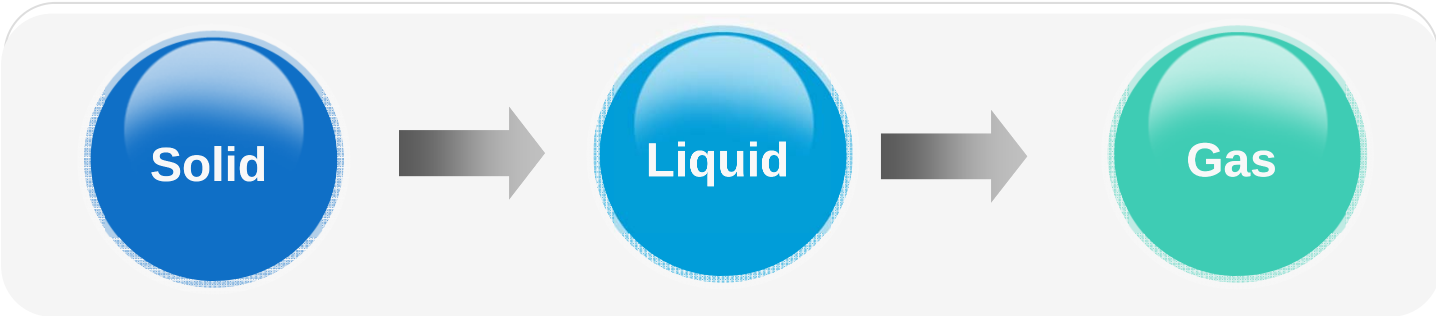
Thought simulation -Coke





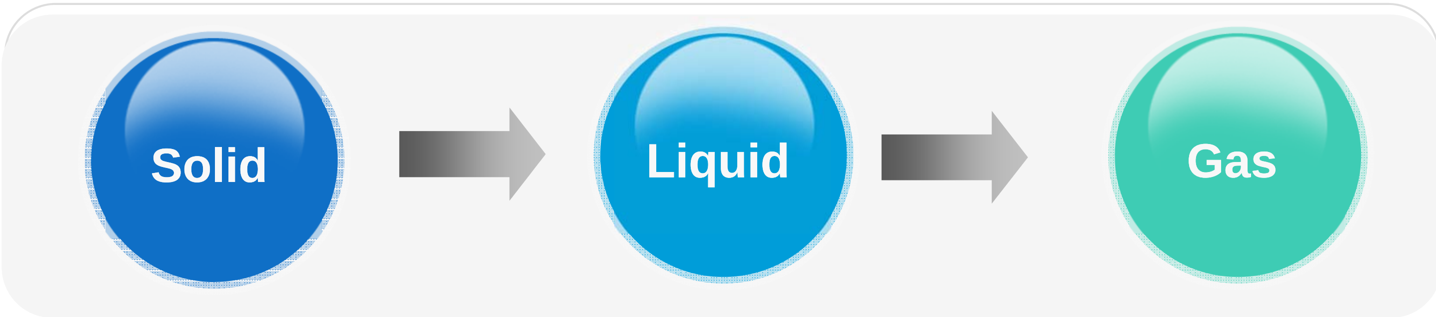
Physics behind Modelling

States of matter (Simplified)



Latent heat

DEF: The amount of energy released or absorbed by a chemical substance during a change of state that occurs without changing its temperature



Courtesy of http://www.gabrielweinberg.com/blog/2010/11/code-icebergs.html

Latent heat – Quantitative point of view

Energy released or absorbed during the change of phase.
Unit(J)

Specific latent heat
Unit(J/kg)

$$Q = mL$$

Mass
Unit(kg)

Example

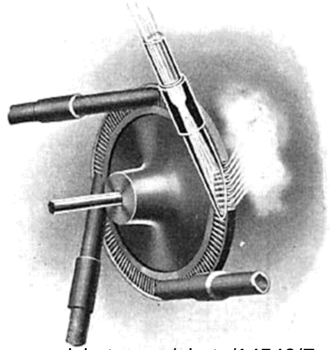
Specific Latent heat of water fusion is

334000 J/kg

Specific heat capacity

DEF: Specific heat capacity is the measure of heat (i.e. thermal energy) required to increase the temperature of a unit quantity of a substance by one unit

Temperature Temperature Temperature



Courtesy of http://www.pxlshots.com/photo/14540/Frozen-thermometer.html

Heat capacity– Quantitative point of view

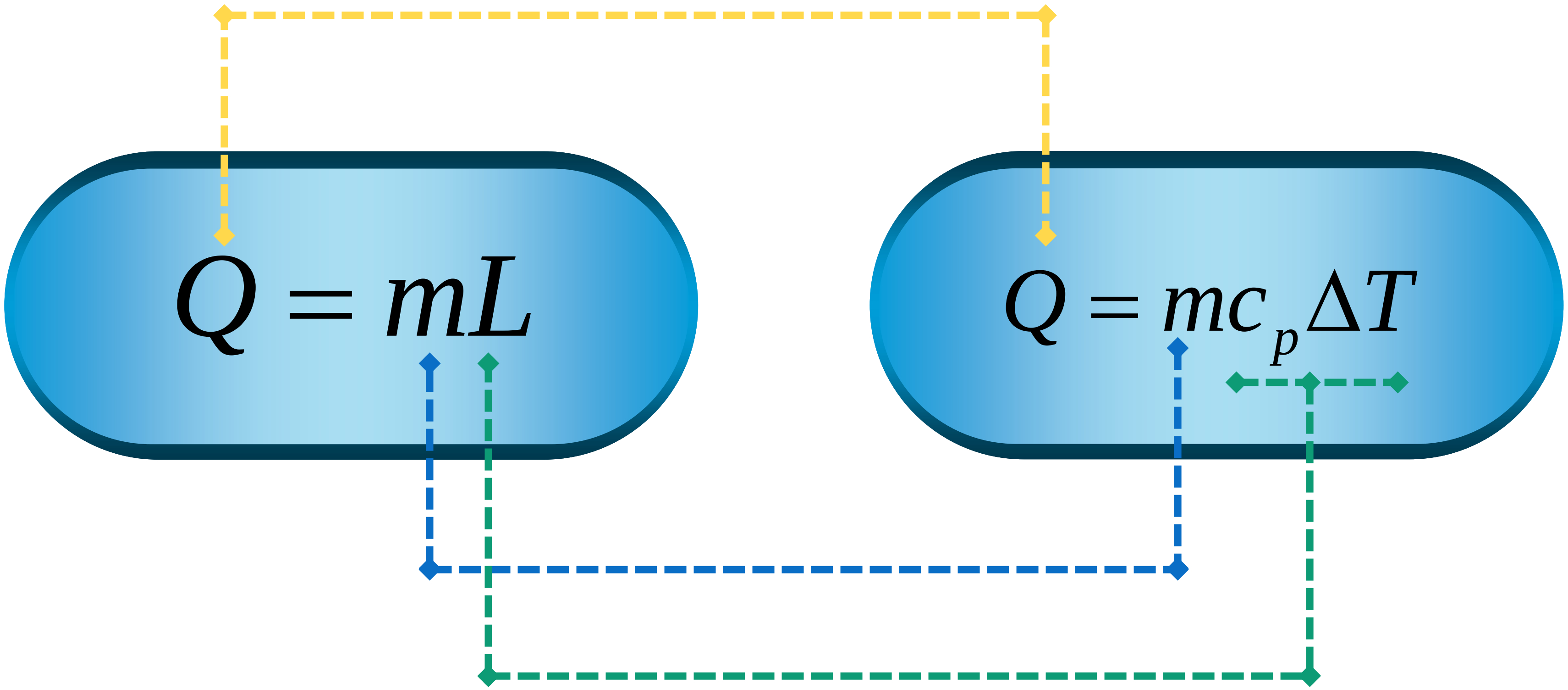
$$Q = mc_p\Delta T$$

- Q: Energy released or absorbed, Unit(J)
- m: Mass, Unit(kg)
- c_p: Specific heat capacity, Unit: J/(kg*k)
- ΔT: Temperature differences, Kelvin(K)

