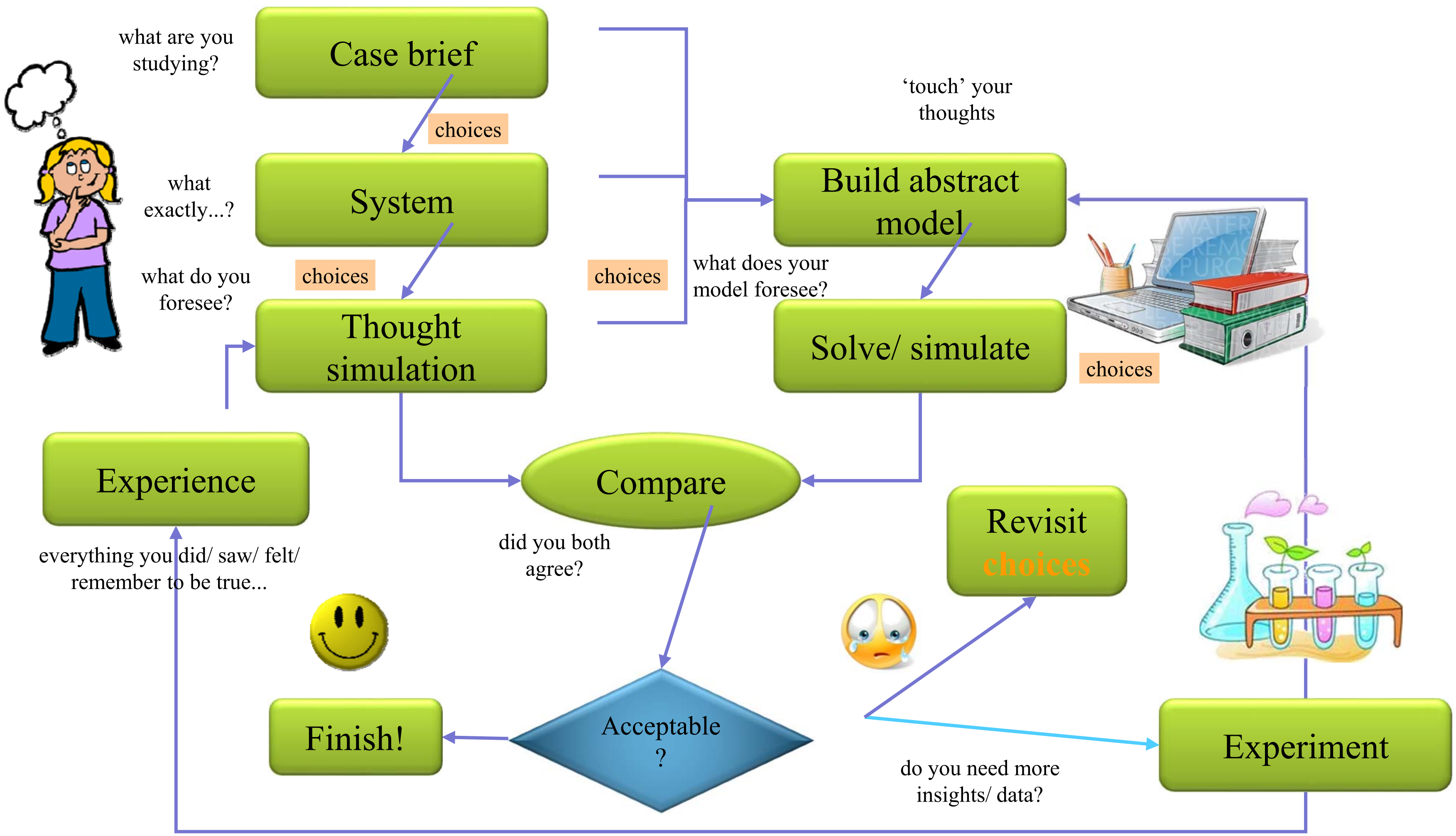
G-L-6:

Review

Contents

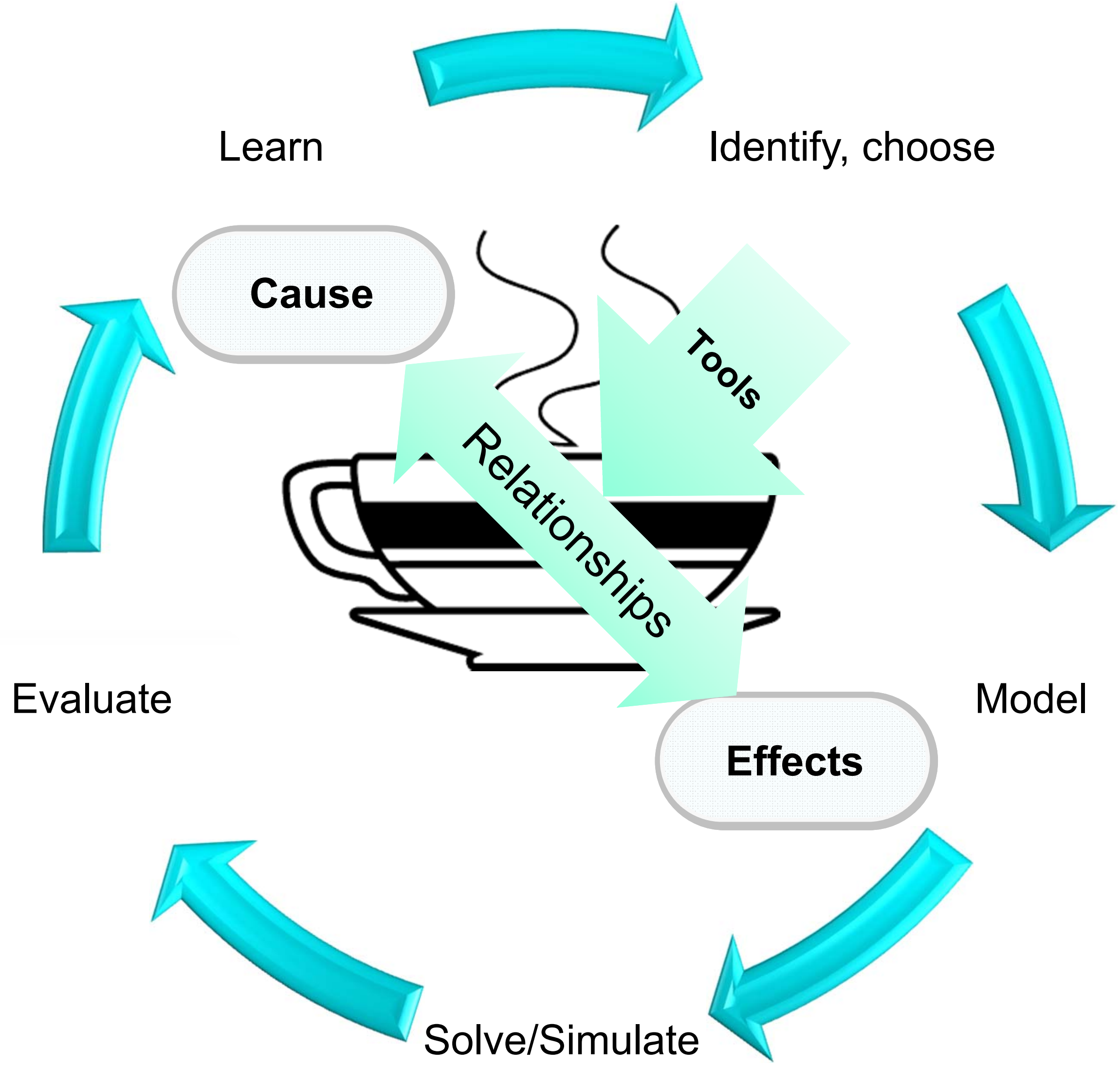
Modelling in Design



Cause ~ Effect

Cause	Cup falls off a table, hits the floor
Identify cause-effect relationship	What is special about falling off a table? Long distance, high speed, high acceleration at impact? etc
Understand cause-effect relationship	'It is not the *fall* that kills you, but the sudden stop at the end'... Acceleration causes large forces...
Quantify cause-effect relationship	Accelerations during impact
Effect	The cup is damaged

Modelling: The basic loop



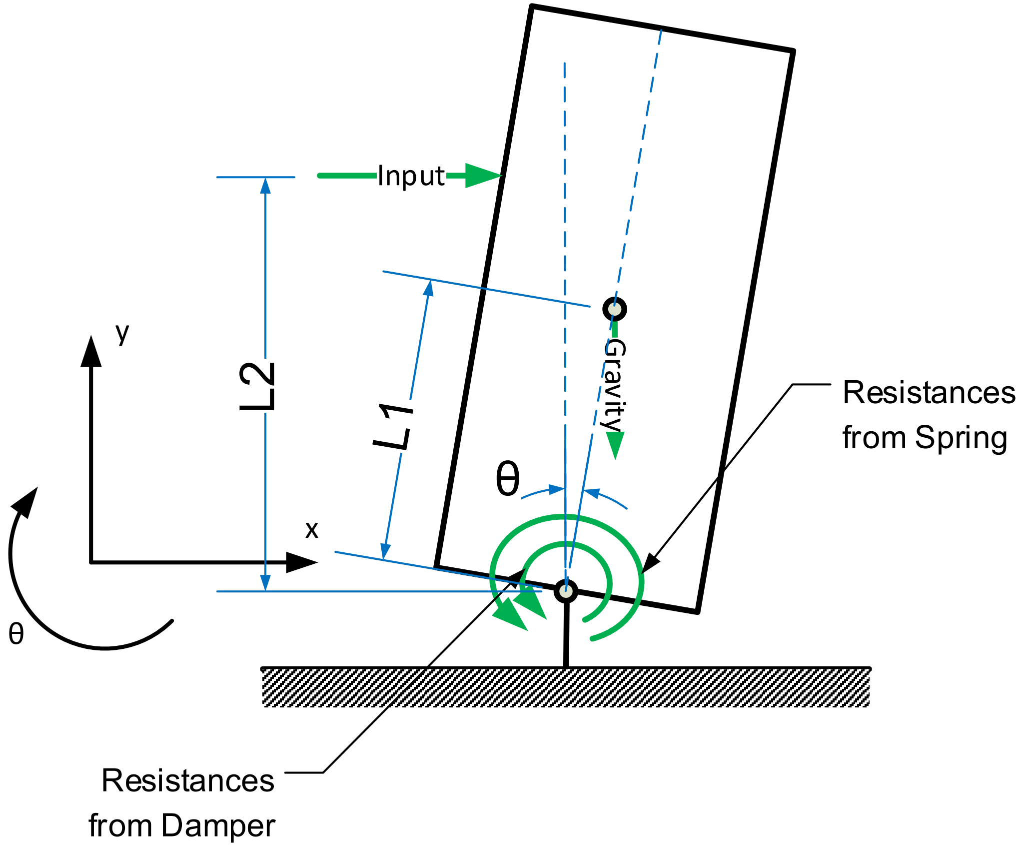
Case study: Free standing punching bar

Design a Freestanding Punch Bar

Make sure the amplitude of the vibration is less than 0.1 meter in 2 seconds after a big punch.

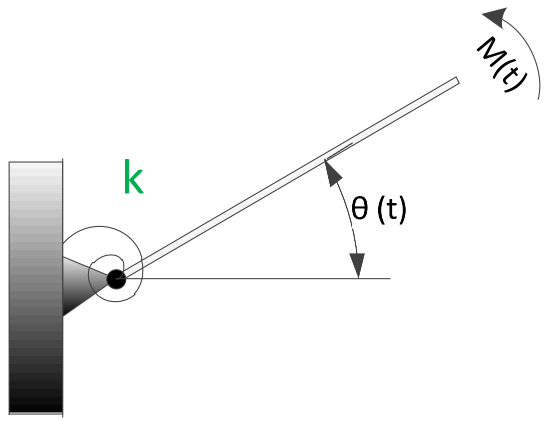
Courtesy of http://www.comparestoreprices.co.uk/keep-fit/fitness-free-standing-punch-bag.asp

A simplified diagram



Physics behind – Angular Spring

Sketch



Model

Torque[Nm]

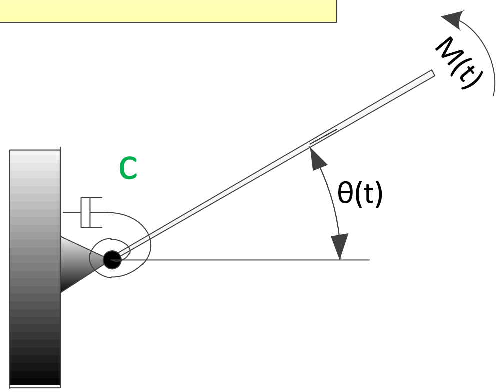
$$M(t) = -k\theta(t)$$

Spring Constant
[Nm/radian]

Angular
displacement
[radian]

Physics behind – Angular damper

Sketch



Model

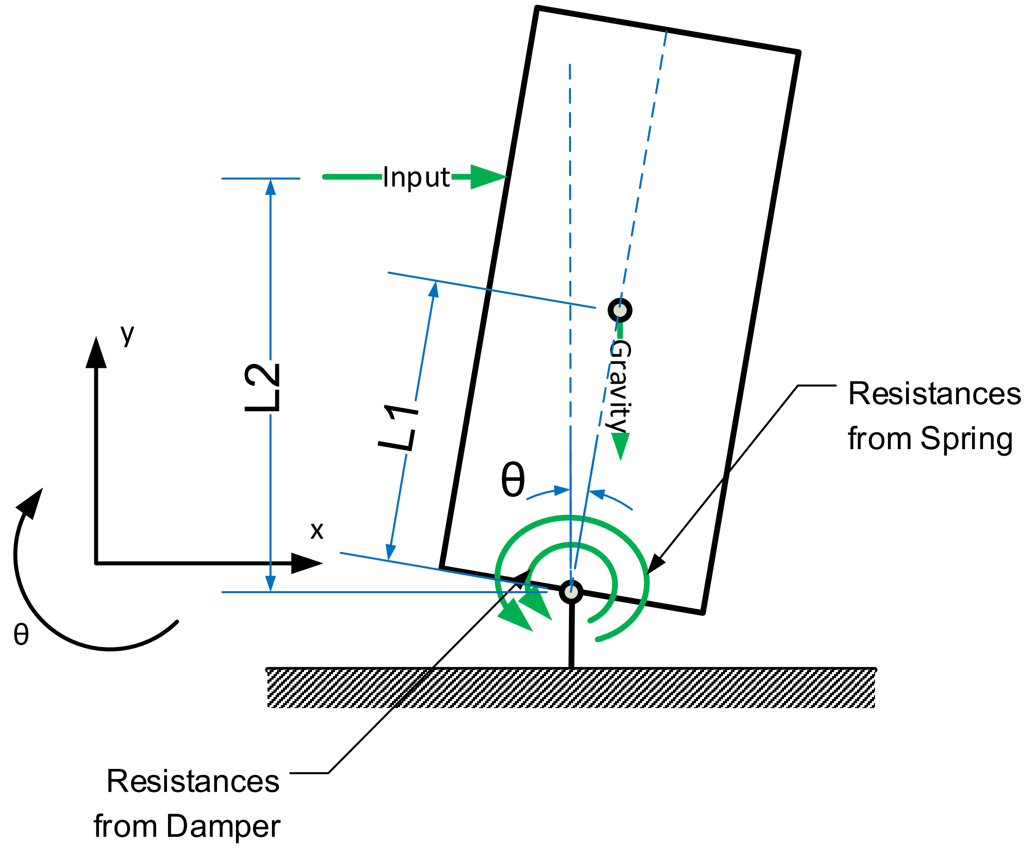
Torque[Nm]

$$M(t) = -c\,\frac{d\theta(t)}{dt}$$

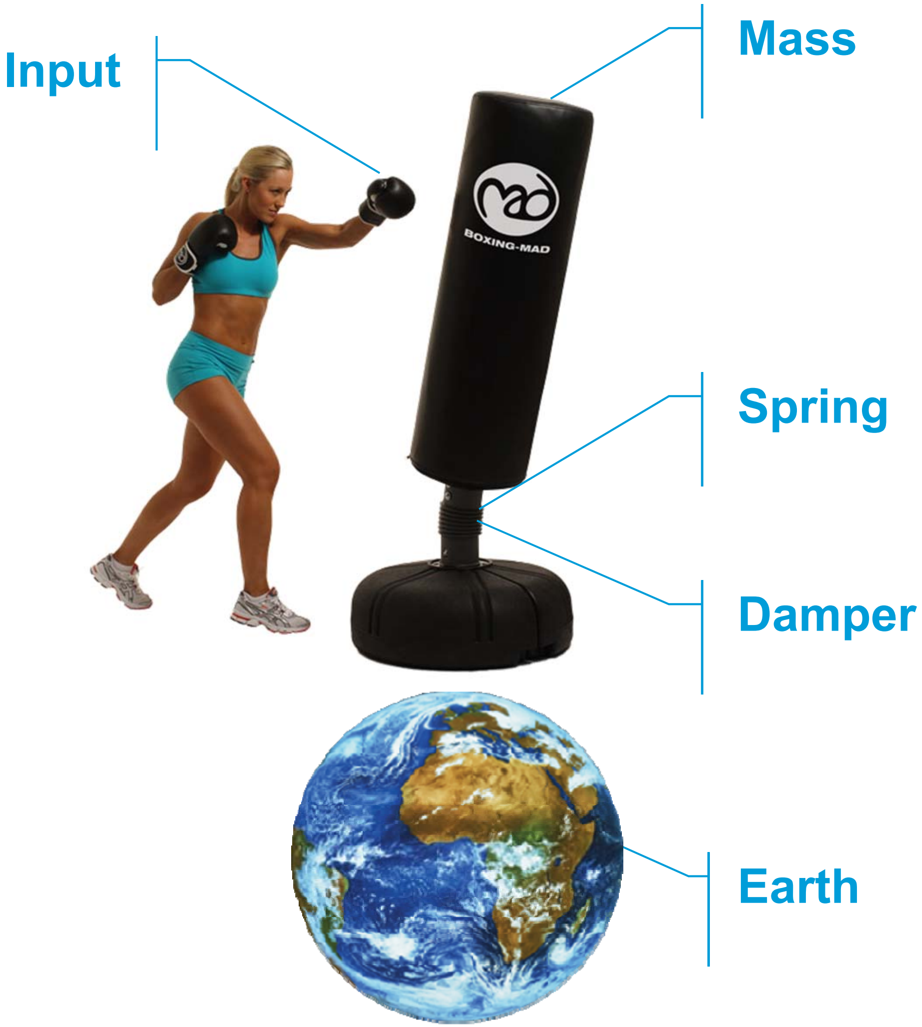
Damping coefficient [Nms/radian]

Angular Velocity [radian/s]

A simplified diagram



System components



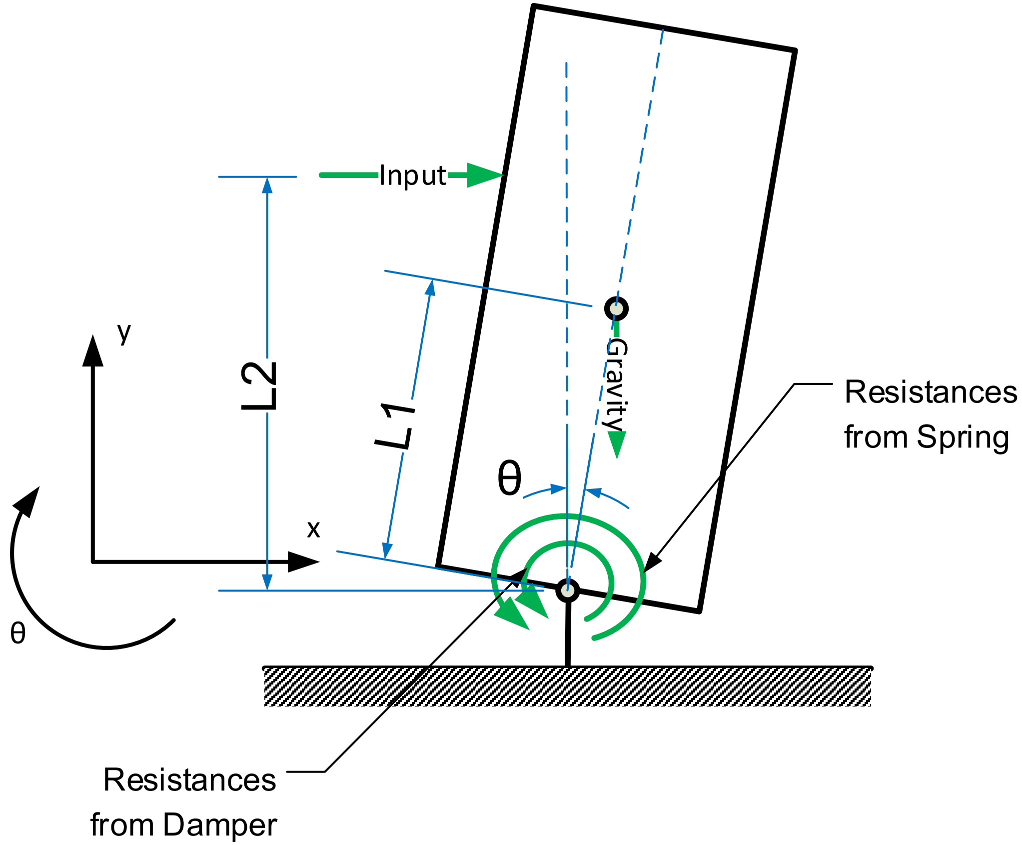
Courtesy of http://www.comparestoreprices.co.uk/keep-fit/fitness-free-standing-punch-bag.asp

Cause-Effect

Cause	Effect
The lady hits the freestanding punch bag	The freestanding punch bag starts to swing
The freestanding punch bag starts to swing	The inertia moment slows its movement
The freestanding punch bag starts to swing.	The angular spring prevents its movement
The freestanding punch bag starts to swing.	The angular damper prevents its motion
The freestanding punch bag starts to swing	The gravity of the bag tends make the bag fall down

Modelling

$$\sum M = 0$$



$$-m \cdot L1^2 \cdot \frac{d^2\theta(t)}{dt^2} + F_{input} \cdot L2 - k \cdot \theta(t) - c \cdot \frac{d\theta(t)}{dt} + m \cdot g \cdot L1 \cdot \sin(\theta(t)) = 0$$

We choose

Input

$$F_{input} = \begin{cases} 1000 & 0 < t < 0.1 \\ 0 & 0 \end{cases}$$

Mass

$m = 10$

Spring

$k = 10000$

Damper

$c = 20$

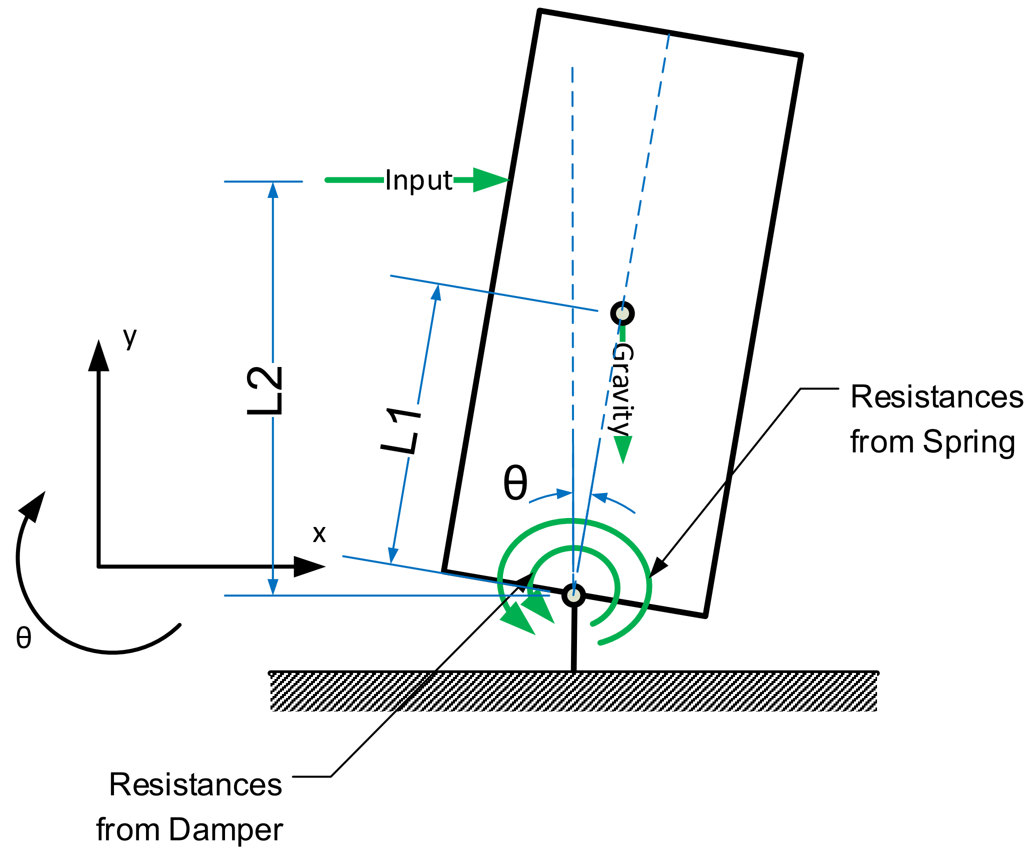
$$-m \cdot L1^2 \cdot \frac{d^2\theta(t)}{dt^2} + F_{input} \cdot L2 - k \cdot \theta(t) - c \cdot \frac{d\theta(t)}{dt} + m \cdot g \cdot L1 \cdot \sin(\theta(t)) = 0$$

Courtesy of http://www.comparestoreprices.co.uk/keep-fit/fitness-free-standing-punch-bag.asp

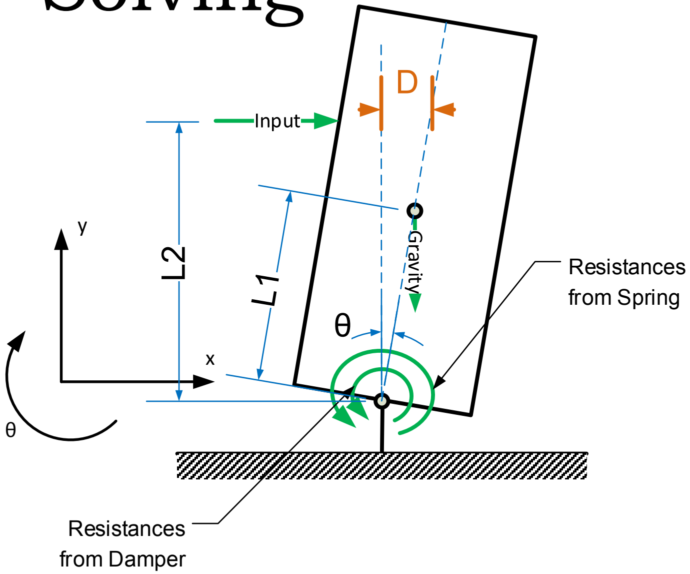
A simplified diagram- Choices

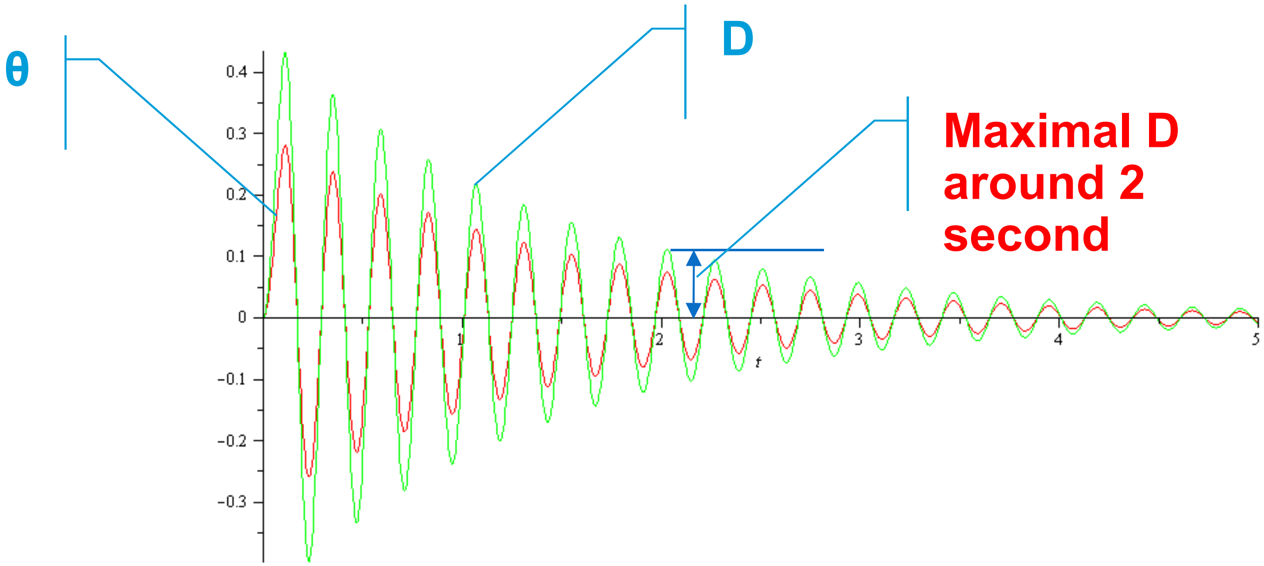
Choices:

We only can change the mass and the damper in the system



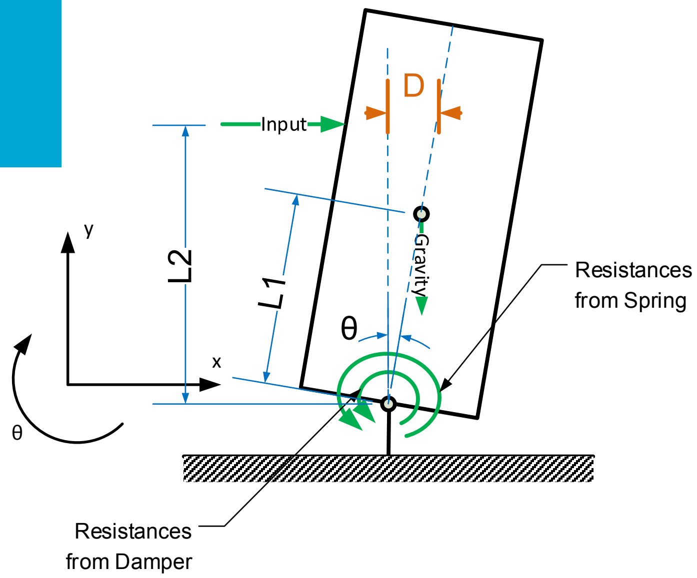
Solving



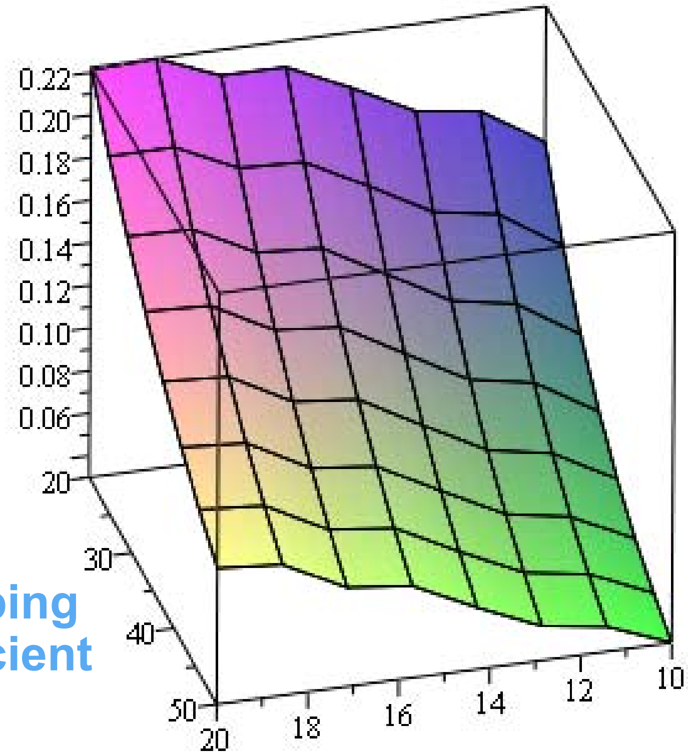


Maximal D around 2 second

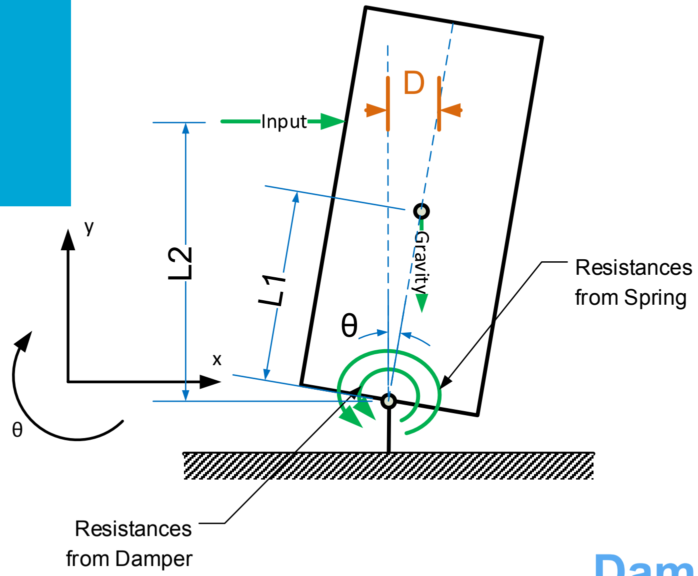
Evaluation

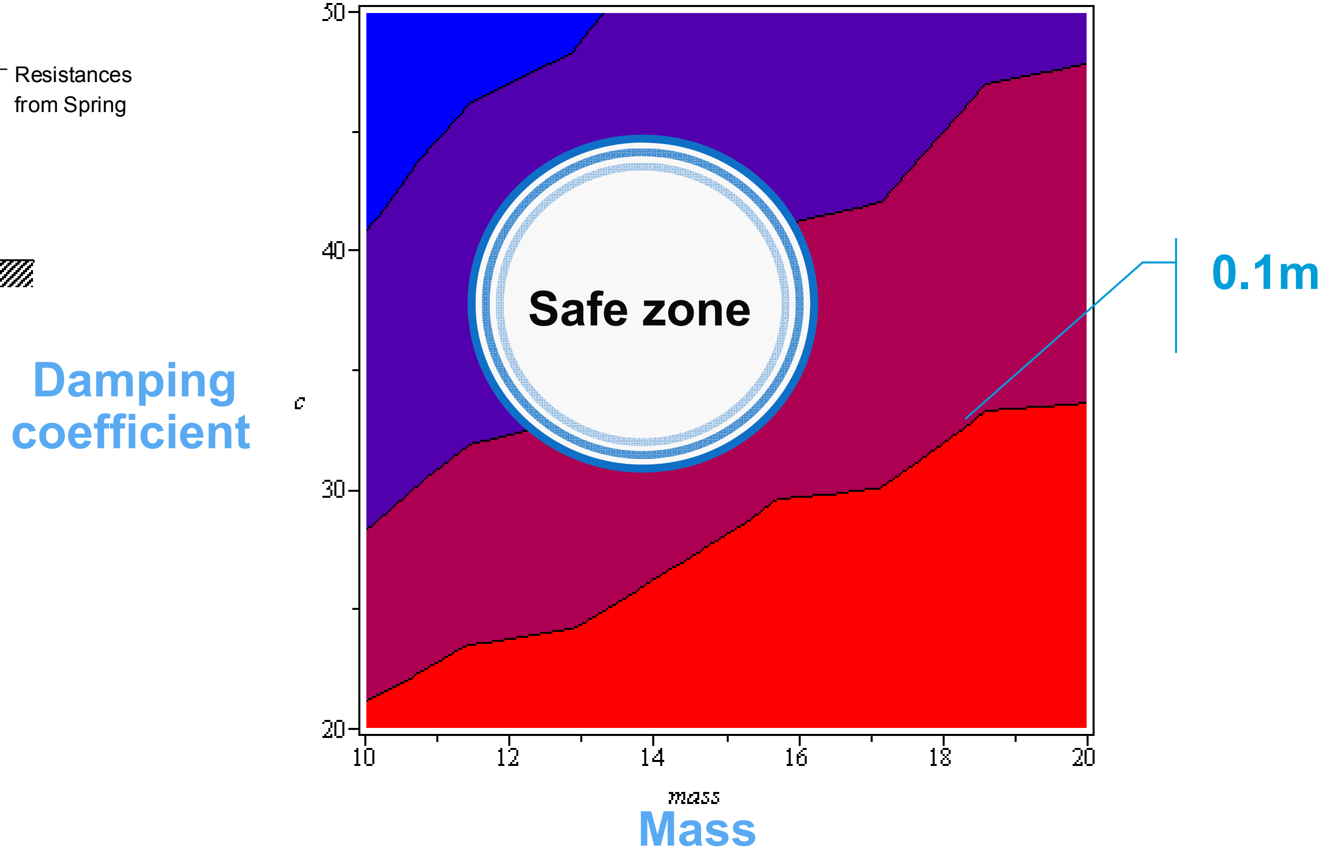


Maximal D around 2 second

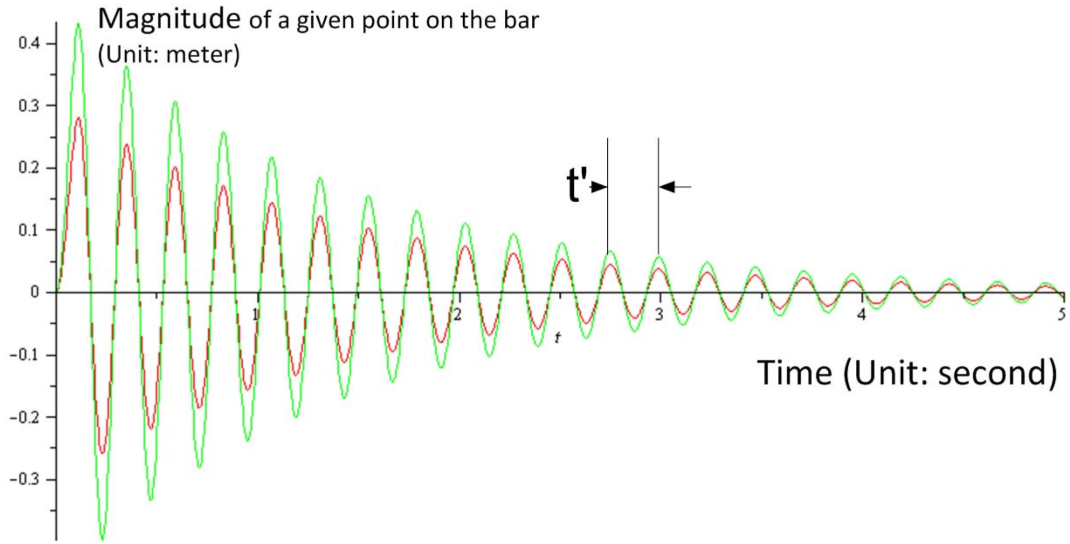


Evaluation


D
Input
L2
L1
Gravity
θ
y
x
θ
Resistances from Spring
Resistances from Damper


Safe zone
0.1m
Damping coefficient
c
mass
Mass
50
40
30
20
10
12
14
16
18
20

Further thought



Questions

The client wants you to adjust the design parameter(s) in order to reduce the frequency of the vibration, e.g., enlarge time **t'** in the figure.