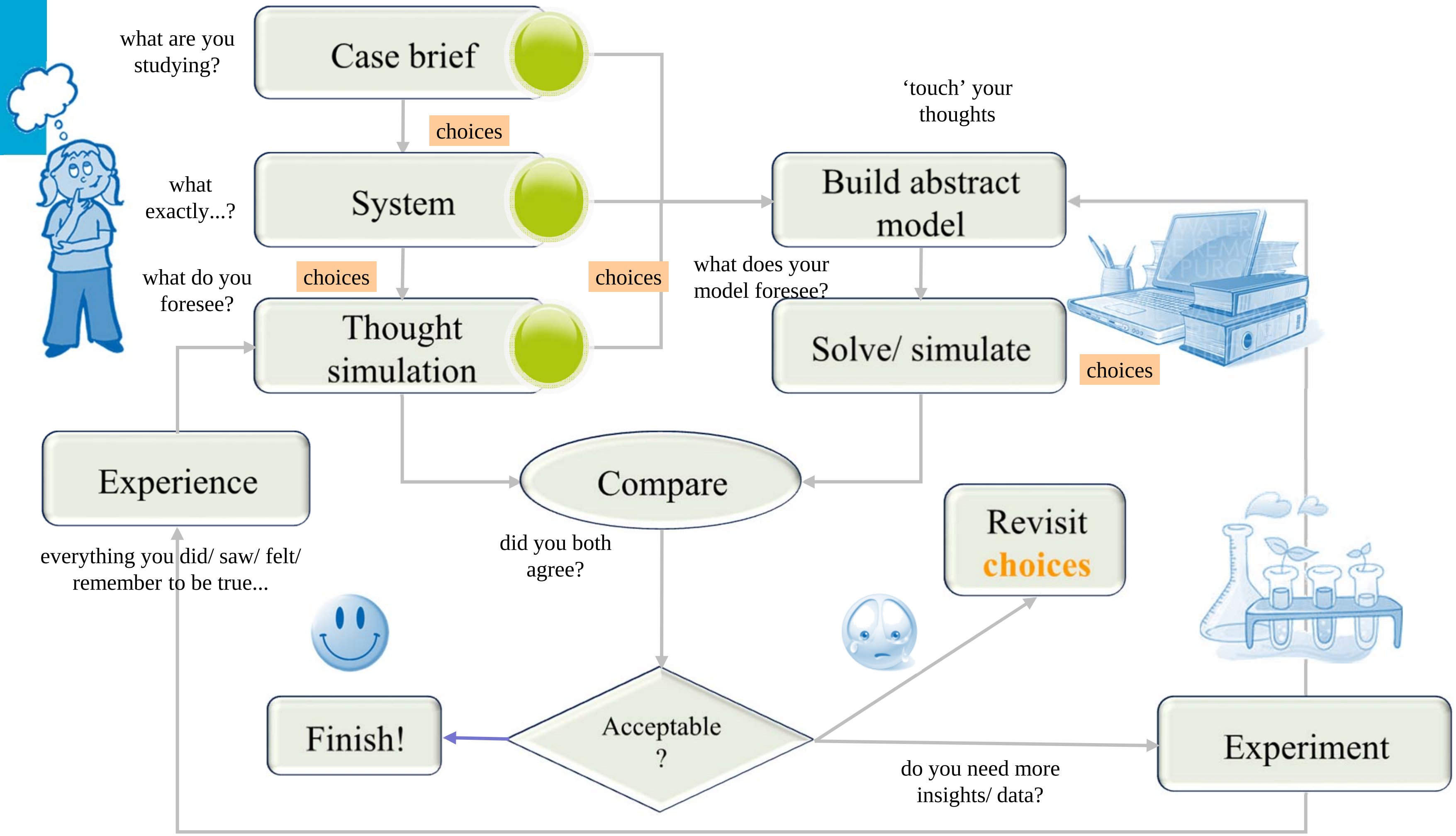
Example

Ice	$c_p = 2050$
Water	$c_p = 4180$
Steam	$c_p = 2080$

What is the similarity / difference between the models



The next step



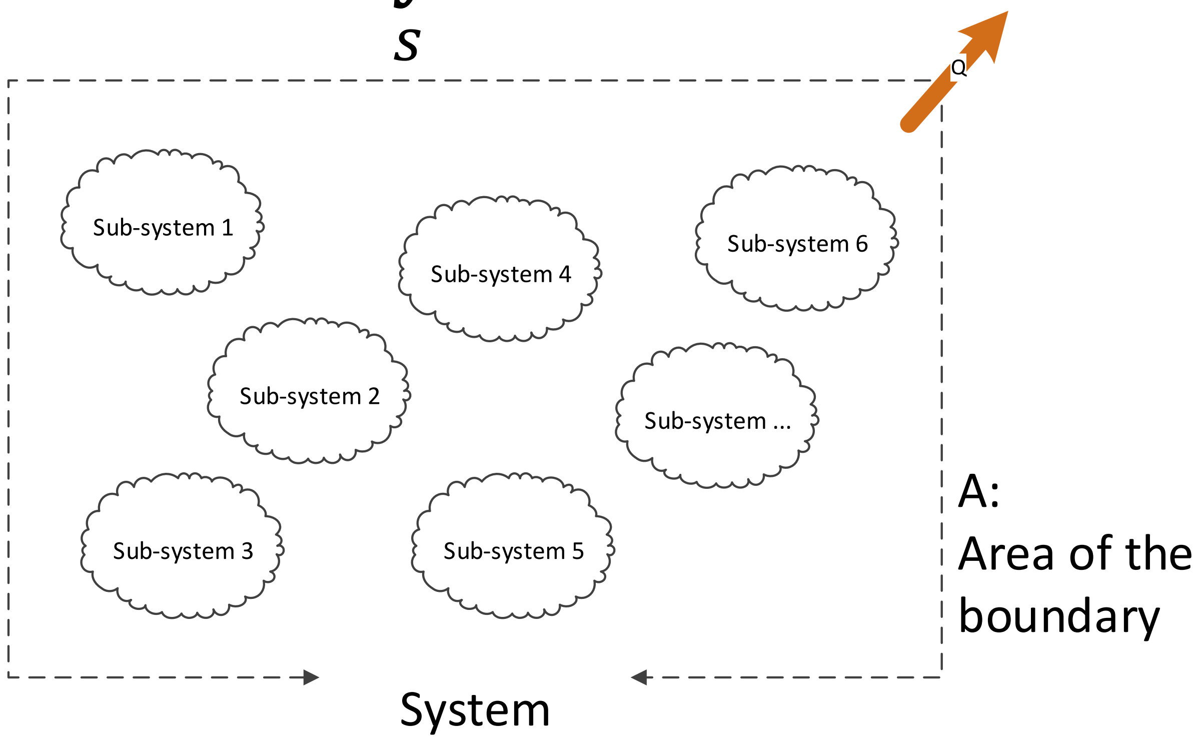
Courtesy of centech.com.pl and http://www.clipsahoy.com/webgraphics4/as5814.htm

Conservation of energy

Any other energy change

Heat exchanged through the boundary

$$\Delta E + \int_{S} Q dA = 0$$



keep your eyes on the ball™

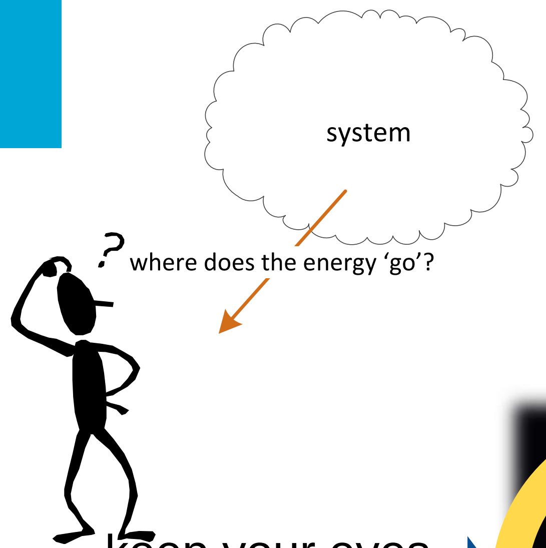
How to master basketball

keep your eyes
on the ball

no, do
not look
here!