Which parameter(s) do you suggest to change?

Case study: Honda® U3-X®

Solid rubber wheel

Honda® U3-X®

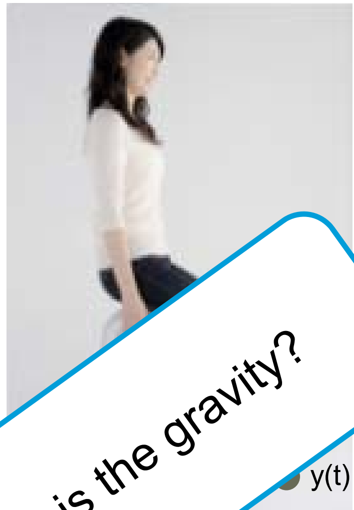
The Honda® U3-X® is a self-balancing one-wheeled electric vehicle.

Question A: Build a mathematical model to describe the vertical movement of the vehicle when it passes a step.

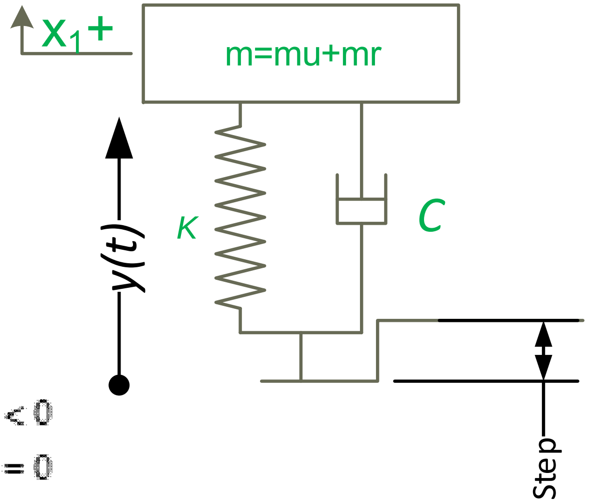
We choose:
1. the mass of the U3-X® is $\mathbf{m_u}$; the mass of the rider is $\mathbf{m_r}$;
2. the vehicle is rigid except the rubber wheel; There is **NO** suspension system;
3. the spring constant of the rubber wheel is **K**;
4. the damping coefficient of the rubber wheel is **C**;
5. the step can be described as a function of time as **y(t)** in the vertical direction;
6. to neglect the air friction.

Question B: Which parameter(s) do you suggest to change in order to reduce the amplitudes of the vibrations after it passes the step? And why?

Case study: Honda® U3-X®



Where is the gravity?

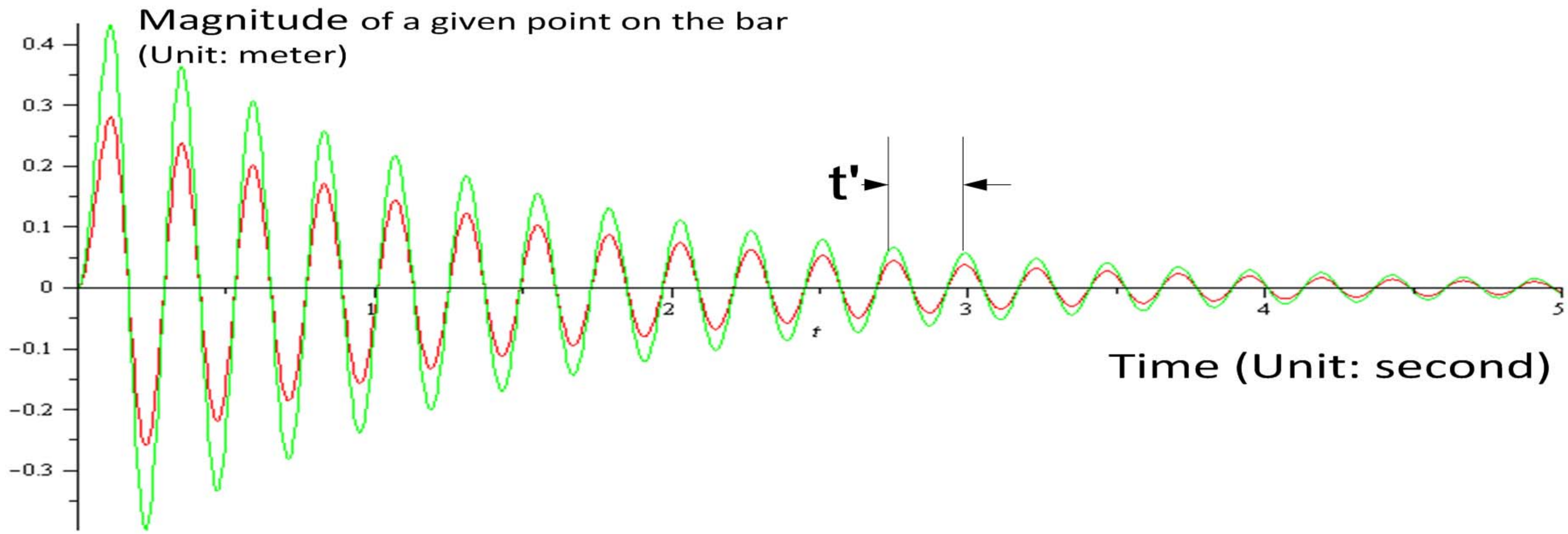


$$y(t) = \begin{cases} 0 & t < 0 \\ undefined & t = 0 \\ Step & t > 0 \end{cases}$$

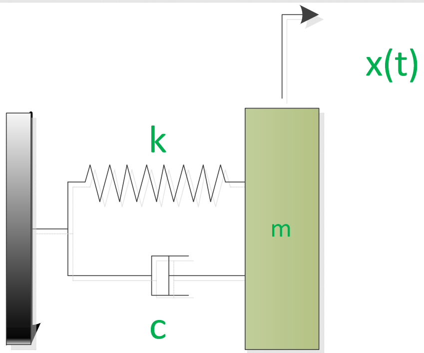
$$\sum F = 0$$

$$-m\frac{d^2x_1(t)}{dt^2} - k\ (x_1(t) - y(t)) - c(\frac{dx_1(t)}{dt} - \frac{dy(t)}{dt}) = 0$$

The natural frequency



Given a mass-spring damper system



It can be modeled as:

$$-m\frac{d^2x(t)}{dt^2} - c\frac{dx(t)}{dt} - kx(t) = 0$$

Introduce two parameters

$\omega_0 = \sqrt{\frac{k}{m}}$	Name: Natural frequency	Unit: Radians/Second
$\zeta = \frac{c}{2\sqrt{mk}}$	Name: Damping Ratio	Unit: dimensionless

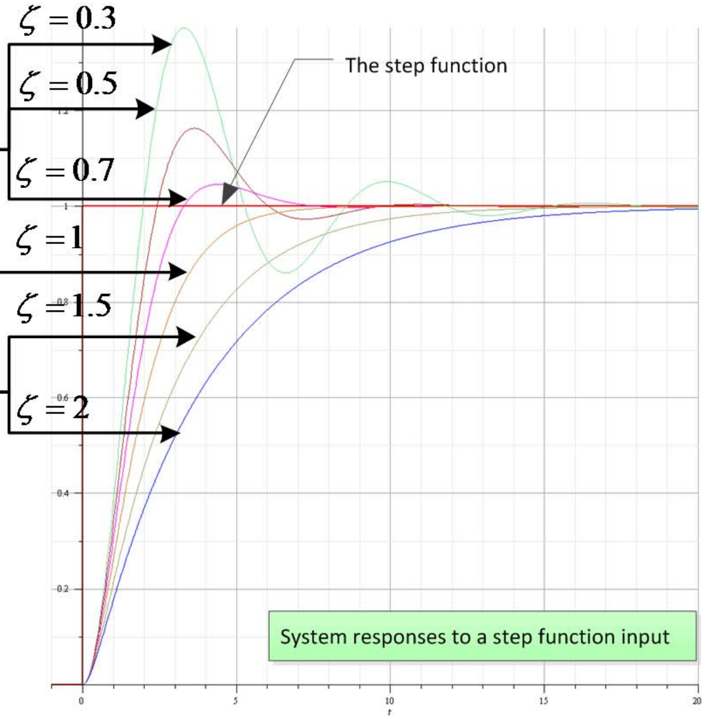
The model turns to:

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_0\frac{dx(t)}{dt} + \omega_0^2 x(t) = 0$$

The damping ratio

$$\zeta = \frac{c}{2\sqrt{mk}}$$

$0 \le \zeta < 1$	Under-damping	Solution is a complex, system will oscillate at natural frequency
$\zeta = 1$	Critical-damping	System converges to the input in the fastest way
$\zeta > 1$	Over-damping	System converges to the input longer than critical damping, but it is more stable

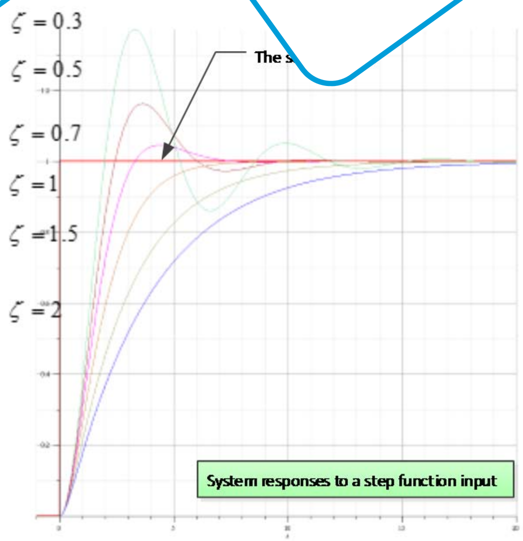


Case study: Honda® U3-X®

Enlarge or reduce the damping ratio?

If we do not want to change the frequency?

$$\zeta = \frac{c}{2\sqrt{mk}}$$



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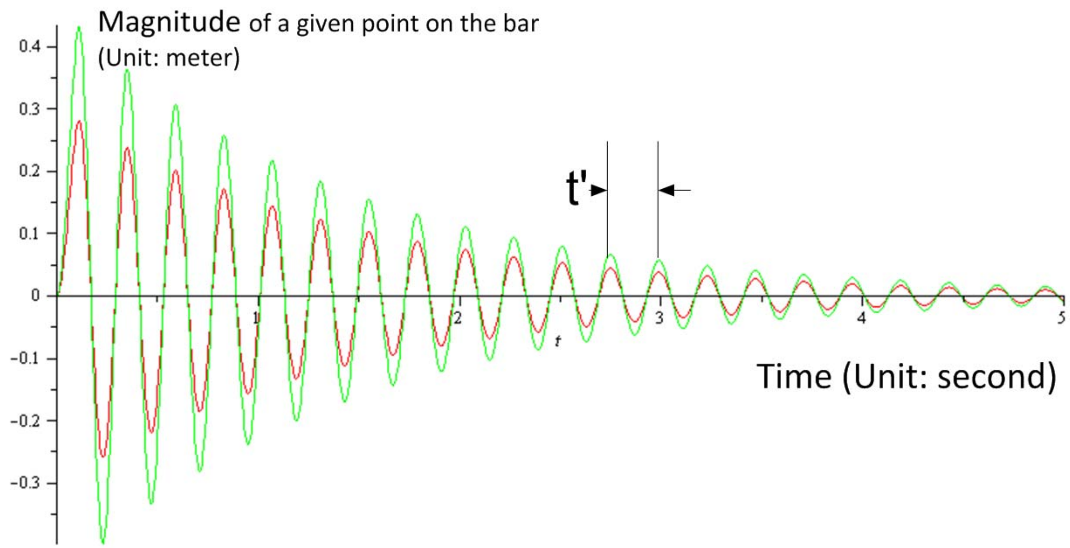
The free standing punching bar



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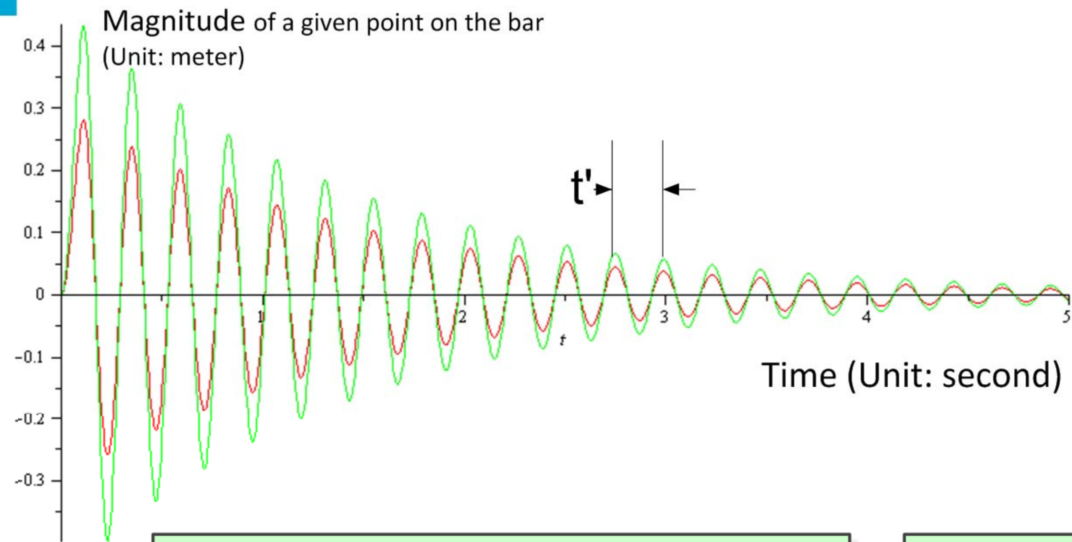
The free standing punching bar



Questions

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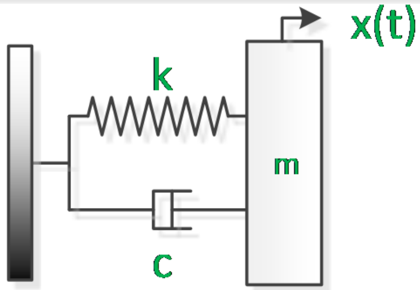
Which parameter(s) do you suggest to change?