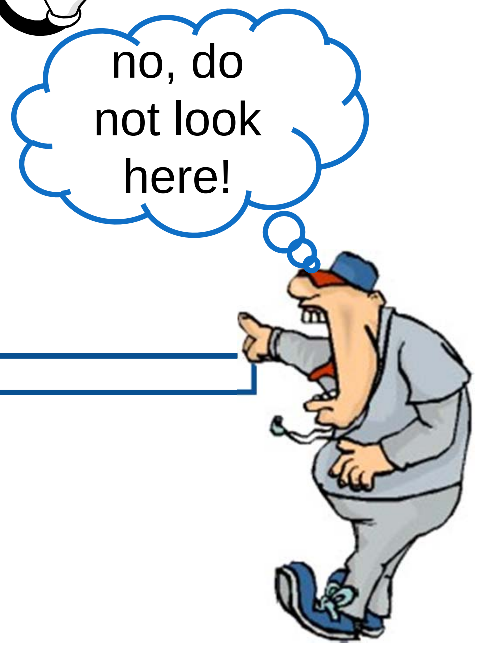
Courtesy of http://www.lpl.arizona.edu/~umpire/softball/aiasoftball00.html

How to master ~~basketball~~ modelling



$$\Delta E + \int_S Q dA = 0$$

keep your eyes
on the ball

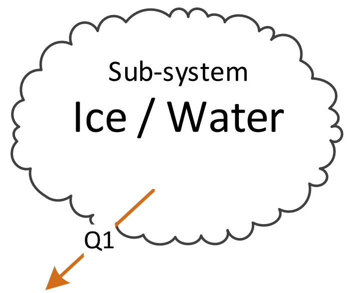


Courtesy of http://www.lpl.arizona.edu/~umpire/softball/aiasoftball00.html

Sub-system Ice:

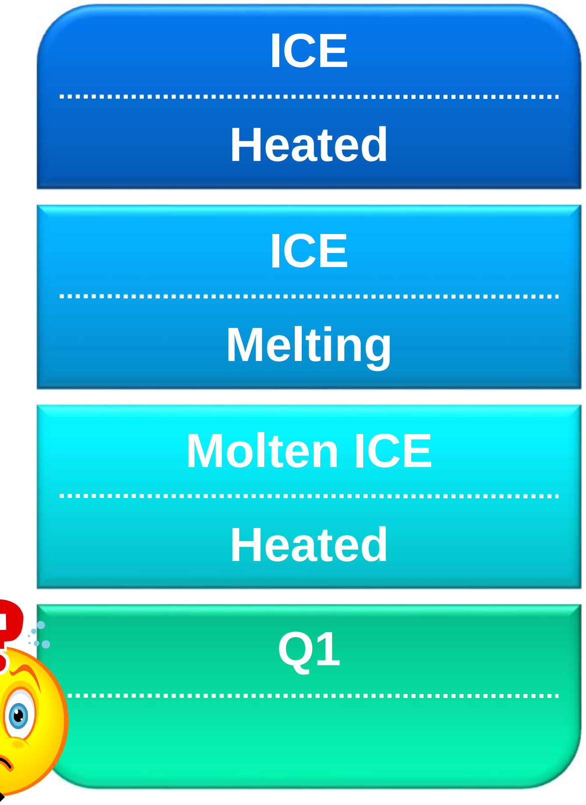
$$\Delta E + \int_S Q dA = 0$$

1. Ice temperature increase
2. Ice melting
3. Water (Molten ice) temperature increase
4. Heat released through the boundary



Courtesy of http://www.eriding.net/media/states.shtml, http://www.coffeee.net/Kettles-And-Toasters/Kettles/Red-Kettles, http://cater4you.blogspot.com/2009/01/new-products-paper-coke-cups.html

Modelling
-Sub-system Ice



Change of energy state

$$m_{ice} c_{p_{ice}} (0 - T_{ice\_initial})$$

Change of energy state

$$m_{ice} L$$

Change of energy state

$$m_{ice} c_{p_{water}} (T_{final} - 0)$$

Release energy through boundary

$$Q_1$$

Energy taken out of circulation

$$Q + m \; c \; (0 - T \;) + m \; L + m \; c \; (T \; - 0) = 0$$

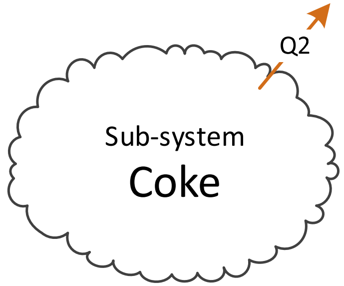
Sub-system Coke:

1

$$\Delta E + \int_{S} Q\,dA = 0$$

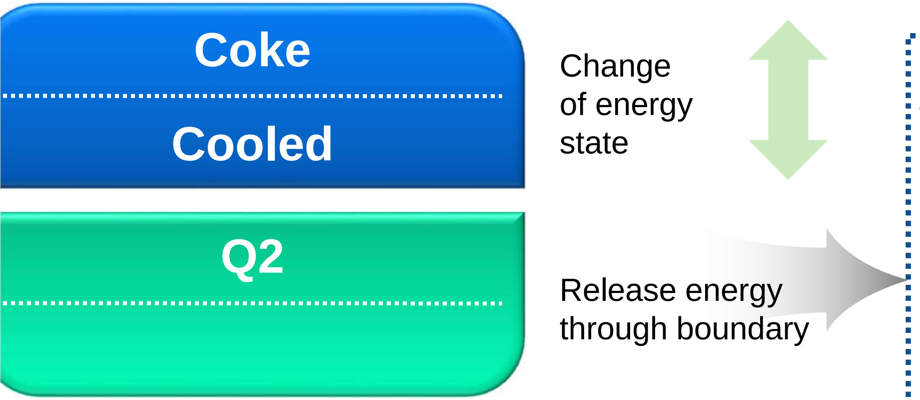
Heat released through boundary

2



Modelling
-Sub-system Coke

Energy taken out of circulation



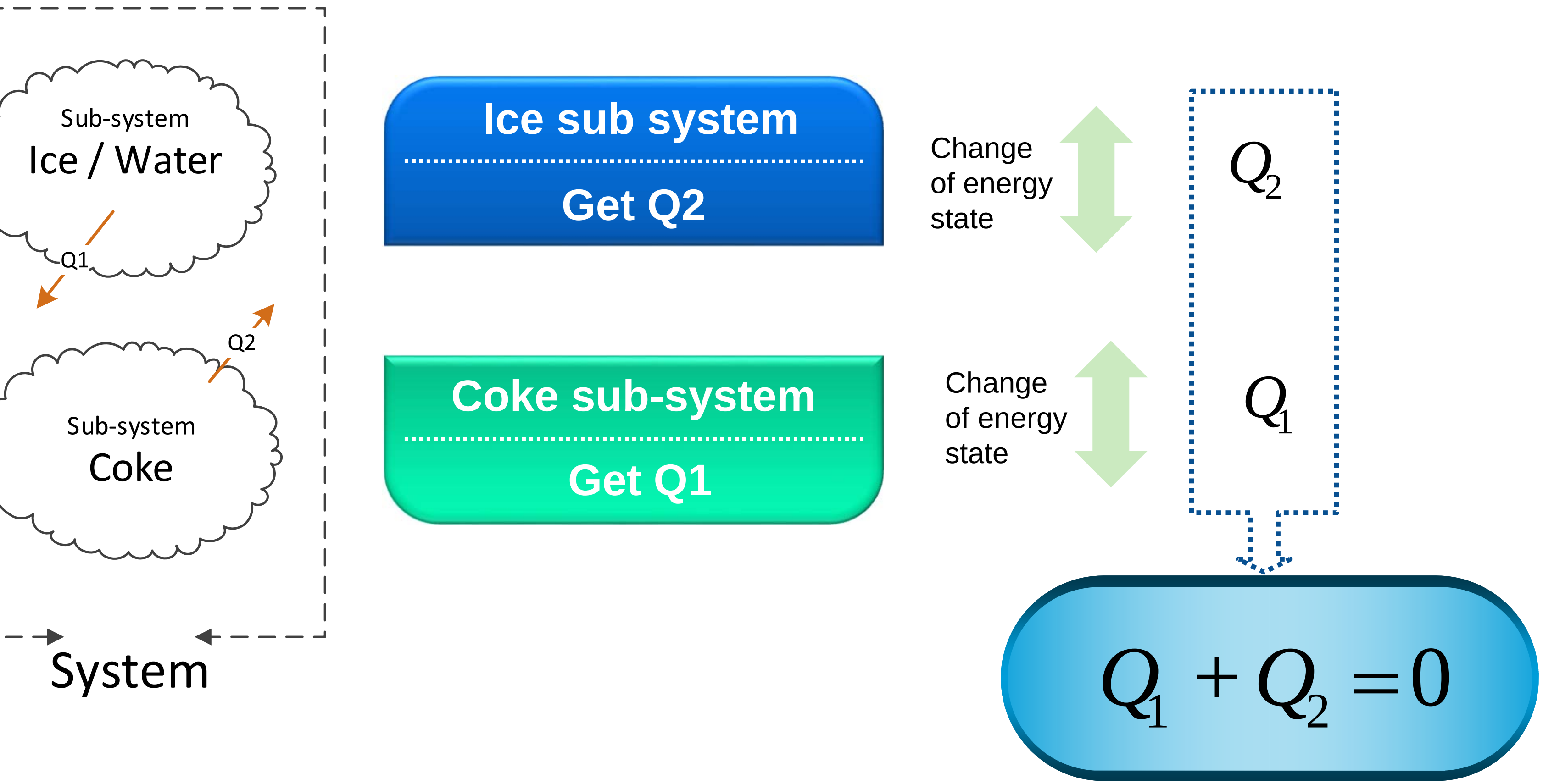
$$m_{coke} c_{p_{coke}} (T_{final} - T_{coke\_initial})$$

$$Q_2$$

$$Q_2 + m_{coke} c_{p_{coke}} (T_{final} - T_{coke\_initial}) = 0$$

Building a mathematical model

System



Modelling:

In the mathematical form

Ice

$$\ldots m_{ice} c_{p_{ice}} (0 - T_{ice\_initial}) + m_{ice} L_{ice} + m_{ice} c_{p_{water}} (T_{final} - 0) = 0$$

$$Q_1 + Q_2 = 0$$

Coke

$$\ldots m_{coke} c_{p_{coke}} (T_{final} - T_{coke\_initial}) = 0$$

$$\ldots (0 - T_{ice\_initial}) + m_{ice} L_{ice} + m_{ice} c_{p_{water}} (T_{final} - 0)] + m_{coke} c_{p_{coke}} (T_{final} - T_{coke\_initial}) = 0$$

Modelling

$$\ldots_{ice}(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0$$

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

Modelling

$$\ldots_{ice}(0 - T_{ice\_initial}) + m_{ice}L_{ice} + m_{ice}c_{p_{water}}(T_{final} - 0)] + m_{coke}c_{p_{coke}}(T_{final} - T_{coke\_initial}) = 0$$

$$n_{cubes}\rho_{ice}V_{cube}$$

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

Modelling

$$ {}_{ice}(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0 $$

$$ c_{p_{ice}} = 2050 $$

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

Modelling

$$\ldots_{ice}(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0$$

$$L_{ice}=334000$$

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

Modelling

$$\ldots_{ice}(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0$$

$$c_{p_{water}}=4181$$

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

Modelling

$$\ldots_{ice}(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0$$

$$\ldots = \rho_{water}V_{coke}$$

Europe	• 330ml
American	• 16oz. ~ 453.6g

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

Modelling

$$\ldots_{ice}(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0$$

$\ldots_{al}$

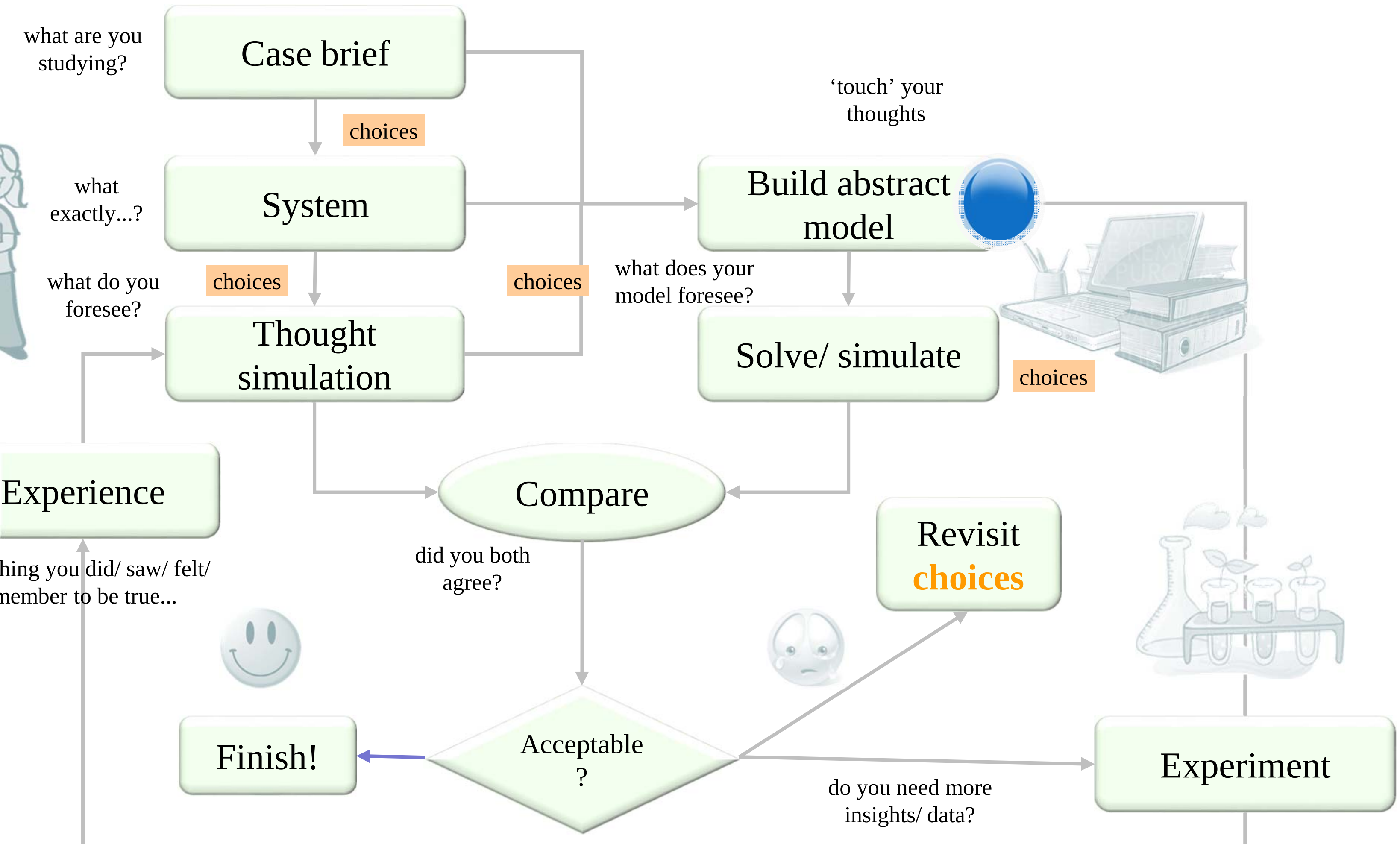
n_{cubes}

5?

We choose

- The initial temperature of ice is -15 °C
- The specific heat capacity of ice is 2050 J/kg*k
- The density of ice is 916 kg/m^3
- The length of ice cube is 1.6 cm, ice melts at 0°C
- The specific latent heat of ice changing to water is 334 000J/kg
- The specific heat capacity of water is 4181 J/kg*k
- The density and the heat capacity of coke are same as water
- The initial temperature of coke is the same as tap water, which is 10°C

The next step



Solving – we try 5 ice cubes

Modeling

> $restart:$

> $equ := m_{ice} \cdot C_{ice} \cdot T_{d1} + m_{ice} \cdot L + m_{water} \cdot C_{water} \cdot T_{d2} + m_{coke} \cdot C_{coke} \cdot T_{d3} = 0:$

Solving

For europe

> $\rho_{coke} := 1000 : \rho_{ice} := 916 : V_{ice} := 0.016 \cdot 0.016 \cdot 0.016 : m_{coke} := \rho_{coke} \cdot 330 \cdot 1e{-}6 : m_{ice} := n \cdot \rho_{ice} \cdot V_{ice}; m_{water} := m_{ice}:$

$$m_{ice} := 0.003751936\, n \qquad (3.1)$$

> $C_{water} := 4181.3 : C_{coke} := C_{water} : L := 334000 : C_{ice} := 2050 :$

> $T_{d1} := 15 : T_{d2} := T_{final} - 0 : T_{d3} := T_{final} - 10 :$

> $n := 5 :$

> $solve(equ, T_{final})$

$$4.769831516 \qquad (3.2)$$

For American

> $m_{cokeA} := \rho_{coke} \cdot 453.6 \cdot 1e{-}6 : equ2 := m_{ice} \cdot C_{ice} \cdot T_{d1} + m_{ice} \cdot L + m_{water} \cdot C_{water} \cdot T_{d2} + m_{cokeA} \cdot C_{coke} \cdot T_{d3} = 0 :$