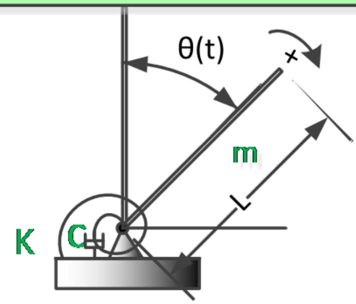
Linear movement



It can be modeled as: $-m\frac{d^2x(t)}{dt^2} - c\frac{dx(t)}{dt} - kx(t) = 0$

$\omega_0 = \sqrt{\frac{k}{m}}$	Name: Natural frequency
	Unit: Radians/Second

Rotation

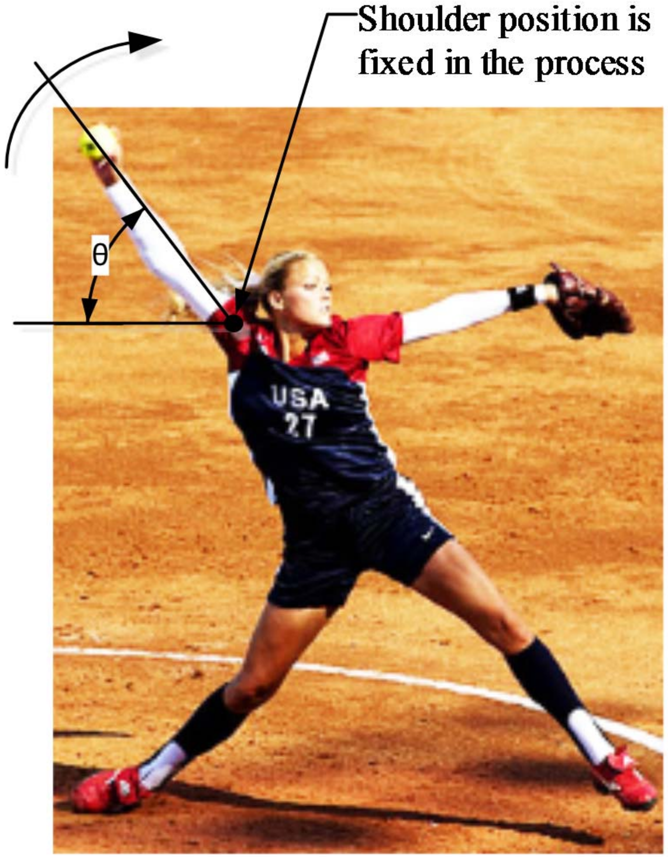


It can be modeled as (neglect the Gravity): $-I\frac{d^2\theta(t)}{dt^2} - c\frac{d\theta(t)}{dt} - k\theta(t) = 0$

Natural frequency is defined based on **I** and **K**

Changing **M, L, or K**

Case studies: The Softball Pitcher



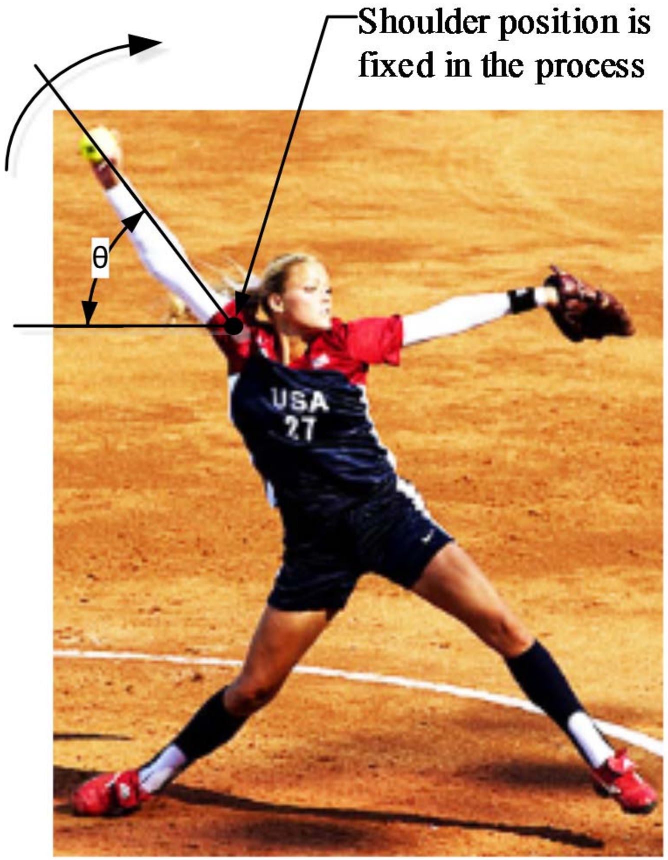
The Softball Pitcher

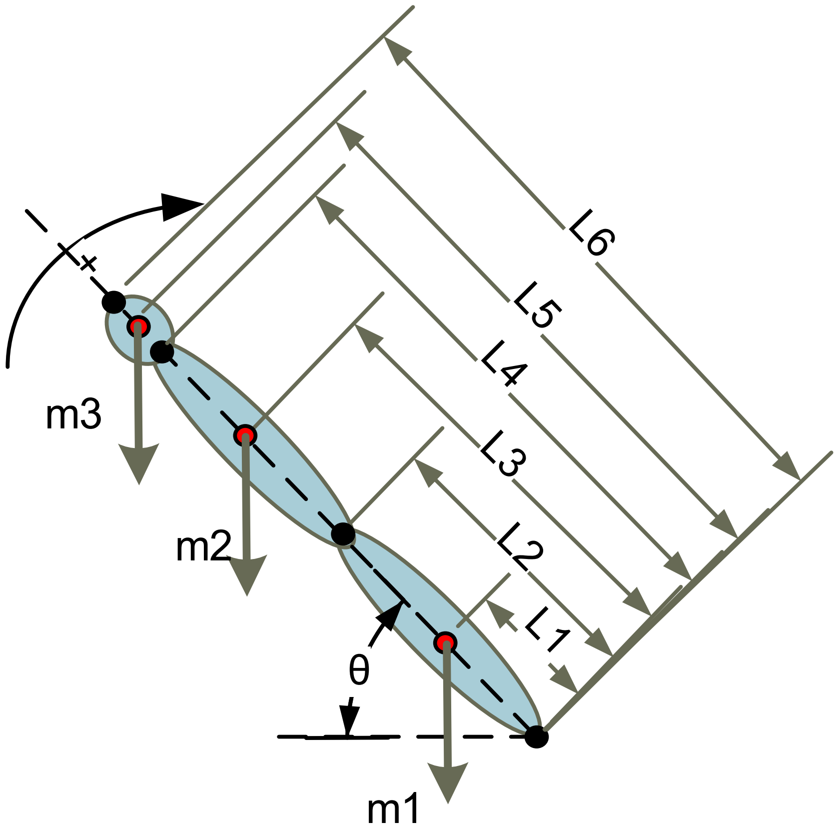
Consider a softball pitcher throws a ball:

Question:
via a motion capture software, it is identified that when $\theta = \boldsymbol{\pi/4}$, the angular velocity of her right arm is **2** rad/s, the angular acceleration is **1** rad/s. What is the torque she applied on her right shoulder joint in this moment?

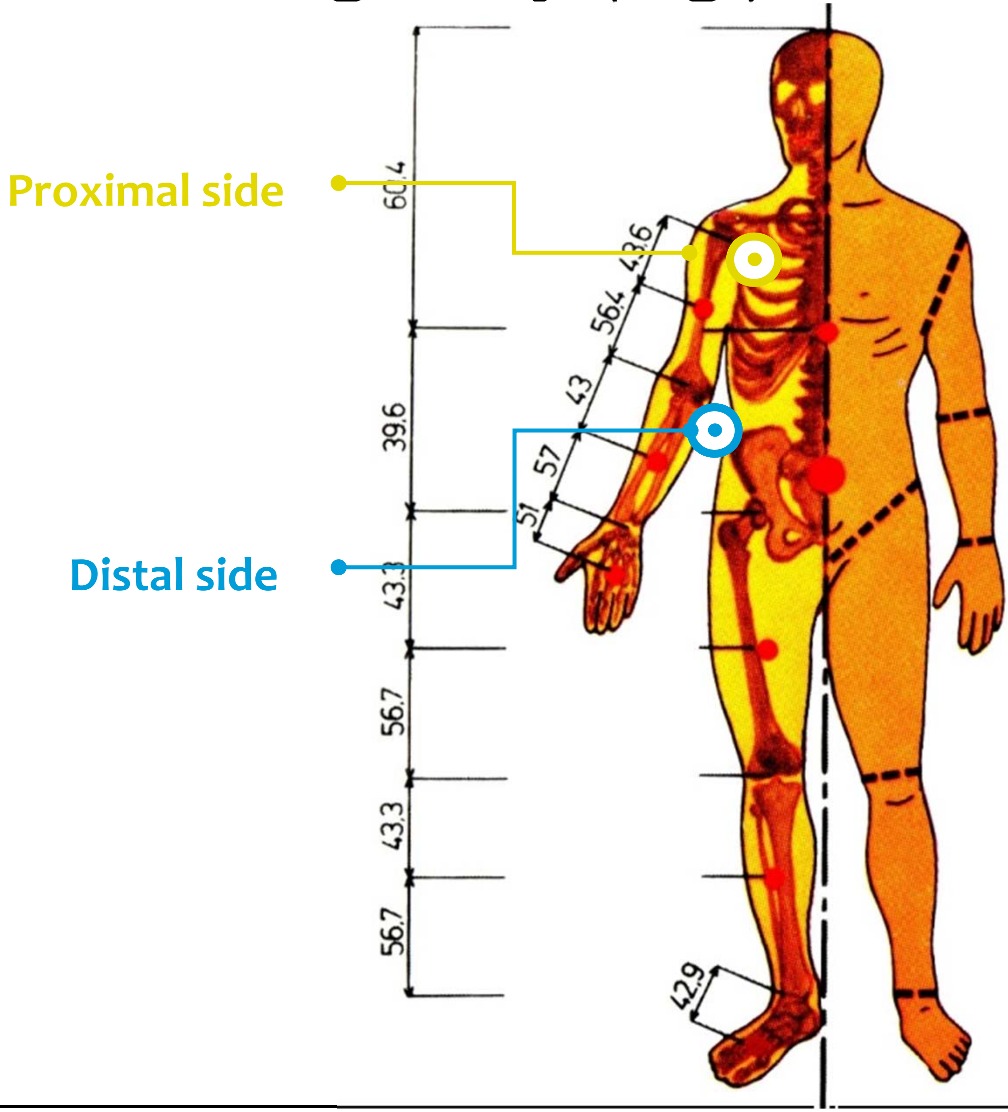
We choose:
1. her mass is **65** kg;
2. the length of her upper arm is **0.27** m;
3. the length of her forearm is **0.24** m;
4. the length of her hand is **0.1** m;
5. the mass of the softball is **0.2** kg;
6. the position of her right shoulder joint is fixed in the movement;
7. her right elbow joint and her wrist joint is not moving in the process;
8. to use point mass to approximate the mass moment of inertia.

Case studies: The Softball Pitcher





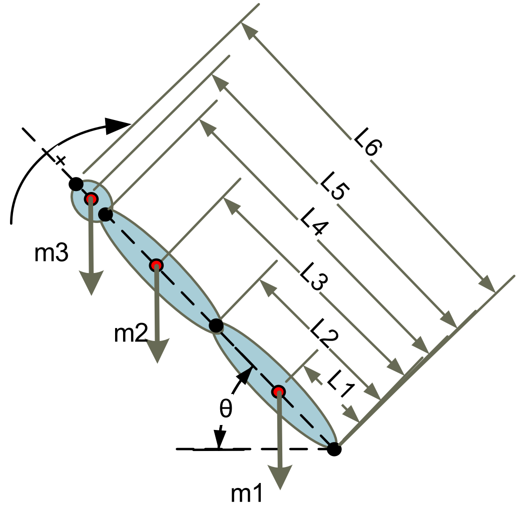
Centre of gravity (c.g.) of body segments



Mass of body segment

Following Biomechanics and Motor Control of Human Movement

Segment	Segment Total Body Weig	Centre of Mas Segment leng	Centre of M Segment le
		Proximal	Distal
Hand	0.006	0.506	0.494
Forearm	0.016	0.43	0.57
Upper arm	0.028	0.436	0.564
F'arm+hand	0.022	0.682	0.318
Upper limb	0.05	0.53	0.47
Foot	0.0145	0.5	0.5
Shank	0.0465	0.433	0.567
Thigh	0.1	0.433	0.567
Foot + shank	0.061	0.606	0.394
Lower Limb	0.161	0.447	0.553
Head, neck, trunk	0.578	0.66	0.34
Head, neck, arms, trunk	0.678	0.626	0.374
Head and neck	0.081		



$$m1 = 70 * 0.028 = 1.96$$

$$m2 = 70 * 0.016 = 1.12$$

$$m3 = 70 * 0.006 + 0.2 = 0.62$$

$$L1 = 0.436 * 0.27 = 0.11772$$

$$L2 = 0.27$$

$$L3 = L2 + 0.43 * 0.24 = 0.3732$$

$$L4 = L2 + 0.24 = 0.51$$

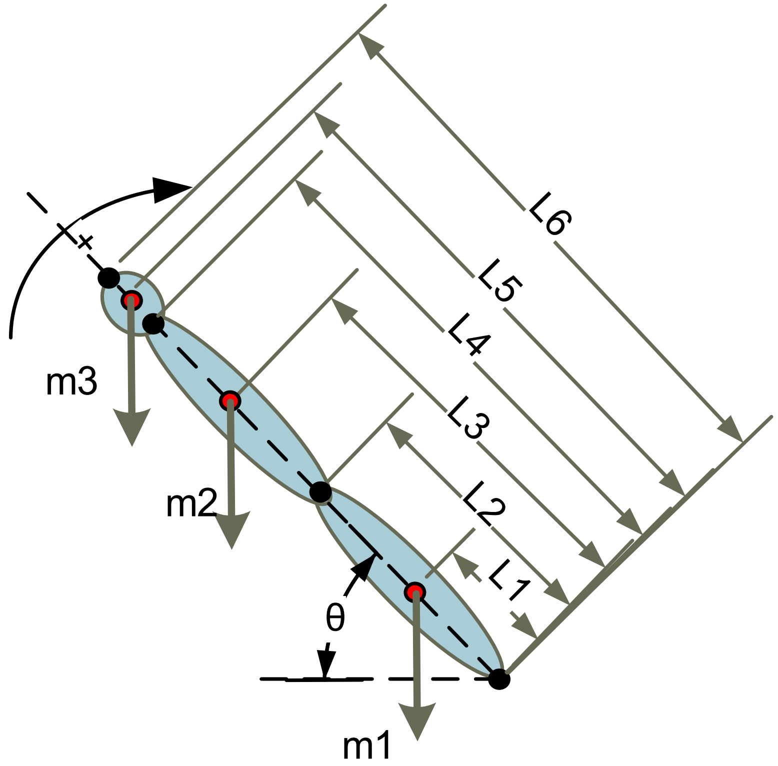
$$L5 = L4 + 0.1 * 0.506 = 0.5606$$

Ref. http://books.google.nl/books?id=_bFHL08IWfwC&printsec=frontcover&source=gbs_ge_summary_r&cad=0#v=snippet&q=mass%20segment&f=false

Case studies: The Softball Pitcher

Is it better to use cylinder to approach the arm

$$\sum M = 0$$



$$-(m1L1^2 + m2L3^2 + m3L5^2)\frac{d^2\theta(t)}{dt^2} - (m1L1 + m2L3 + m3L5)g\cos\theta(t) + M = 0$$

Case studies: The coke can

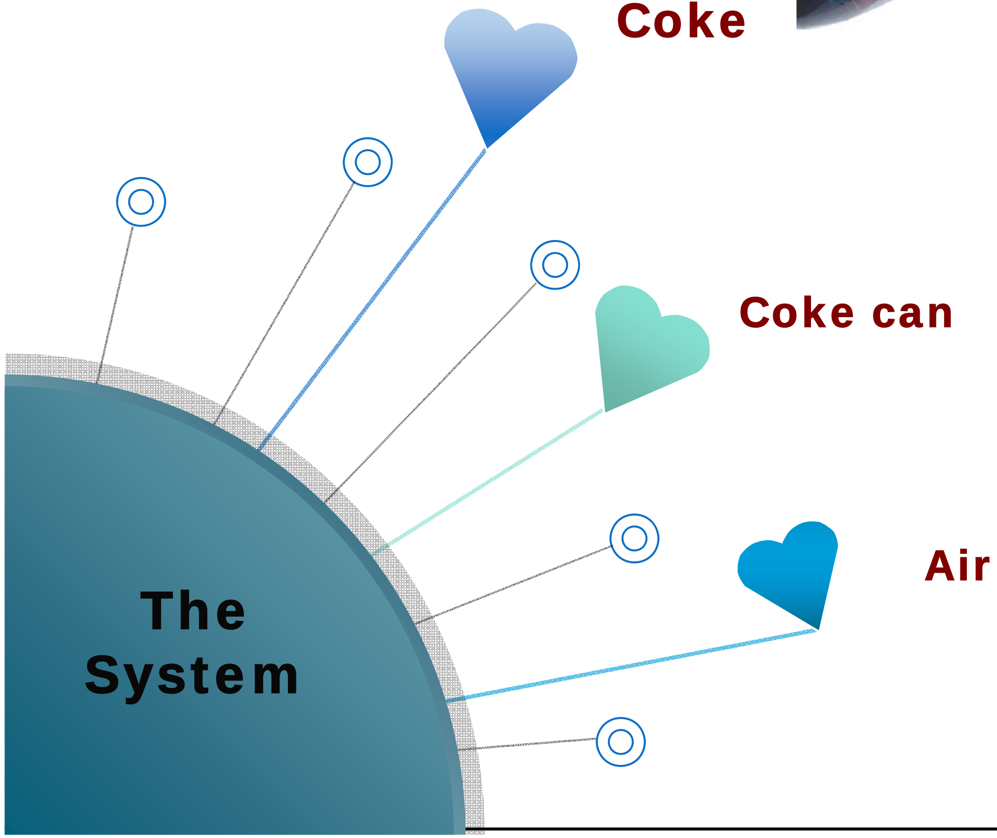
Design brief

Take a can of Coca-Cola® (**0.355**Liter**)** out from the refrigerator and put it on the table:

Question:
Make a *quick estimation* of the time span during which the Coca-Cola® stays cool (≤ **8**°C, which is nice to drink).

Courtesy of http://www.coca-cola.com/

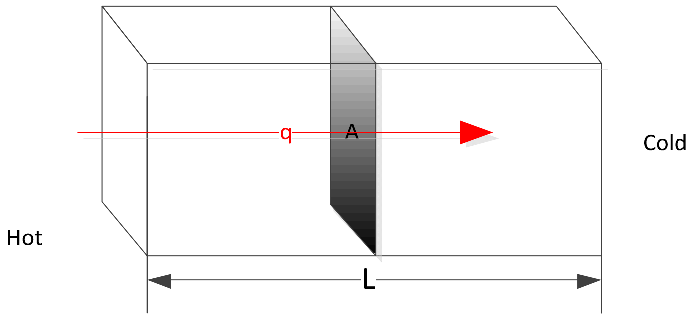
Components



Cause-effect

Physic behind - Conduction

Conduction



$$q = -\frac{kA}{L}\Delta T$$

Thermal conductivity
Unit: W/(m·K)

Cross sectional area
Unit: m^2

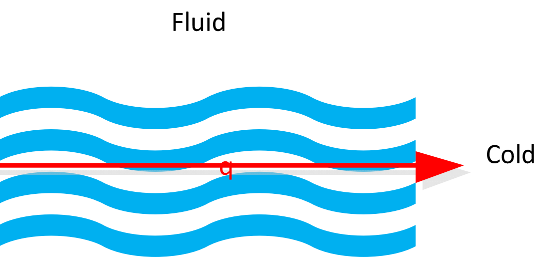
Length
Unit: m

Temperature difference between two ends.
Unit: K

Physic behind - Convection

nvection

e process in which heat is
rried by the bulk movement
a fluid/gas



Heat transfer coefficient.
Unit: W/(m^2·K)

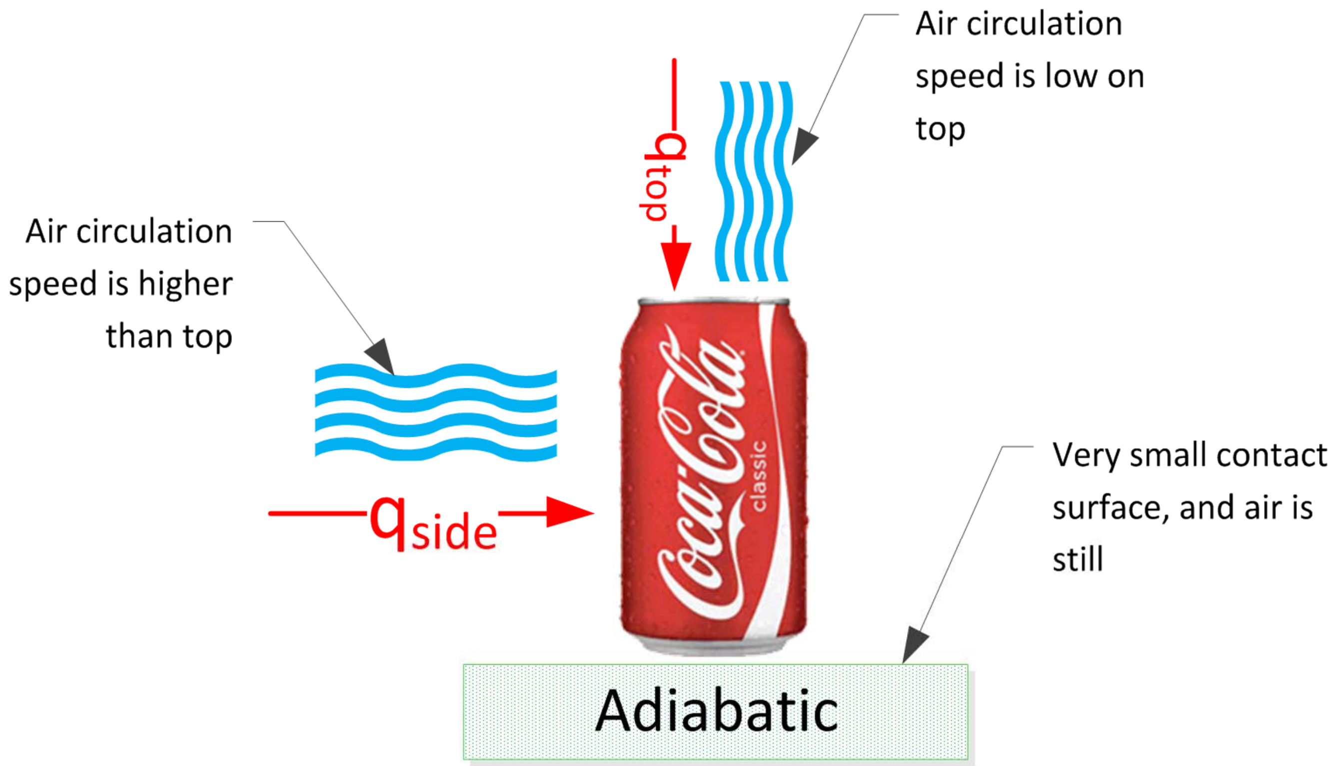
Contact area
Unit: m^2

$$q = -hA\Delta T$$

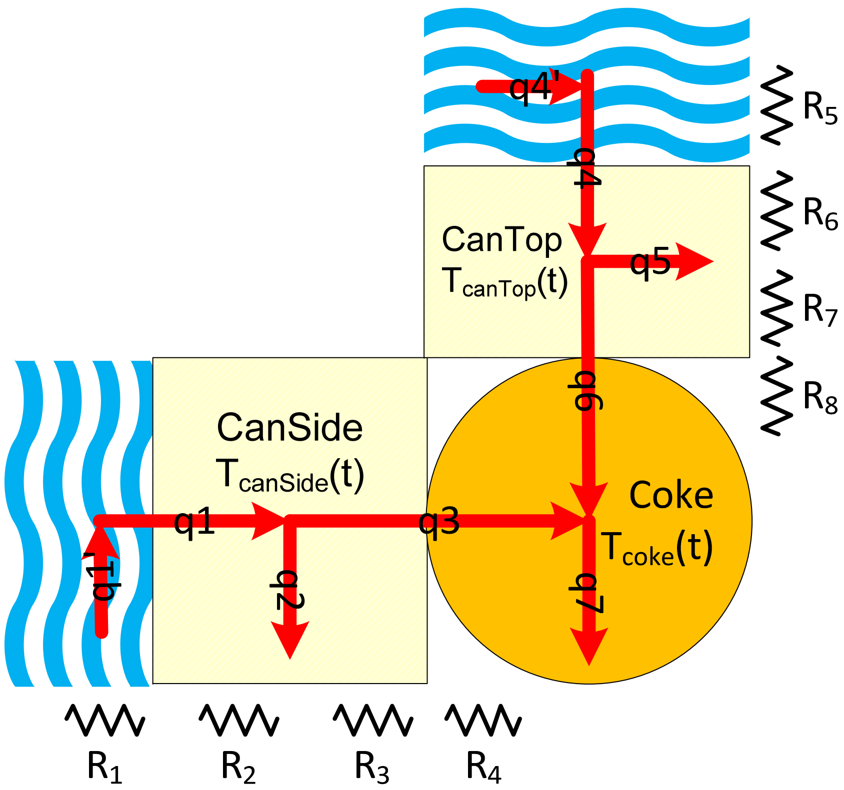
Temperature difference between two ends.
Unit: K

h	Fluid properties The velocity of Fluid
The typical values of h	Air: 10~100 Water: 500 to 10,000

A simple sketch



Understand the problem



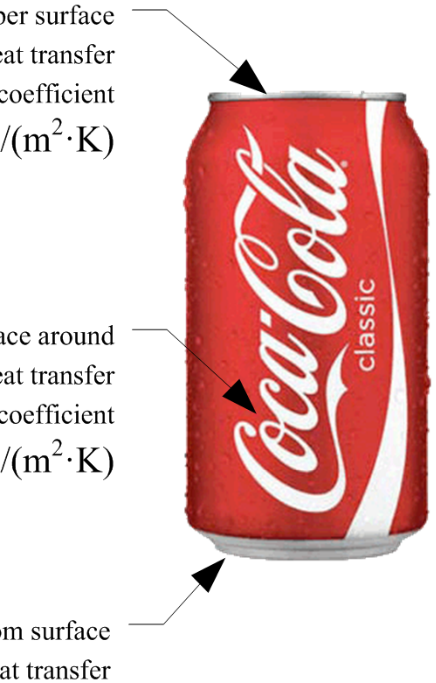
$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_1 = \frac{1}{h_{airSide} A_{side}} \quad R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}} \quad R_4 = \frac{1}{h_{water} A_{side}}$$

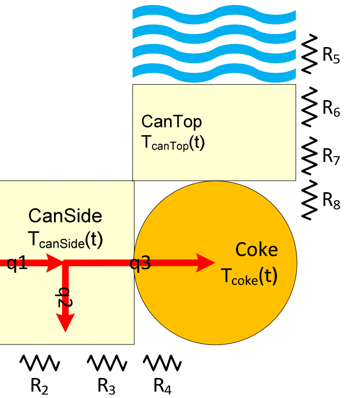
Our choices



We choose:

1. The can is made of aluminum, the specific heat capacity of aluminum is **897** J/(kg·K);
2. The thermal conductivity of aluminum is **237** W/(m·K);
3. The area of the top of the can is **0.003167** m^2;
4. The area of the surface around of the can is **0.0226** m^2;
5. The thickness of the can is **0.28** mm;
6. The can is full filled with **0.335** kg of Coca-Cola®;
7. The specific heat of the Coca-Cola® is **4181** J/(kg·K);
8. The initial temperature of the Coca-Cola® is **4**°C;
9. The environment (air) temperature is **20**°C;
10. The heat transfer coefficient between the Coca-Cola® and the can is **500** $W/(m^2 \cdot K)$;

Model



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}} \quad R_4 = \frac{1}{h_{water} A_{side}}$$

Cause

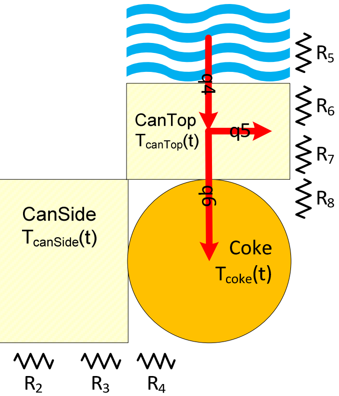
Air temperature is higher than can	The coke can absorbs heat	The can(side) temperature is higher than Coke

Effect

Heat transferred from air to the can: q1	Temperature rises: q2	Heat transferred to coke: q3

$$\frac{T_{air} - T_{canSide}(t)}{\ } - m_{canSide} C_{canSide} \frac{dT_{canSide}(t)}{\ } - \frac{T_{canSide}(t) - T_{Coke}(t)}{\ } = 0$$

Model



$$R_5 = \frac{1}{h_{airTop}A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can}A_{top}}$$

$$R_8 = \frac{1}{h_{water}A_{top}}$$

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can}A_{side}} \quad R_4 = \frac{1}{h_{water}A_{side}}$$

Cause

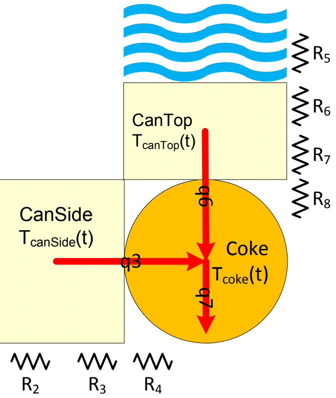
Air temperature is higher than can top	The coke can top absorbs heat	The can top temperature is higher than Coke

Effect

Heat transferred from air to the can: q4	Temperature rises: q5	Heat transferred to coke: q6

$$\frac{T_{air} - T_{canTop}(t)}{\ldots} - m \ldots C \ldots \frac{dT_{canTop}(t)}{\ldots} - \frac{T_{canTop}(t) - T_{Coke}(t)}{\ldots} = 0$$

Model



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}} \qquad R_4 = \frac{1}{h_{water} A_{side}}$$

Cause

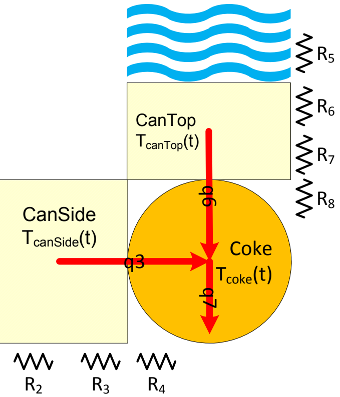
The can(side) temperature is higher than Coke	The can(top) temperature is higher than Coke	The temperature of coke arises

Effect

Heat transferred to coke: q3	Heat transferred to coke: q6	Heat absorbed by coke: q7

$$\frac{T_{canSide}(t) - T_{Coke}(t)}{} + \frac{T_{canTop}(t) - T_{Coke}(t)}{} - m_{Coke} C_{Coke} \frac{dT_{Coke}(t)}{} = 0$$

Review



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}} \qquad R_4 = \frac{1}{h_{water} A_{side}}$$

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ol for Mathematics and Modeling

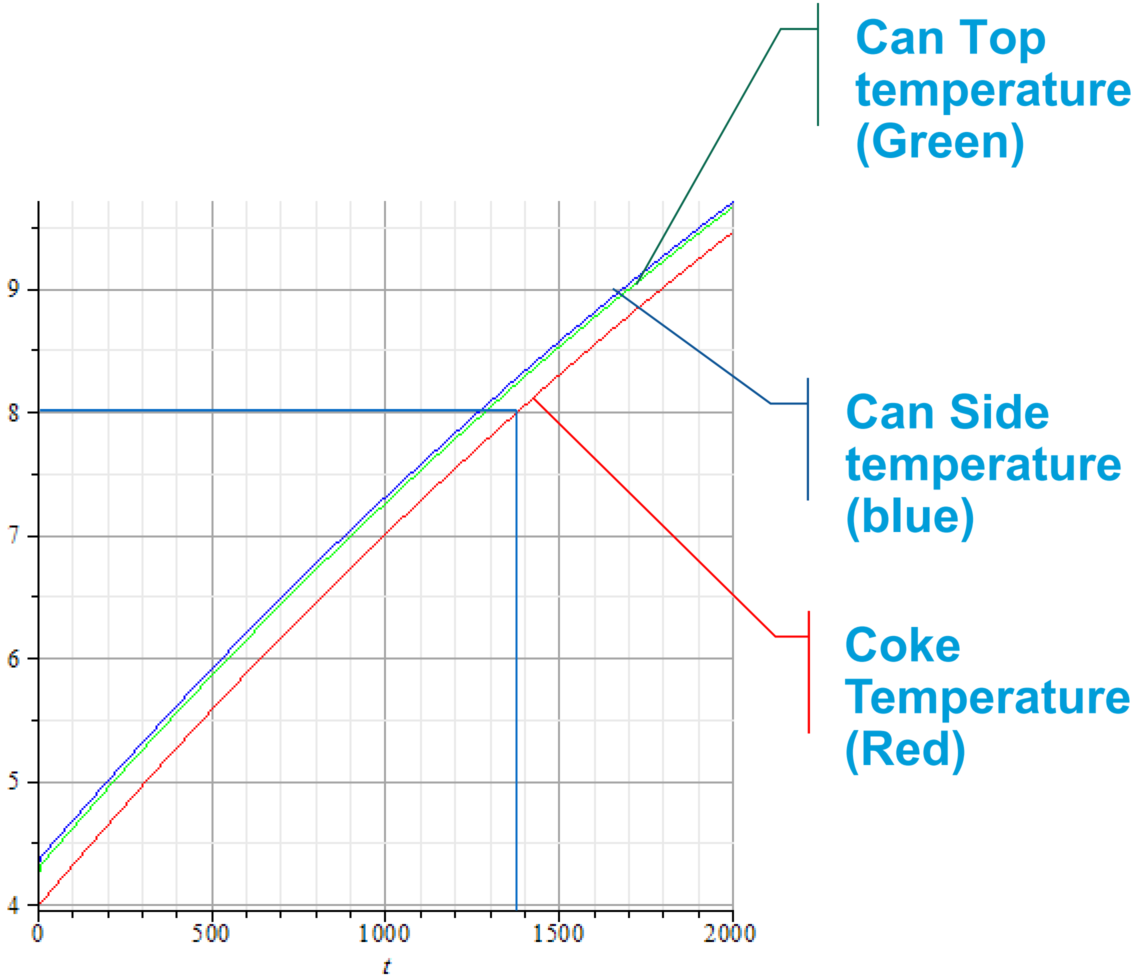
$$\frac{T_{air} - T_{canSide}(t)}{R_1 + R_2} - m_{canSide} C_{canSide} \frac{dT_{canSide}(t)}{dt} - \frac{T_{canSide}(t) - T_{Coke}(t)}{R_3 + R_4} = 0$$

$$\frac{T_{air} - T_{canTop}(t)}{R_5 + R_6} - m_{canTop} C_{canTop} \frac{dT_{canTop}(t)}{dt} - \frac{T_{canTop}(t) - T_{Coke}(t)}{R_7 + R_8} = 0$$

$$\frac{T_{canSide}(t) - T_{Coke}(t)}{} + \frac{T_{canTop}(t) - T_{Coke}(t)}{} - m \; C \; \frac{dT_{Coke}(t)}{} = 0$$

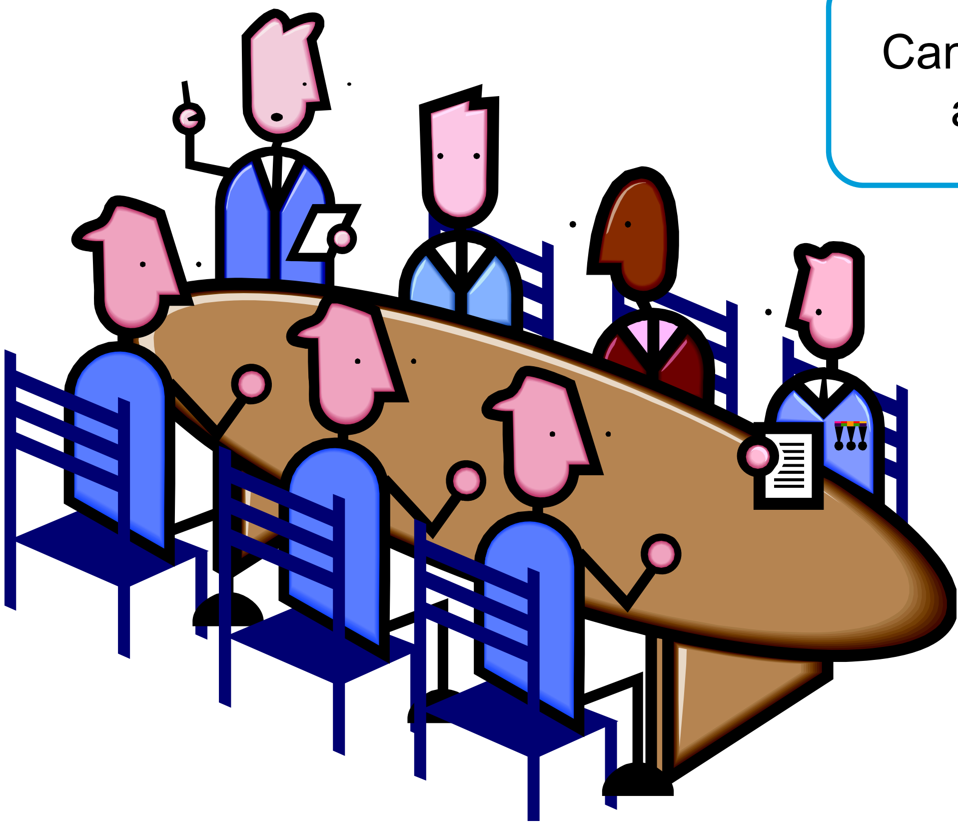
Solving

8 degrees



1575 seconds

Can we find a quick solution?



Can I make a step advantage?