> $solve(equ2, T_{final})$

$$6.138383601 \qquad (3.3)$$

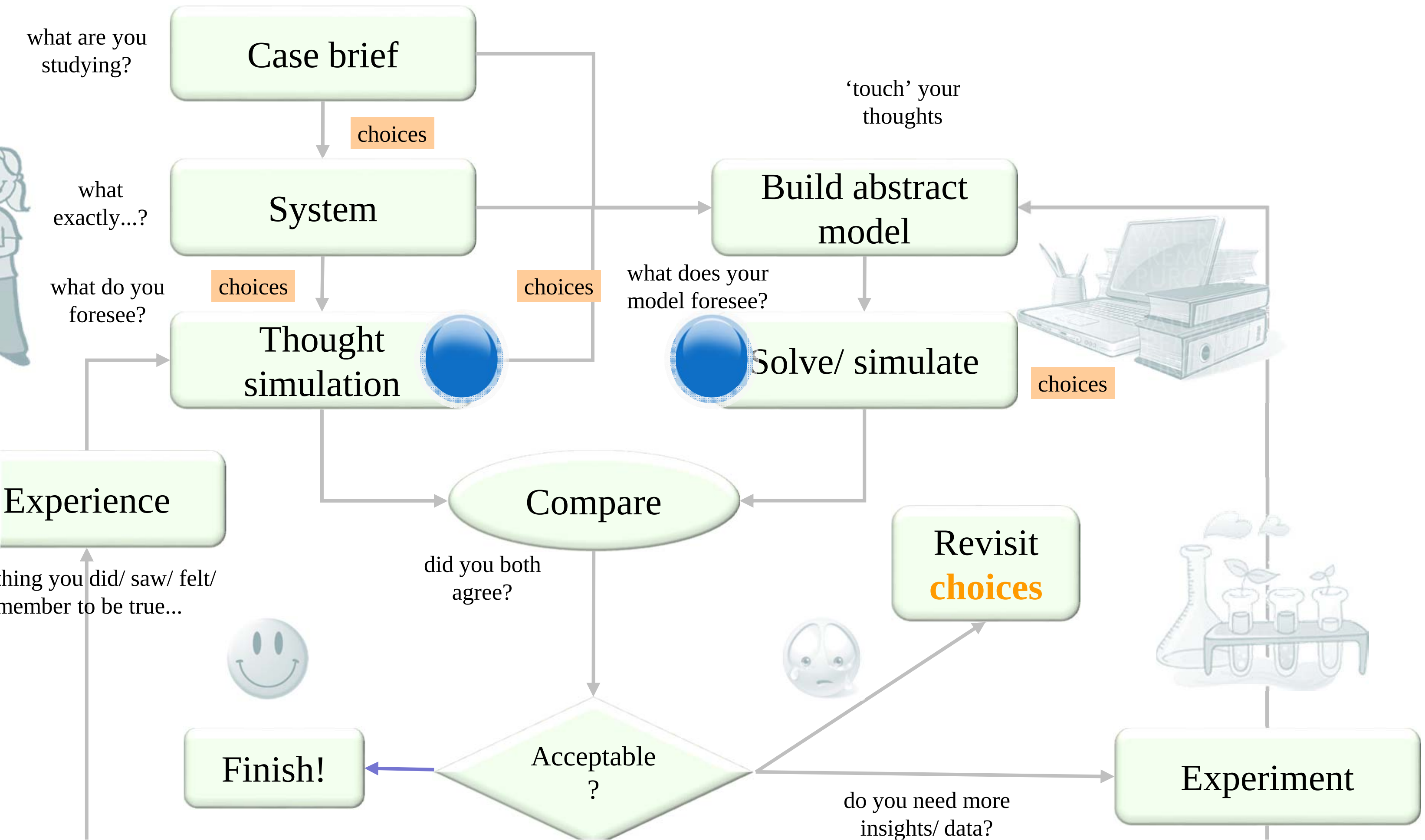
European size

5 cubes → 4.77 °C

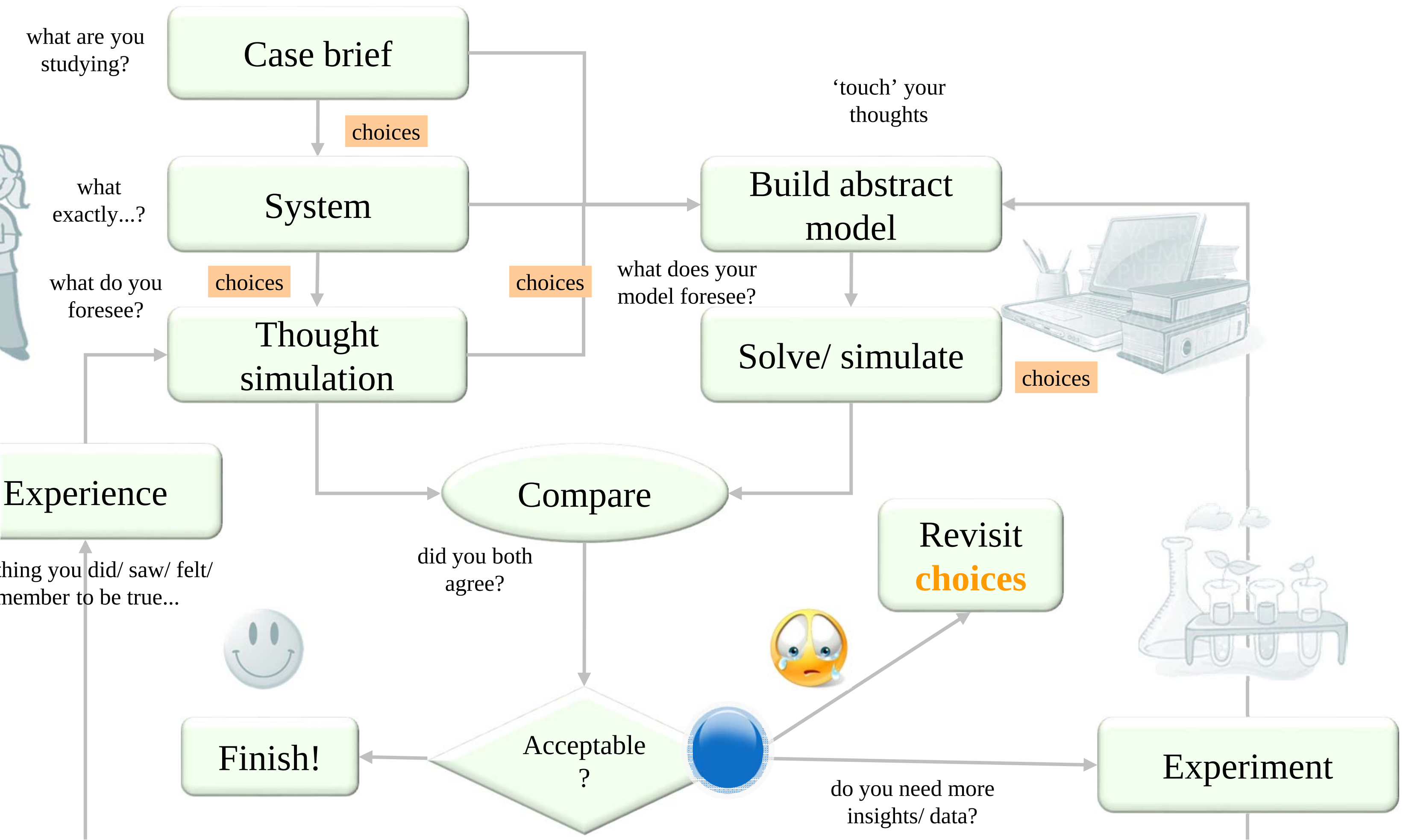
American size