Revisit the model

Heat absorbed

1.26%

Heat absorbed

temperature + 1 degree

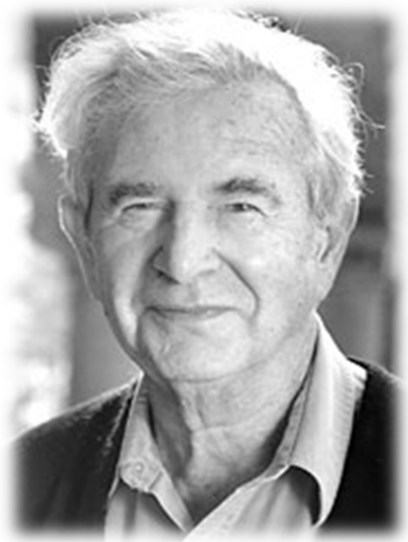
$$m_{Coke} \cdot C_{Coke} \cdot \Delta T_{Coke}(t)$$
$$= 0.33 \cdot 4181 = 1400.6$$

$$m_{can} \cdot C_{can} \cdot \Delta T_{can}(t)$$
$$= 0.0195 \cdot 900 = 17.6$$

The purpose of models

ne purpose of models is not to fit the

*ata but to **sharpen** the questions.*



Samuel Karlin

Revisit the model

Heat absorbed

1.26%

Heat absorbed

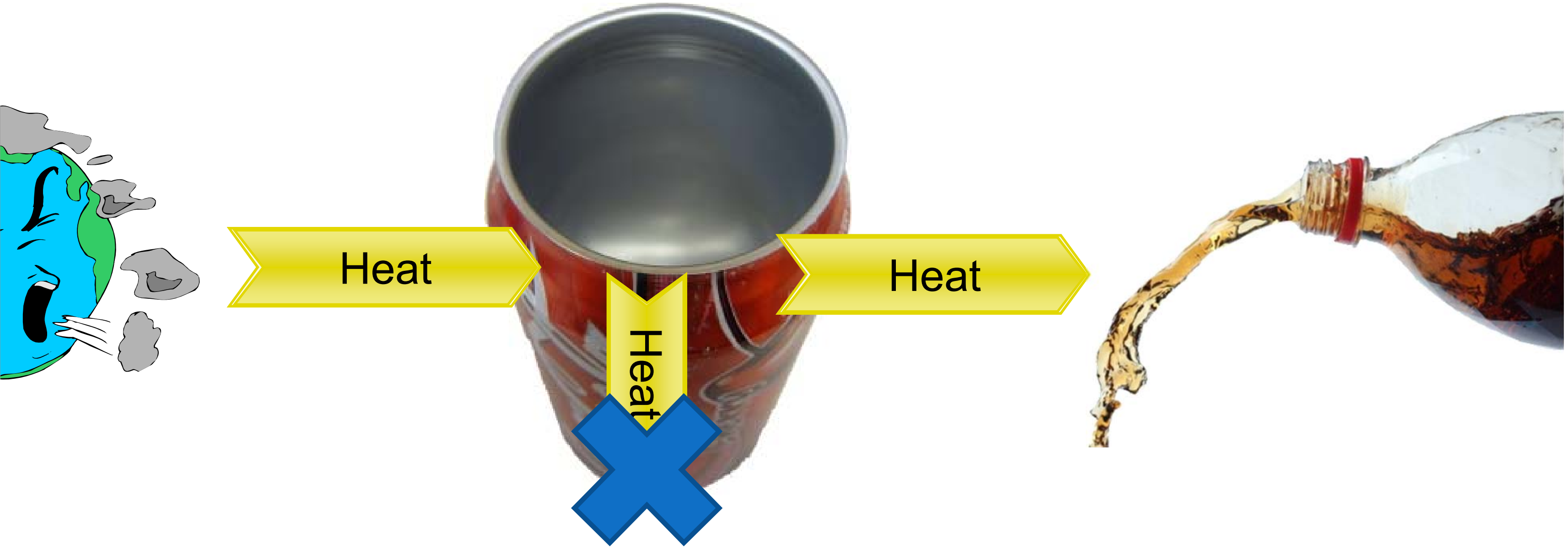
temperature + 1 degree

$$m_{Coke} \cdot C_{Coke} \cdot \Delta T_{Coke}(t) = 0.33 \cdot 4181 = 1400.6$$

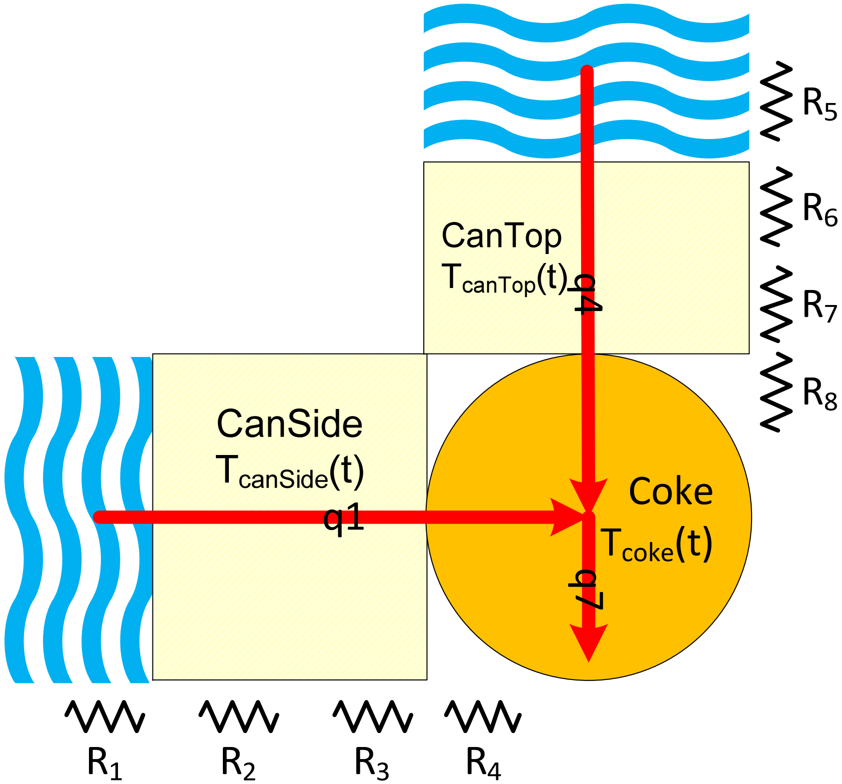
$$m_{can} \cdot C_{can} \cdot \Delta T_{can}(t) = 0.0195 \cdot 900 = 17.6$$

Neglect heat absorbed by the can for a quick estimation

Cause-effect



Neglect the heat absorbed by the can



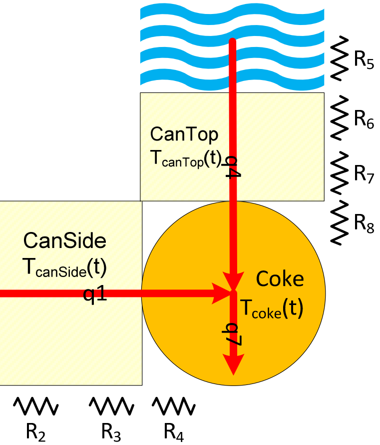
$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_1 = \frac{1}{h_{airSide} A_{side}} \quad R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}} \quad R_4 = \frac{1}{h_{water} A_{side}}$$

The new model: Case 2



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}} \qquad R_4 = \frac{1}{h_{water} A_{side}}$$

Cause

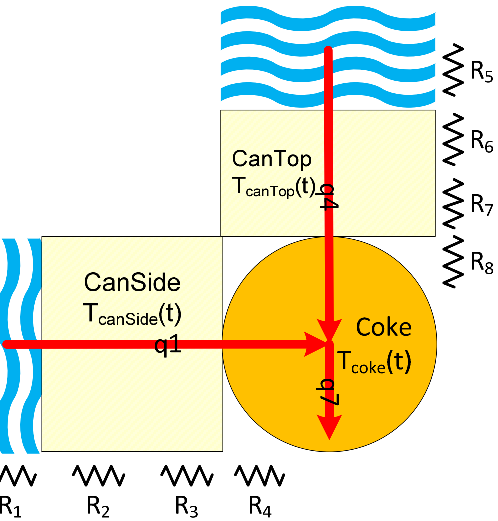
The air(side) temperature is higher than Coke	The air(top) temperature is higher than Coke	The temperature of coke arises

Effect

Heat transferred to coke: q1	Heat transferred to coke: q4	Heat absorbed by coke: q7

$$\frac{T_{air} - T_{Coke}(t)}{\;} + \frac{T_{air} - T_{Coke}(t)}{\;} - m \quad C \quad \frac{dT_{Coke}(t)}{\;} = 0$$

Further:



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$ 31.5

$$R_6 = R_7 = \frac{L_{can}/2}{K_{can} A_{top}}$$ $1.87*10^{-4}$

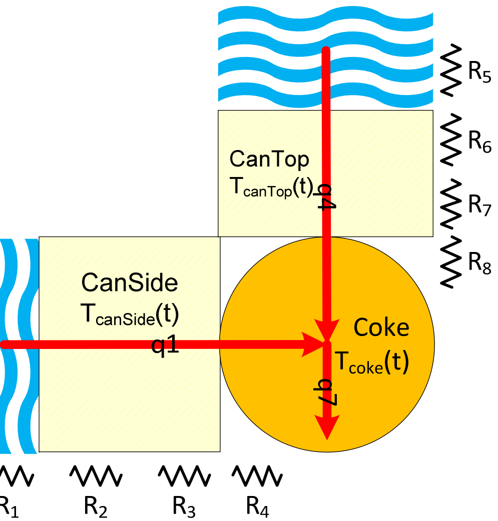
$$R_8 = \frac{1}{h_{water} A_{top}}$$ 0.63

$$\frac{1}{\ldots_e A_{side}}$$ 3.68

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}}$$ $2.6*10^{-5}$

$$R_4 = \frac{1}{h_{water} A_{side}}$$ 0.088

Further:



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$ 31.5

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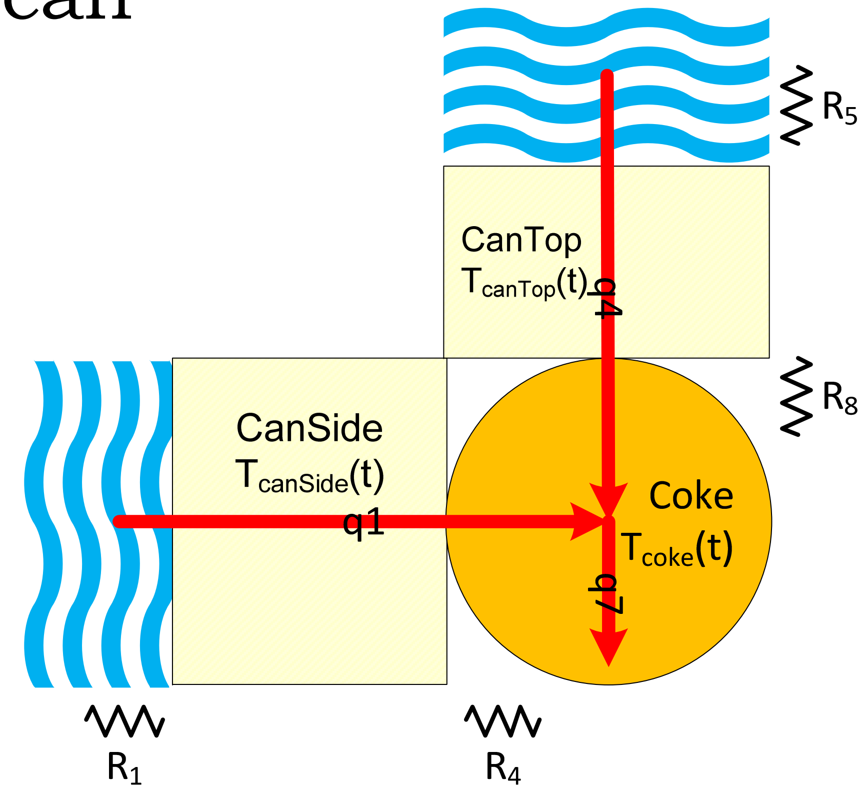
$$\frac{1}{\ldots_{e} A_{side}}$$ 3.68

$$R_2 = R_3 = \frac{L_{can}/2}{K_{can} A_{side}}$$ 2.6*10^-5

$$R_4 = \frac{1}{h_{water} A_{side}}$$ 0.088

Neglect thermal resistance of the can

Neglect the thermal resistances of the can



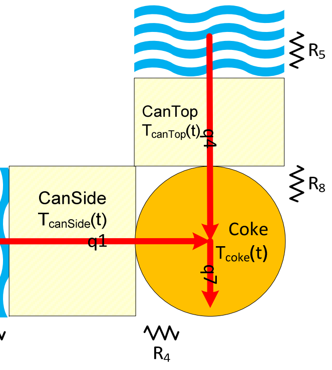
$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_1 = \frac{1}{h_{airSide} A_{side}}$$

$$R_4 = \frac{1}{h_{water} A_{side}}$$

The new model: Case 3



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$$R_4 = \frac{1}{h_{water} A_{side}}$$

Cause

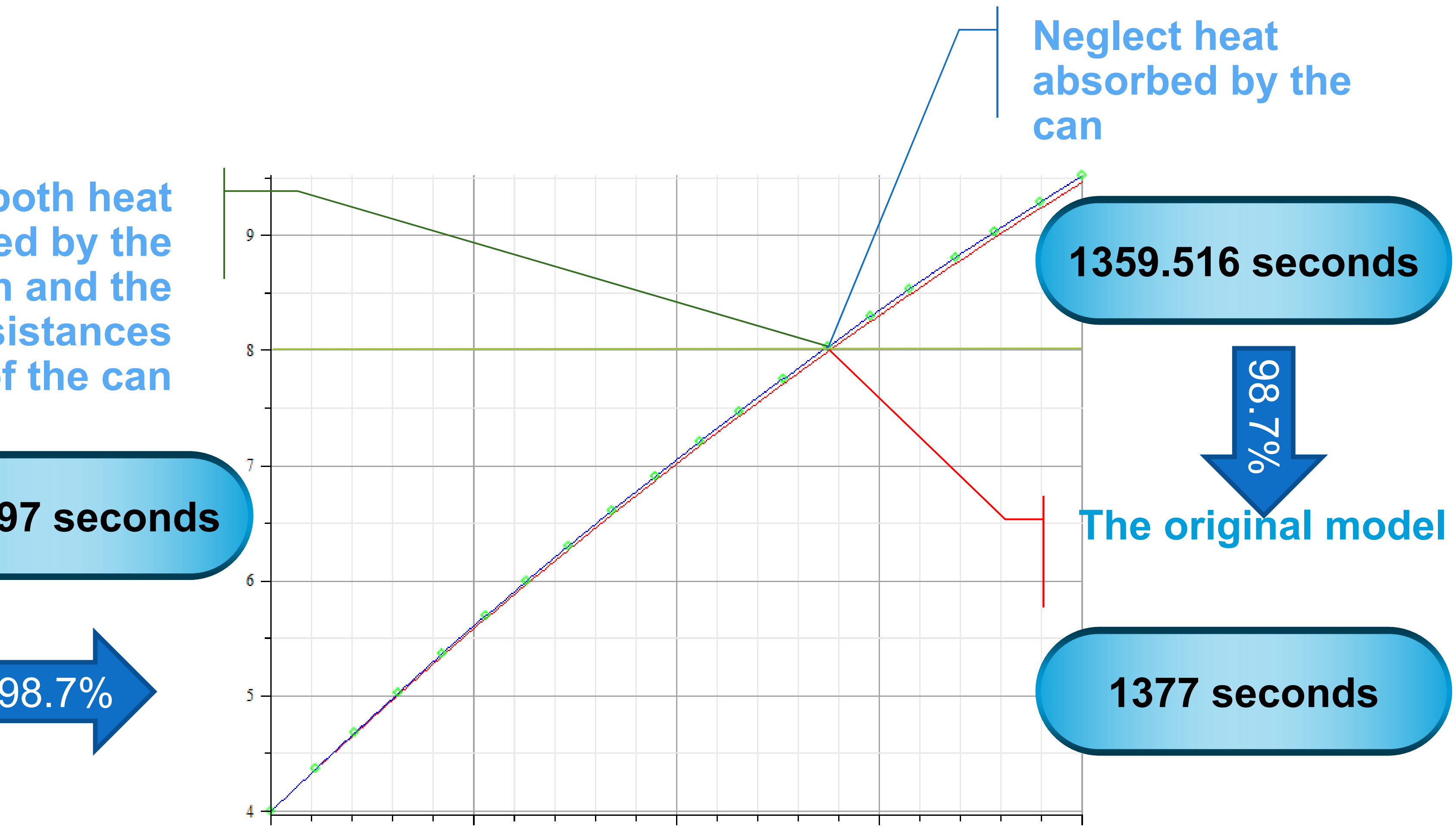
The air(side) temperature is higher than Coke	The air(top) temperature is higher than Coke	The temperature of coke arises

Effect

Heat transferred to coke: q1	Heat transferred to coke: q4	Heat absorbed by coke: q7

$$\frac{T_{air} - T_{Coke}(t)}{R + R} + \frac{T_{air} - T_{Coke}(t)}{R + R} - m_{Coke} C_{Coke} \frac{dT_{Coke}(t)}{dt} = 0$$

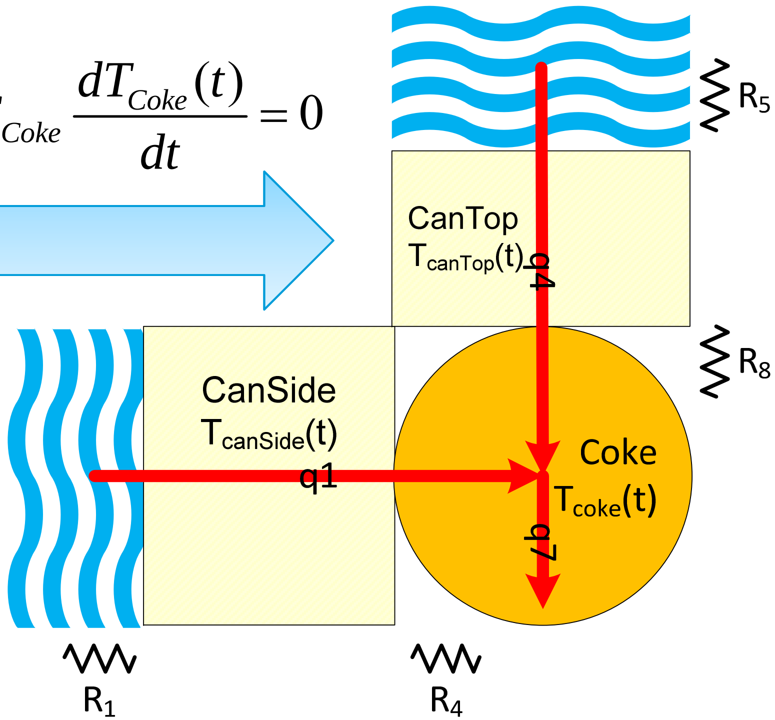
The comparison



A question

Can we solve it by hand?

$$\frac{\ldots T_{Coke}(t)}{R_4} + \frac{T_{air} - T_{Coke}(t)}{R_5 + R_8} - m_{Coke} C_{Coke} \frac{dT_{Coke}(t)}{dt} = 0$$



$$R_5 = \frac{1}{h_{airTop} A_{top}}$$

$$R_8 = \frac{1}{h_{water} A_{top}}$$

$R \ldots \frac{1}{\ldots}$ $R \ldots \frac{1}{\ldots}$

We can solve it by hand

Model

$$\frac{T_{air} - T_{Coke}(t)}{R_1 + R_4} + \frac{T_{air} - T_{Coke}(t)}{R_5 + R_8} - m_{Coke} C_{Coke} \frac{dT_{Coke}(t)}{dt} = 0$$

Simplify

$$C(T_{air} - T_{Coke}(t)) = \frac{dT_{Coke}(t)}{dt}$$

Separation

$$Cdt = \frac{dT_{Coke}(t)}{T_{air} - T_{Coke}(t)}$$

Integration

$$\int Cdt = \int \frac{dT_{Coke}(t)}{T_{air} - T_{Coke}(t)}$$

The result

$$Ct + C1 = -\ln(T_{air} - T_{Coke}(t))$$

Simplify

$$e^{-(Ct+C1)} = T_{air} - T_{Coke}(t)$$

he Heineken beerkeg

ext Week
Tuesday!

Courtesy of www.heineken.com

The Heineken beerkeg

Consider a **5** Liters Heineken® draft beer keg:

Question: *Quickly estimate* the time needed to cool the draft beer to **5**ºC when it is put in a refrigerator.

We know:

1. the keg is made of steel, the specific heat capacity of the steel is **460** J/(kg·K), the thermal conductivity of the steel is **43** W/(m·K), the thickness of the keg is **0.1** mm and the mass of the keg is **130.5** g. The pressure inside the keg is **2** bar;
2. the keg has a cylindrical shape and is filled with beer. The diameter of the keg is **16** cm and the height is **25** cm;
3. most of the bottom part is in contact with the air.
4. the density of the beer is **1060** kg/m^3, the specific heat of the beer is **4181** J/(kg·K), the initial temperature is **20**°C,
5. the temperature inside the refrigerator is **4**°C;
6. the heat transfer coefficient between the beer and the keg is **500** W/(m^2·K);
7. the heat transfer coefficient between the air and the keg is

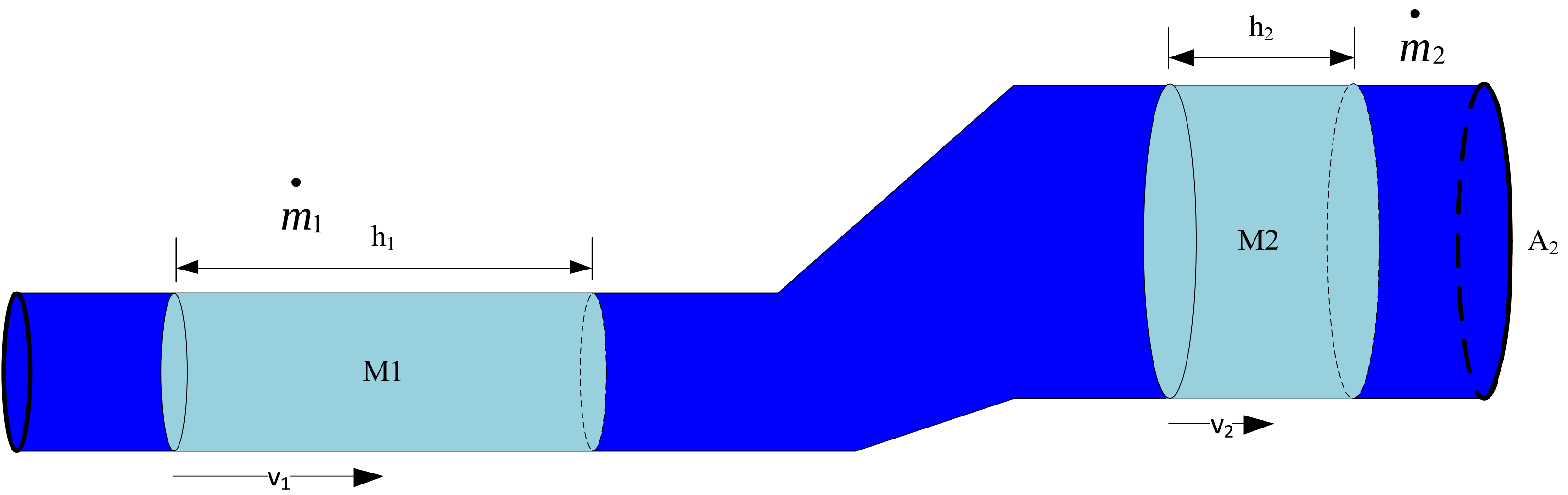
Case study: The bath tub

Design the drain of a bathtub

Specify the radius of 6 holes in the drain to make sure that the bathtub can be emptied within 500 seconds.

Physics behind - CoM

Conservation of Mass	The mass of a closed system (in the sense of a completely isolated system) will remain constant over time



$$\dot{m}_1 - \dot{m}_2 = 0 \rightarrow \rho A_1 v_1 t - \rho A_2 v_2 t = 0 \rightarrow A_1 v_1 - A_2 v_2 = 0$$

Conservation of mass

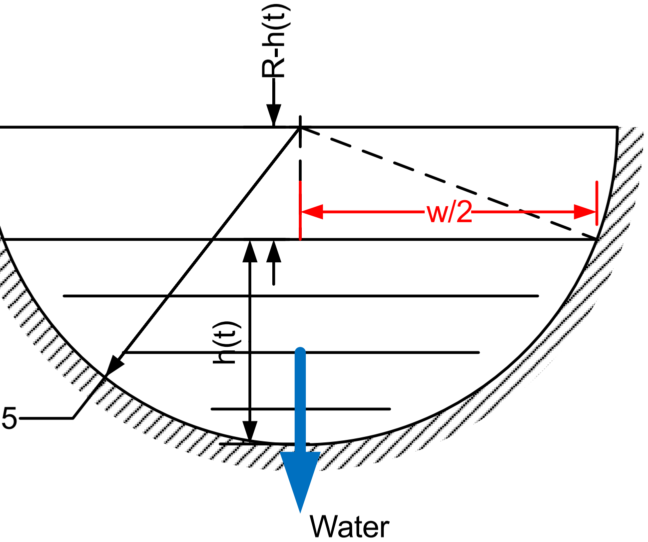
$$\dot{m}_{bathtub} = \dot{m}_{orifice}$$

The area of water in the bathtub

Why "-"

$$-\rho_{water} \cdot A_{water} \cdot \frac{dh(t)}{dt} = \dot{m}_{bathtub}$$

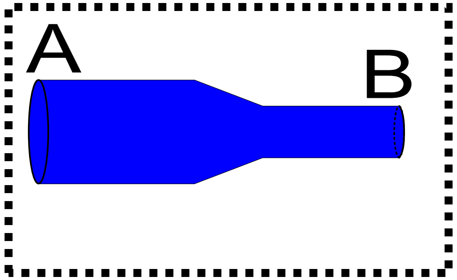
$$A_{water} = Length_{bathtub} \cdot w$$



$$w = 2 \cdot \sqrt{R^2 - (R - h(t))^2}$$

$$\ldots Length \ldots 2 \cdot \sqrt{R^2 - (R - h(t))^2} \, \frac{dh(t)}{dt} = \dot{m}$$

Physics behind - CoE



Bernoulli's Equation

Kinetic Energy

$$\frac{p_a}{\rho} + gh_a + \frac{v_a^2}{2} = \frac{p_b}{\rho} + gh_b + \frac{v_b^2}{2} = \text{Constant}$$

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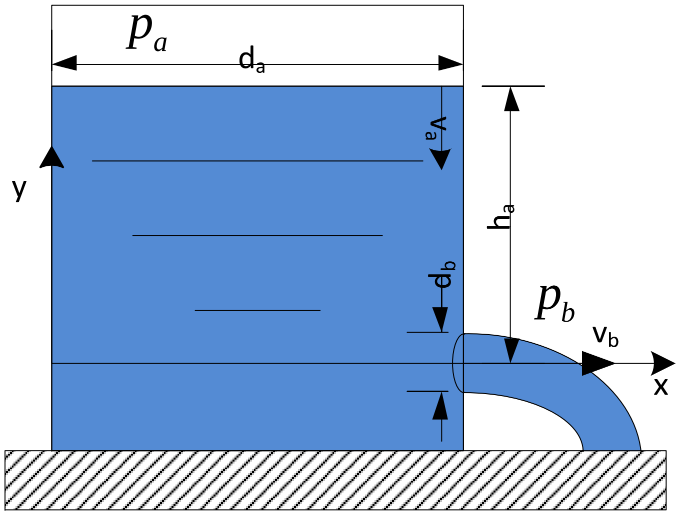
ork

Potential Energy

Physics behind – Torricelli's law



$$\frac{p_a}{\rho} + gh_a + \frac{v_a^2}{2} = \frac{p_b}{\rho} + gh_b + \frac{v_b^2}{2}$$

$$p_a = p_b$$

$$h_b = 0$$

if $d_a >> d_b$,

v_a can be neglected in the energy equation

Torricelli's law(http://en.wikipedia.org/wiki/Torricelli's_law)

$$v_b = \sqrt{2gh_a}$$

Mass flow rate at the orifice

$$\dot{m}_{orifice} = n_{orifices} \cdot \underbrace{C_d}_{\text{Discharge coefficient}} \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)}$$

Solving

$$= -\rho_{water} \cdot Length_{bathtub} \cdot 2 \cdot \sqrt{R^2 - (R - h(t))^2} \cdot \frac{dh(t)}{dt}$$

$$= n_{orifices} \cdot C_d \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)}$$

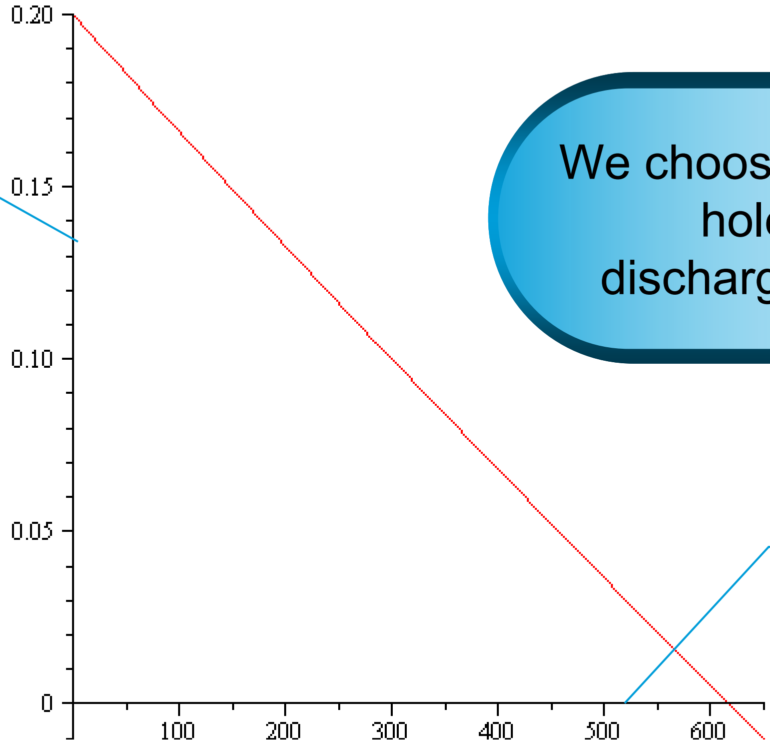
Conservation of Mass

$$\dot{m}_{bathtub} = \dot{m}_{orifice}$$

$$\cdot Length_{bathtub} \cdot 2 \cdot \sqrt{R^2 - (R - h(t))^2} \cdot \frac{dh(t)}{d} = n_{orifices} \cdot C_d \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)}$$

Solving

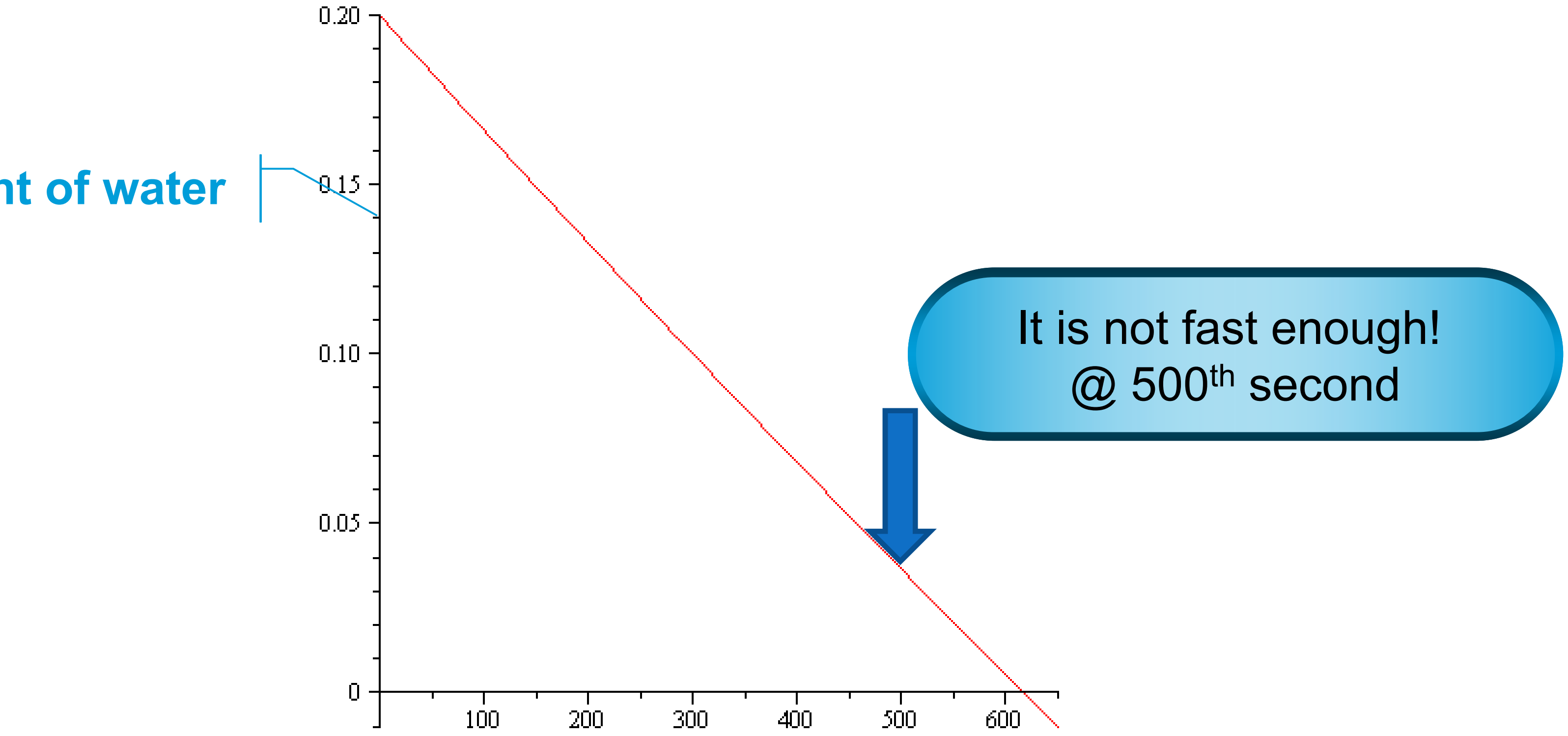
$$\ldots_{ater} \cdot Length_{bathtub} \cdot 2 \cdot \sqrt{R^2 - (R - h(t))^2} \cdot \frac{dh(t)}{dt} = n_{orifices} \cdot C_d \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)}$$



We choose the radius of each hole is 3mm and discharge coefficient is 0.8

Evaluation

$$\ldots_{water} \cdot Length_{bathtub} \cdot 2 \cdot \sqrt{R^2 - (R - h(t))^2} \cdot \frac{dh(t)}{dt} = n_{orifices} \cdot C_d \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)}$$

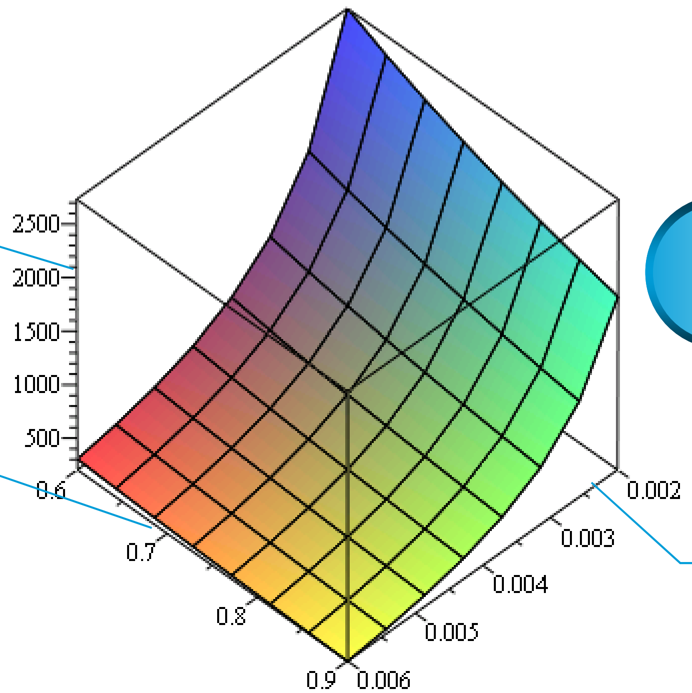


Evaluation

$$_{ater} \cdot Length_{bathtub} \cdot 2 \cdot \sqrt{R^2 - (R - h(t))^2} \cdot \frac{dh(t)}{dt} = n_{orifices} \cdot C_d \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)}$$

me for empty
the bathtub

Discharge
coefficient

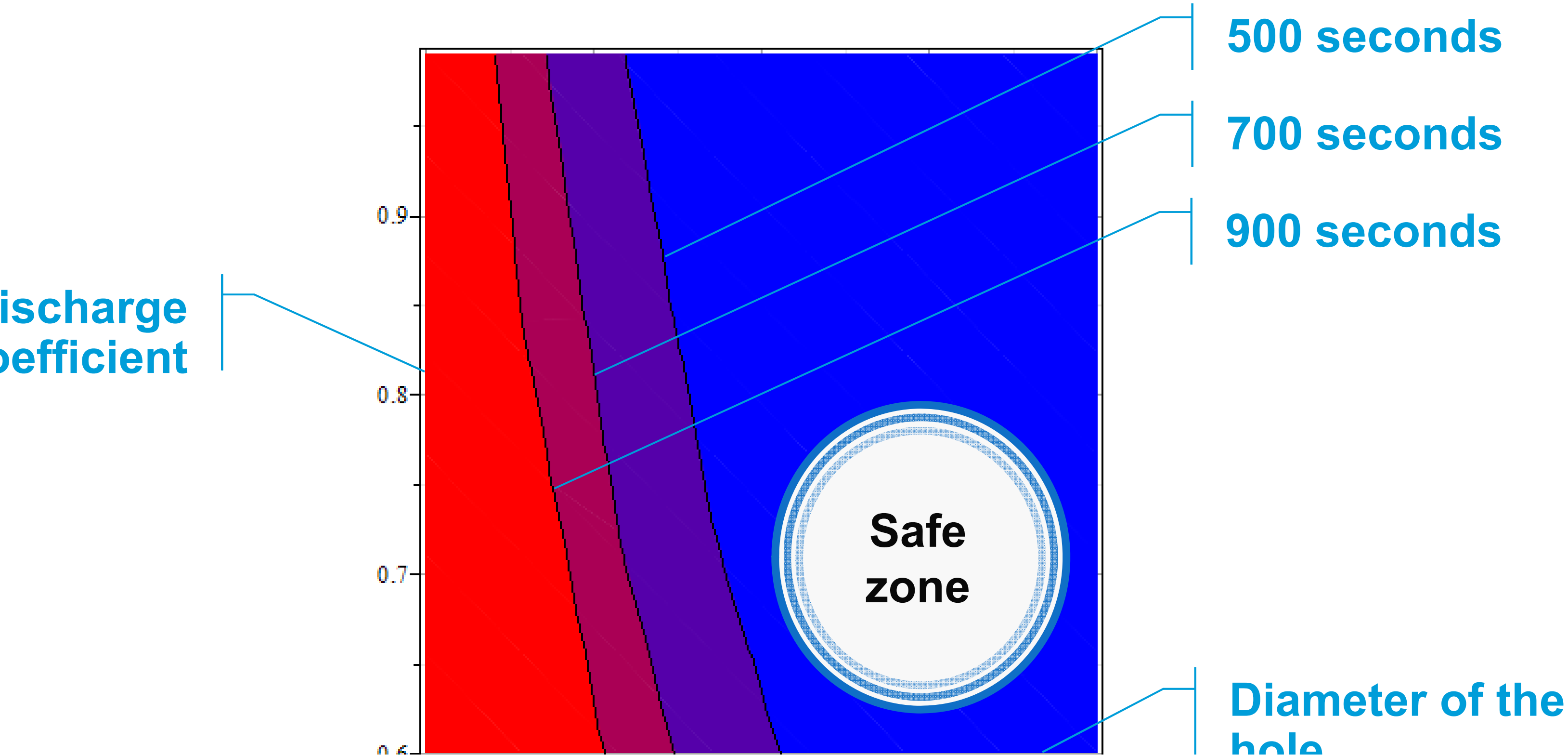


Which parameter is
more effective ?