5 cubes → 6.14 °C

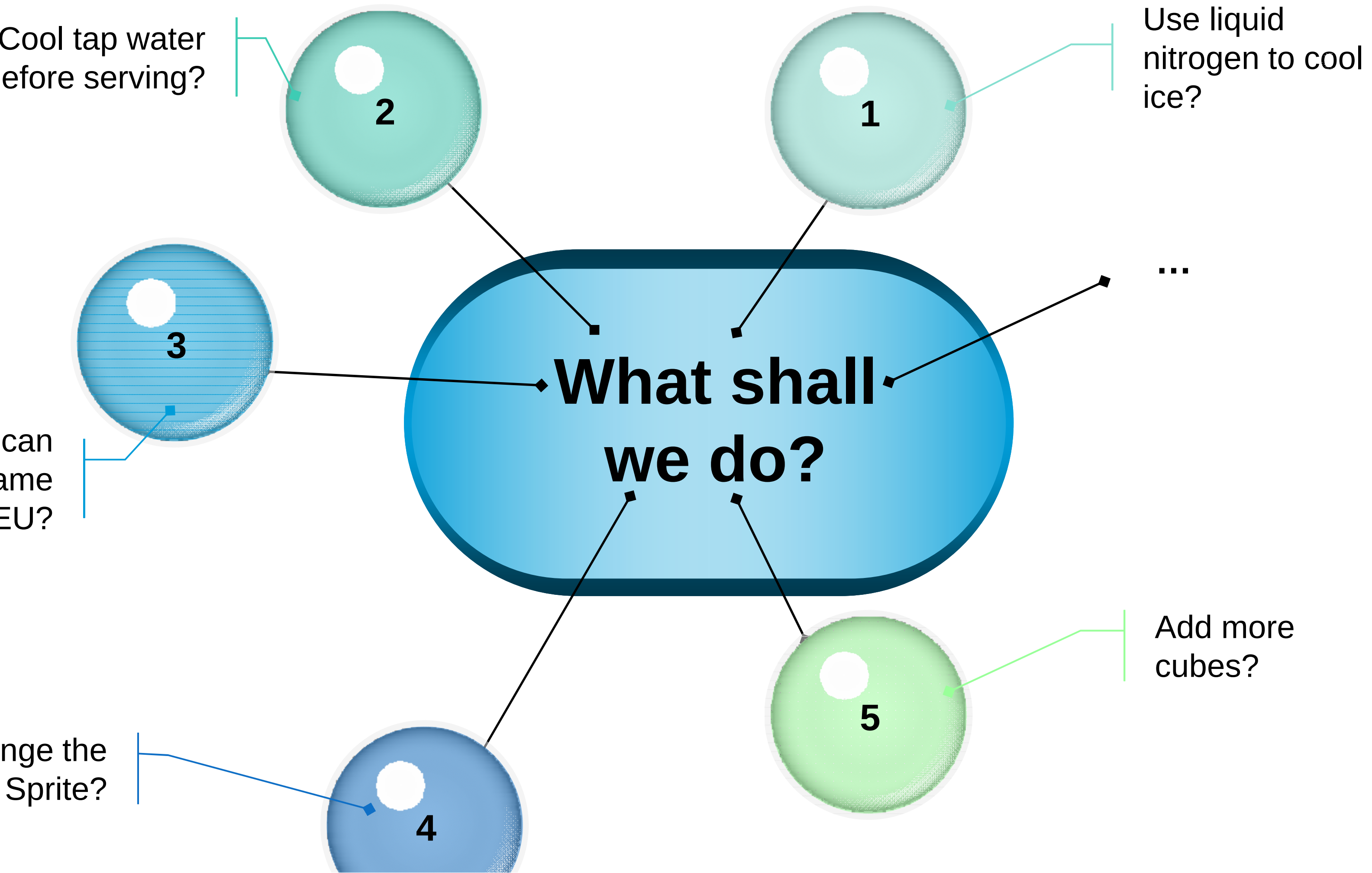
The next step



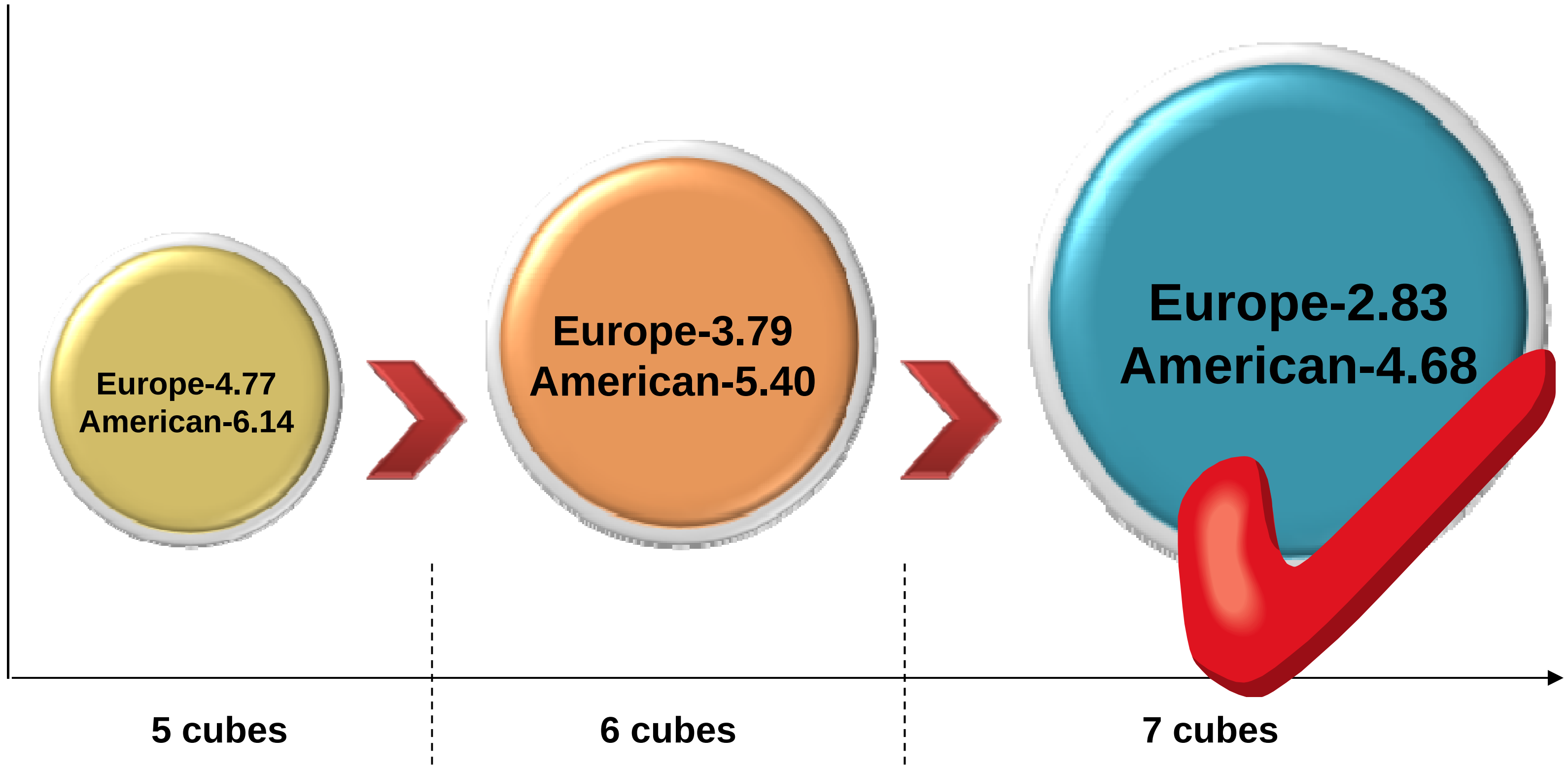
Design Brief



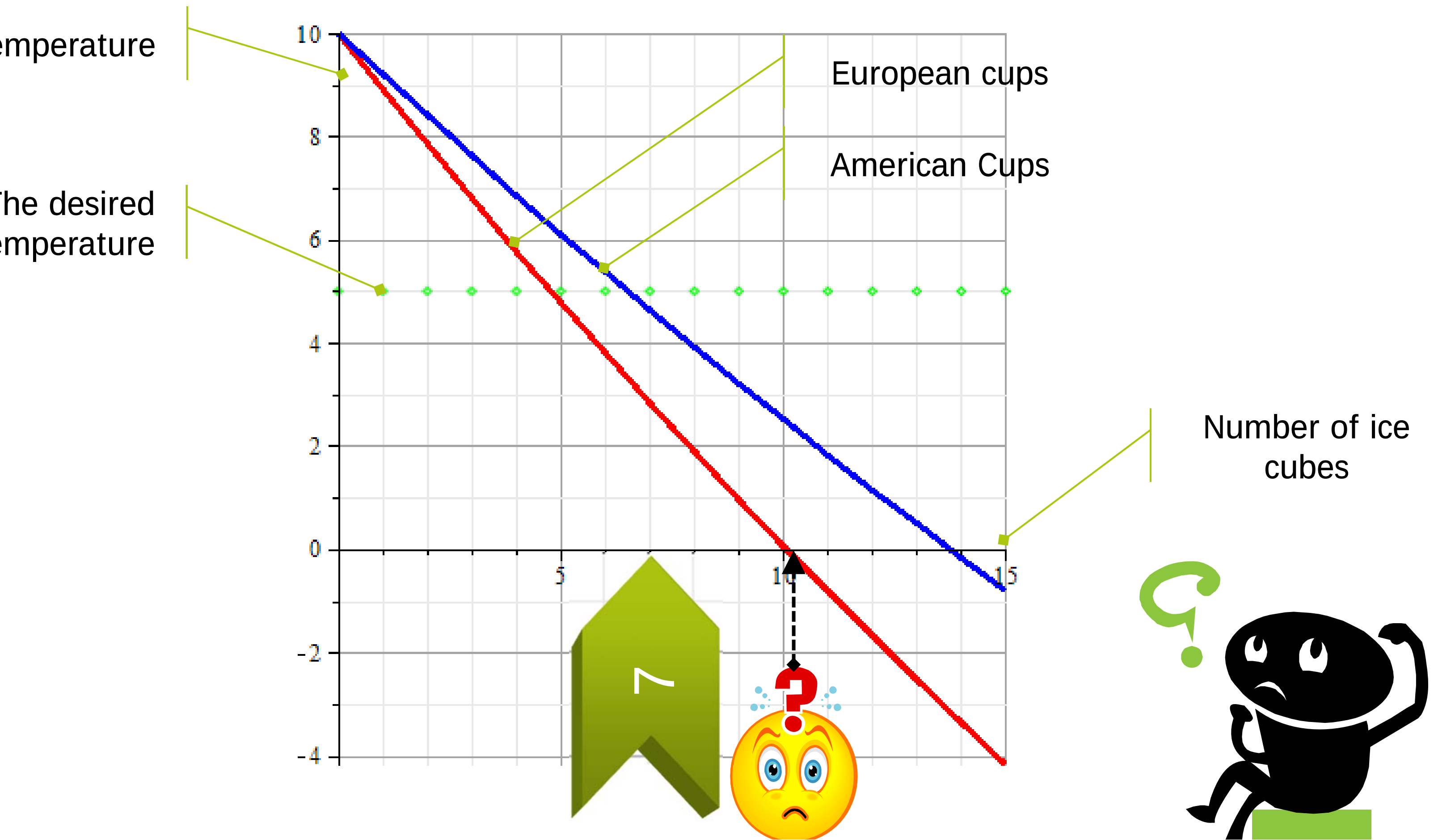
No! American coke is too hot!



Increase the number of ice cubes per touch of the button



The relation between ice cubes & coke temperature according to our model

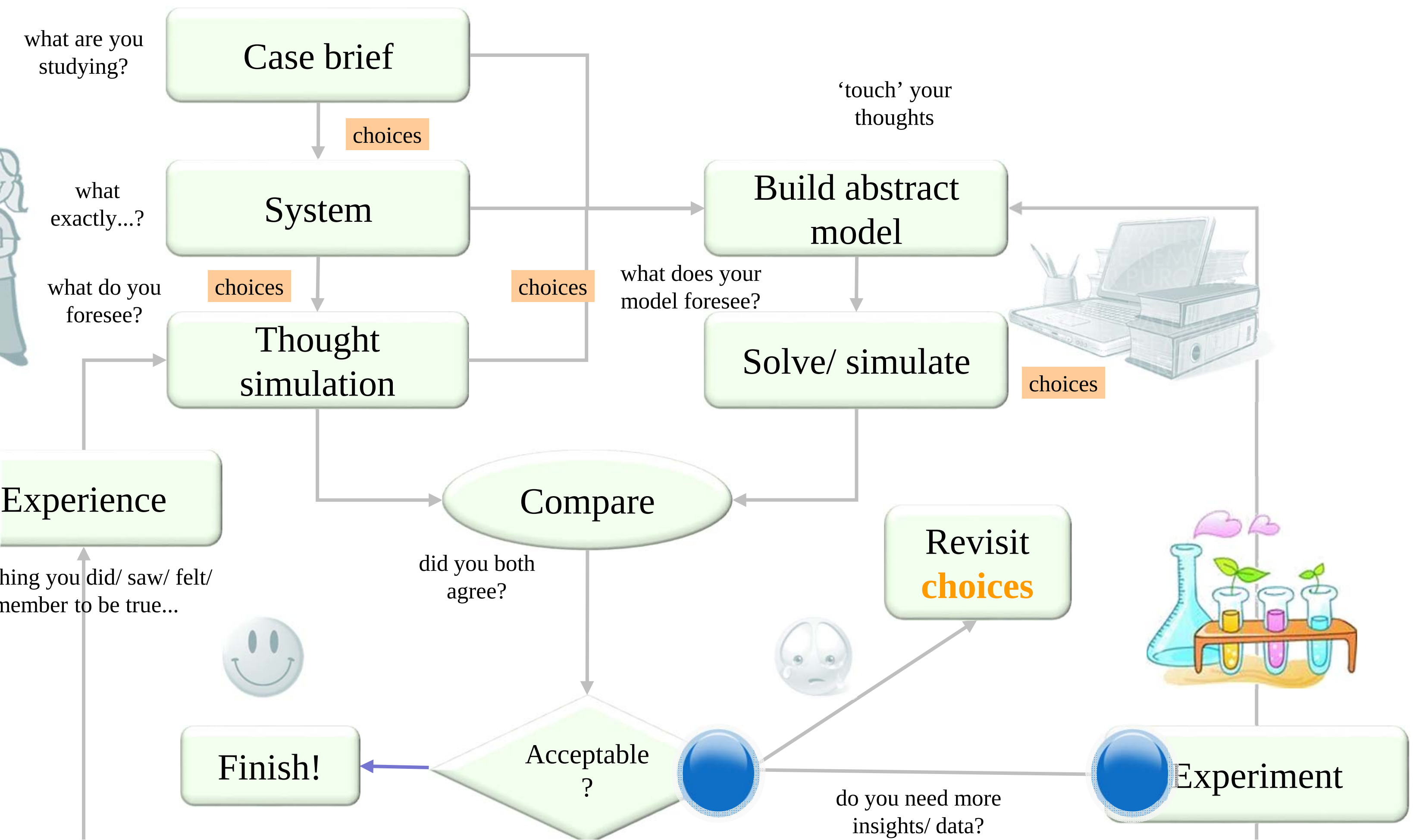


Revisit our models

The ice cubes melt

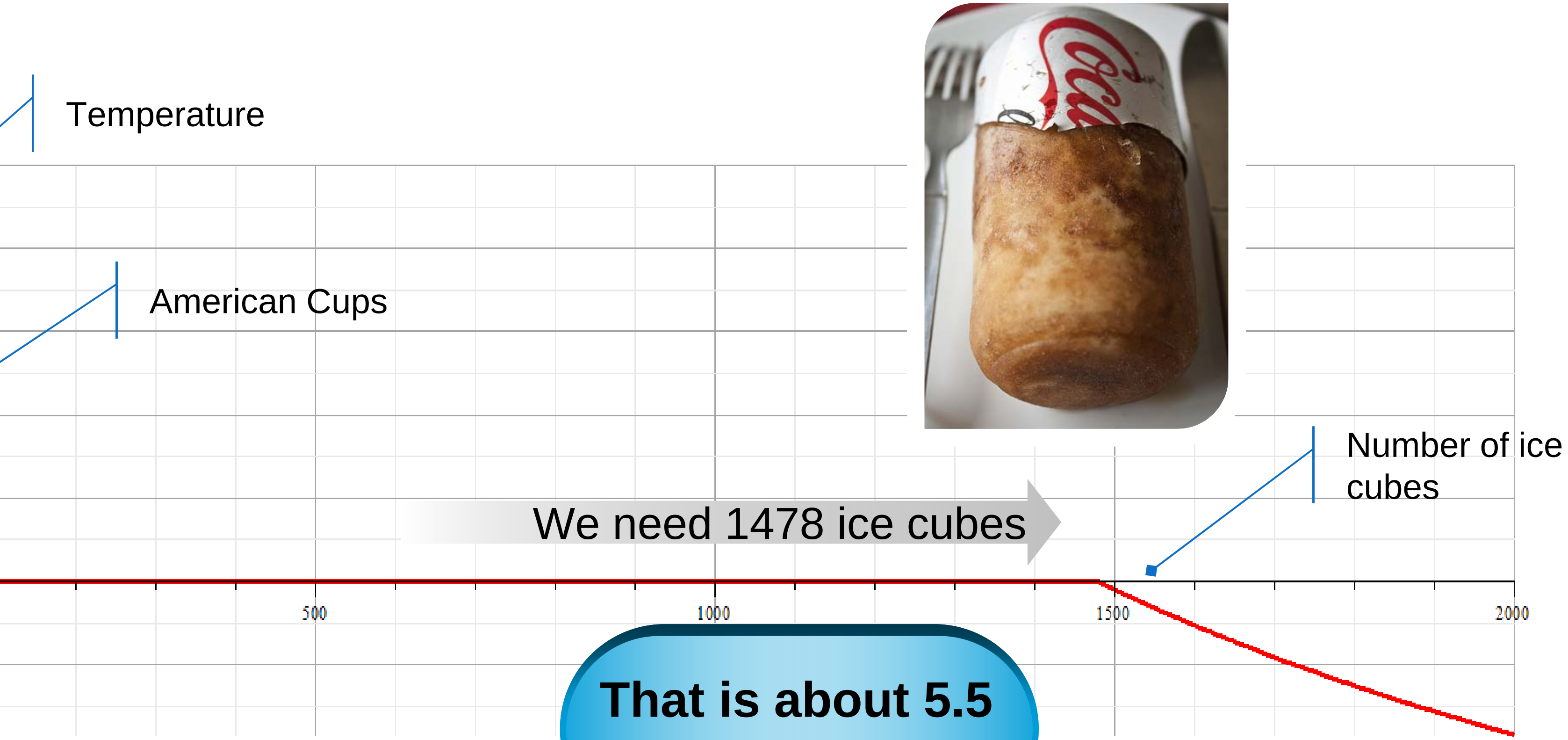
$$(0-T_{ice\_initial})+m_{ice}L_{ice}+m_{ice}c_{p_{water}}(T_{final}-0)]+m_{coke}c_{p_{coke}}(T_{final}-T_{coke\_initial})=0$$

Experiments?



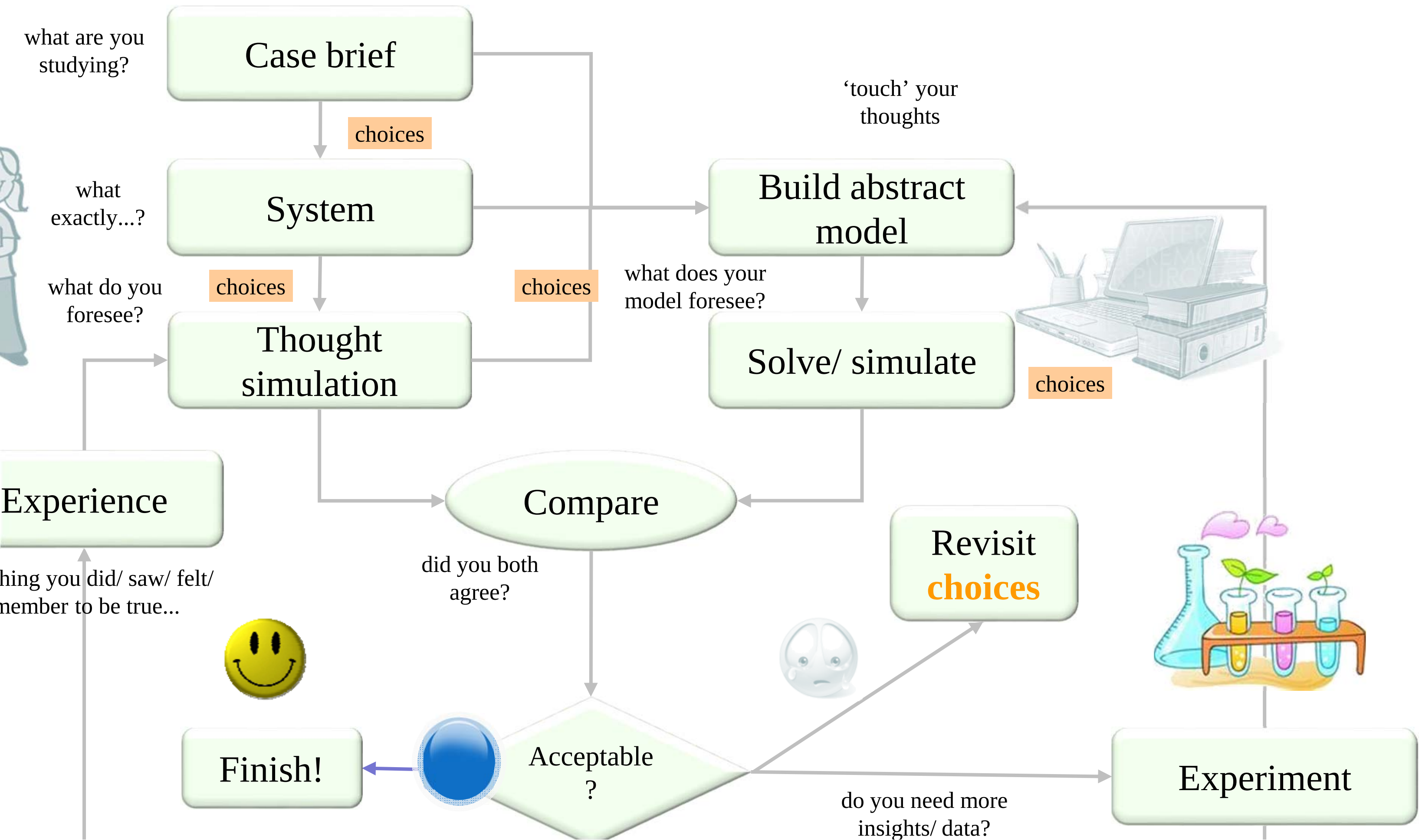
Experiment

If we want to create real “ice” coke

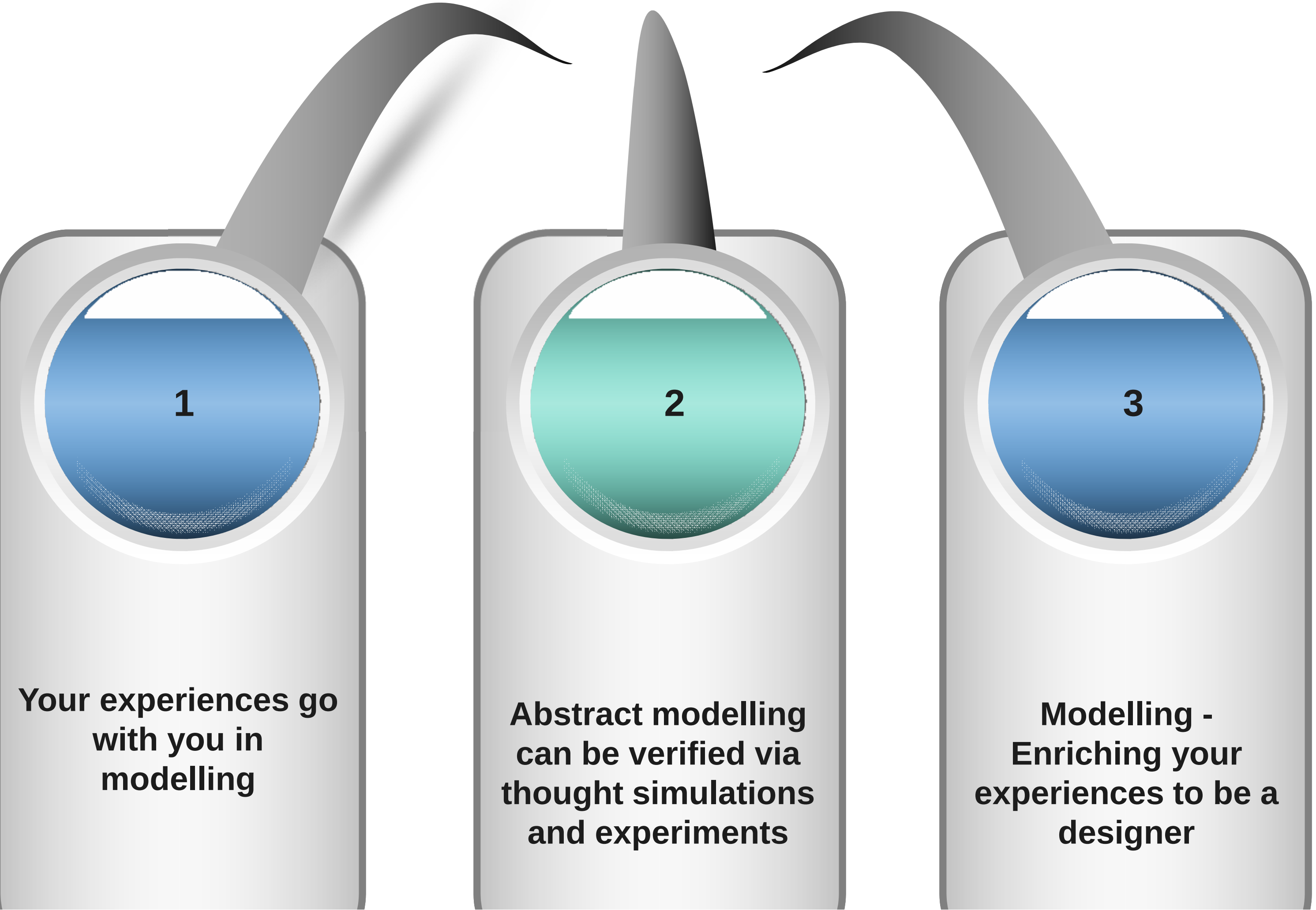


That is about 5.5 litres of water

Or when we are satisfied !



What did we learn?



keep your eyes on the ball™

We think…

curiosity
is fun

knowledge
is power

science
is easy

computers
are tools

experience
can be harnessed

hard work
is the way to success

modelling is the way to wisdom

Succes!