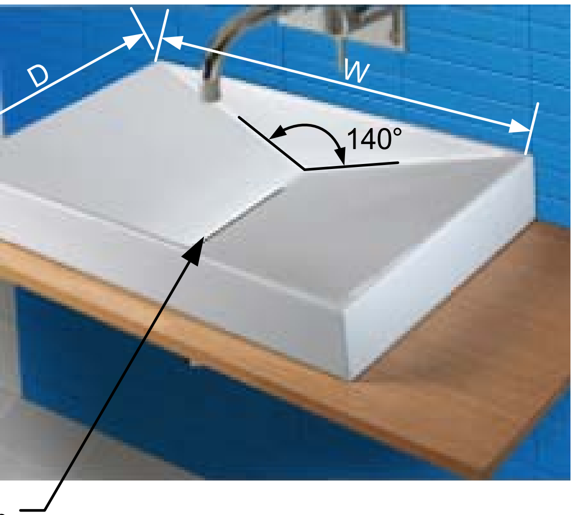
Diameter of the
hole

Evaluation

$$ {}_{er} \cdot Length_{bathtub} \cdot 2 \cdot \sqrt{R^2 - (R - h(t))^2} \cdot \frac{dh(t)}{dt} = -n_{orifices} \cdot C_d \cdot \rho_{water} \cdot A_{orifices} \cdot \sqrt{2 \cdot g \cdot h(t)} $$



Case study: The V-Shaped wash basin



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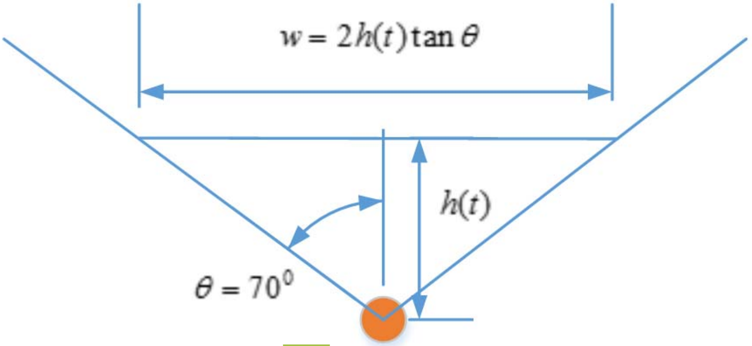
The V-Shaped wash basin

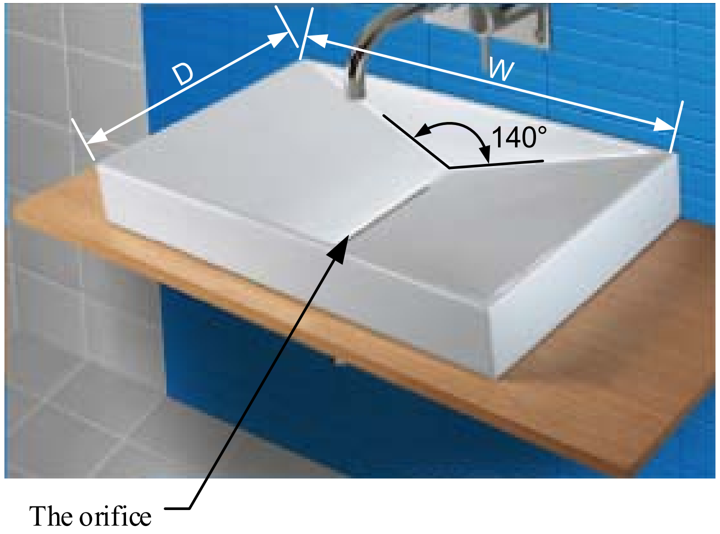
Open the stopper located at the orifice in a V-shaped wash basin and let the water go out:

Questions:
Considering the information below:
Predict the remaining height of water **5** seconds after you open the stopper;

We know:
1. the shape of the basin is a V shape as the figure and the angle is **140°**;
2. the depth **D** of the of wash basin is **0.5**m and the width **W** is **0.85**m, respectively.
3. the initial height of water in the basin is **0.1** m;
4. the area of the orifice is **0.0015** m^2;
5. the discharge coefficient is **0.6** for the orifice;

Case study: The V-Shaped wash basin

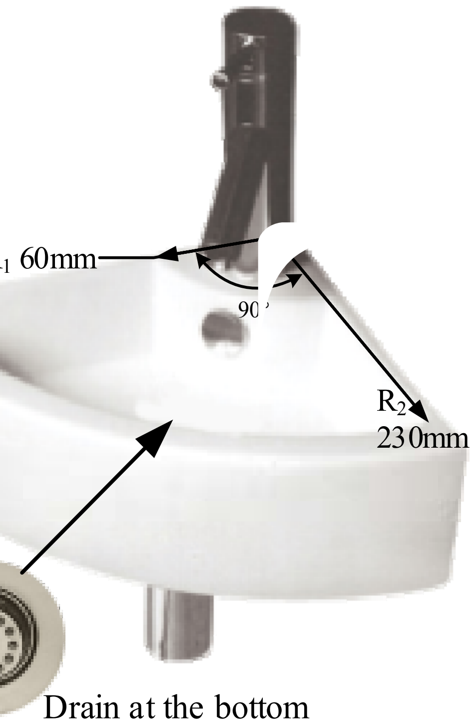




$$\dot{m}_{washbasin} = \dot{m}_{orifice}$$

$$-\rho_{water} \cdot D \cdot \left(2 \cdot h(t) \cdot \tan\theta\right) \cdot \frac{dh(t)}{dt} = C_d \cdot \rho_{water} \cdot A_{orifice} \cdot \sqrt{2 \cdot g \cdot h(t)}$$

Case study: Wash basin



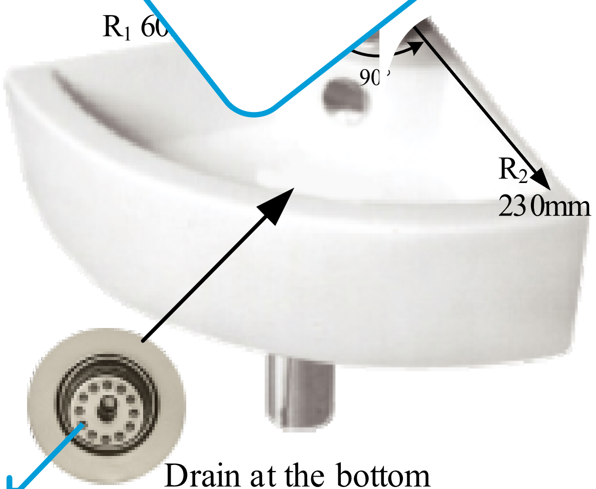
Design the drain of a wash basin

Remove the stopper of the drain in a corner wash basin to let the water go out of the basin.

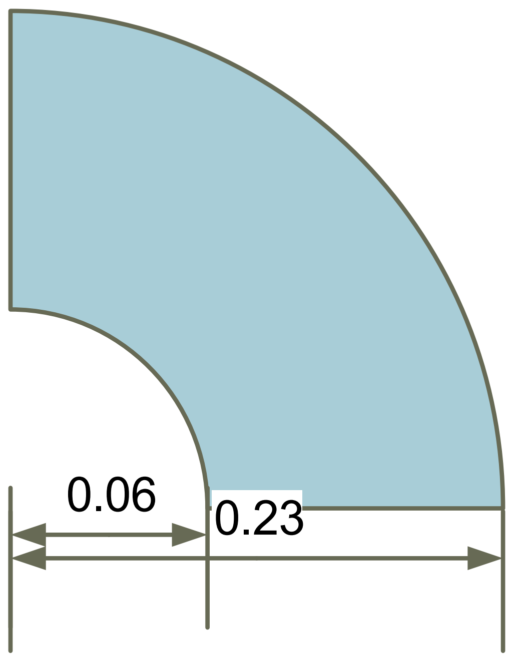
Question A: Predict the remaining height of water after **5** seconds draining;

Question B: Manufacturing errors lead to systematic variation of the diameter of holes in the drain. This will affect the flow rate. Evaluate the sensitivity of the height of the water at **5th** second with respect to the varying **diameter** of holes in the orifice.

Case study: Wash basin



Can we solve it by hand?



$$\dot{m}_{washbasin} = \dot{m}_{orifice}$$

$$-\rho \cdot \frac{1}{\;} \cdot \pi \cdot \left(R2^2 - R1^2\right) \cdot \frac{dh(t)}{\;} = C \cdot \rho \cdot 12 \cdot \pi \cdot R^2 \cdot \sqrt{2 \cdot g \cdot h(t)}$$

Case study: Wash basin

Can you continue?

$$\frac{dh(t)}{dt} = \frac{C_d \cdot 12 \cdot R^2 \cdot \sqrt{2 \cdot g \cdot h(t)}}{-\frac{1}{4} \cdot \left(R2^2 - R1^2\right)}$$

2

$$\frac{dh(t)}{\sqrt{h(t)}} = \frac{C_d \cdot 12 \cdot R^2 \cdot \sqrt{2 \cdot g}}{-\frac{1}{4} \cdot \left(R2^2 - R1^2\right)} dt$$

3

$$\int \frac{dh(t)}{\sqrt{h(t)}} = \int \frac{C_d \cdot 12 \cdot R^2 \cdot \sqrt{2 \cdot g}}{-\frac{1}{4} \cdot \left(R1^2 - R2^2\right)} dt$$

4

Case study: Wash basin

$$= 0.004(R = D/2), t = 5) = (-\frac{C_d \cdot 24 \cdot R^2 \cdot \sqrt{2 \cdot g}}{\left(R1^2 - R2^2\right)} \cdots t + 0.25)^2 = 0.05024$$

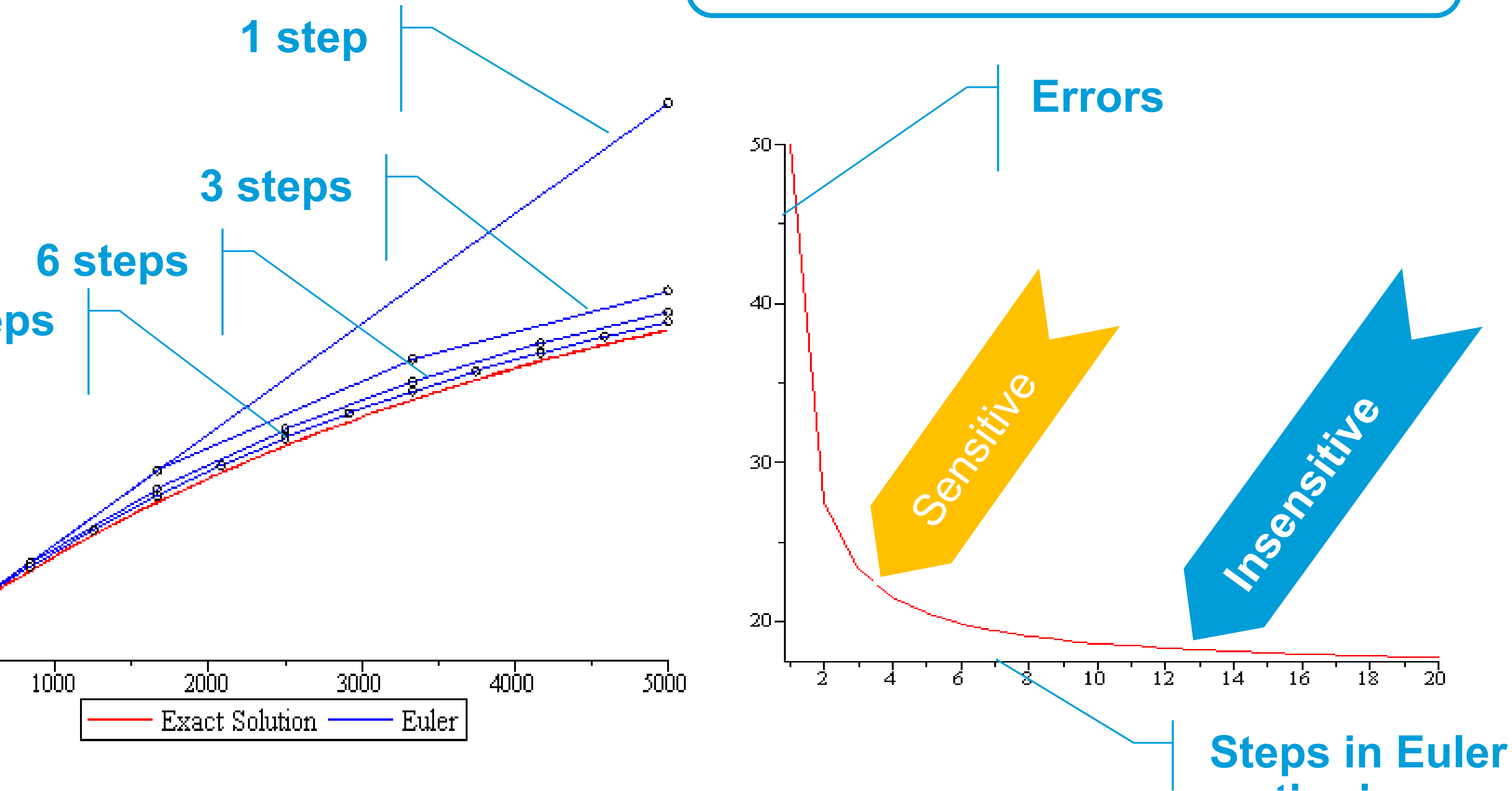
$$= 0.004 * 1.01(R = D/2), t = 5) = (-\frac{C_d \cdot 2 \cdots}{\left(R1^2 - R2^2\right)} \cdots$$

$$Sensitivity = \frac{h(D = 0.004 * 1.01, t = 5) - h(D = 0.004, t = \cdots)}{0.004 * 0.01} = -6$$

What is the meaning of this one point sensitivity?

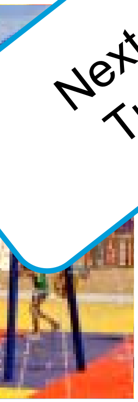
Sensitivity analysis in simulation

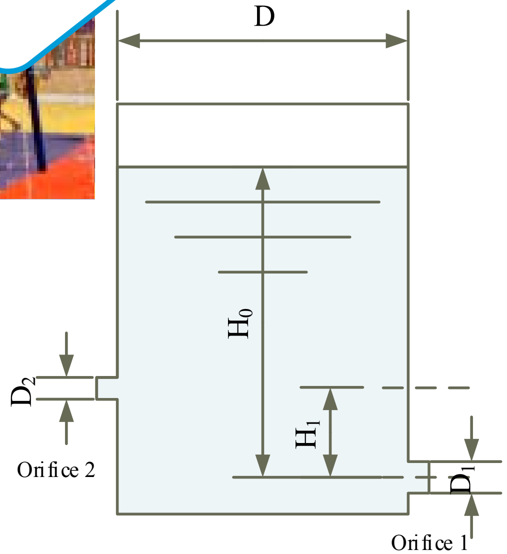
In reality, we need at least 3 points



Case study: The Splash challenger

Next Week Tuesday!





The Splash challenger

With a big water tank, the splash challenger is able to spray water through several orifices to amuse children in the playground.

Questions:
Considering a two-orifice splash challenger as shown in the sketch:
A. Predict the remaining height of water **60** seconds after the start of the play;
B. Manufacturing errors lead to systematic variation of the diameters of orifices. This will affect the flow rate. Evaluate the sensitivity of the height of the water at **60th** second with respect to the varying **diameter** of **orifice 1**.

We know:
1. the shape of the tank is cylindrical and its diameter (**D**) is **1** meter;
2. the height difference (**H_1**) between the two orifices is **30** cm;
3. the initial height of water (**H_0**) in the tank is **1** m regarding **orifice 1**;
4. the shapes of both orifices are circular; the diameter of **orifice 1** (**D_1**) is **3** cm and the diameter of **orifice 2** (**D_2**) is **2** cm; the discharge

This afternoon

The Unicycle

A unicycle is a human-powered, single-track vehicle with one wheel. Unicycles resemble bicycles, but are less complex.

In a market investigation, it was found that two groups of children often use unicycles.
They are:

Group A: Age 6~7 and
Group B: Age 7~8;

Observation research indicates that among those children, **75%** are **Group A** children and **25%** are **Group B** children.

In designing a unicycle, defining the damping ratio is crucial regarding the comfort of the rider. The damping ratio of a unicycle can be specified as:

$$\zeta = \frac{c}{2\sqrt{mk}}$$

where **m** is the sum of the mass of the unicycle and the mass of the rider, **k** is the spring constant of the unicycle and **c** is the damping coefficient of the unicycle .

This afternoon

The Unicycle

Question:
Find the optimal spring constant **k** and the optimal damping coefficient **c** of the unicycle to satisfy **as many children as possible** based on the following wish:

Professor Jørgen Winkel, a senior ergonomics scientist, concluded that: "*For **both** groups of children, the optimal damping ratio of a unicycle is about **0.3***".

In the optimization, we choose:
1. the mass of the unicycle to be **5** kg;
2. the average mass of **Group A** children to be **20** kg;
3. the average mass of **Group B** children to be **31** kg;
4. to use gradient descent method in the optimization;
5. to use the values of the gradient directly in the gradient descent method (**NOT** the unit vector of the gradient);
6. the initial guess of **k** to be **14463** (N/m) and **c** to be **1627** (N·s/m);
7. one step only in the optimization and the step size is $\mathbf{1 \cdot 10^5}$.

We think…

curiosity
is fun

knowledge
is power

science
is easy

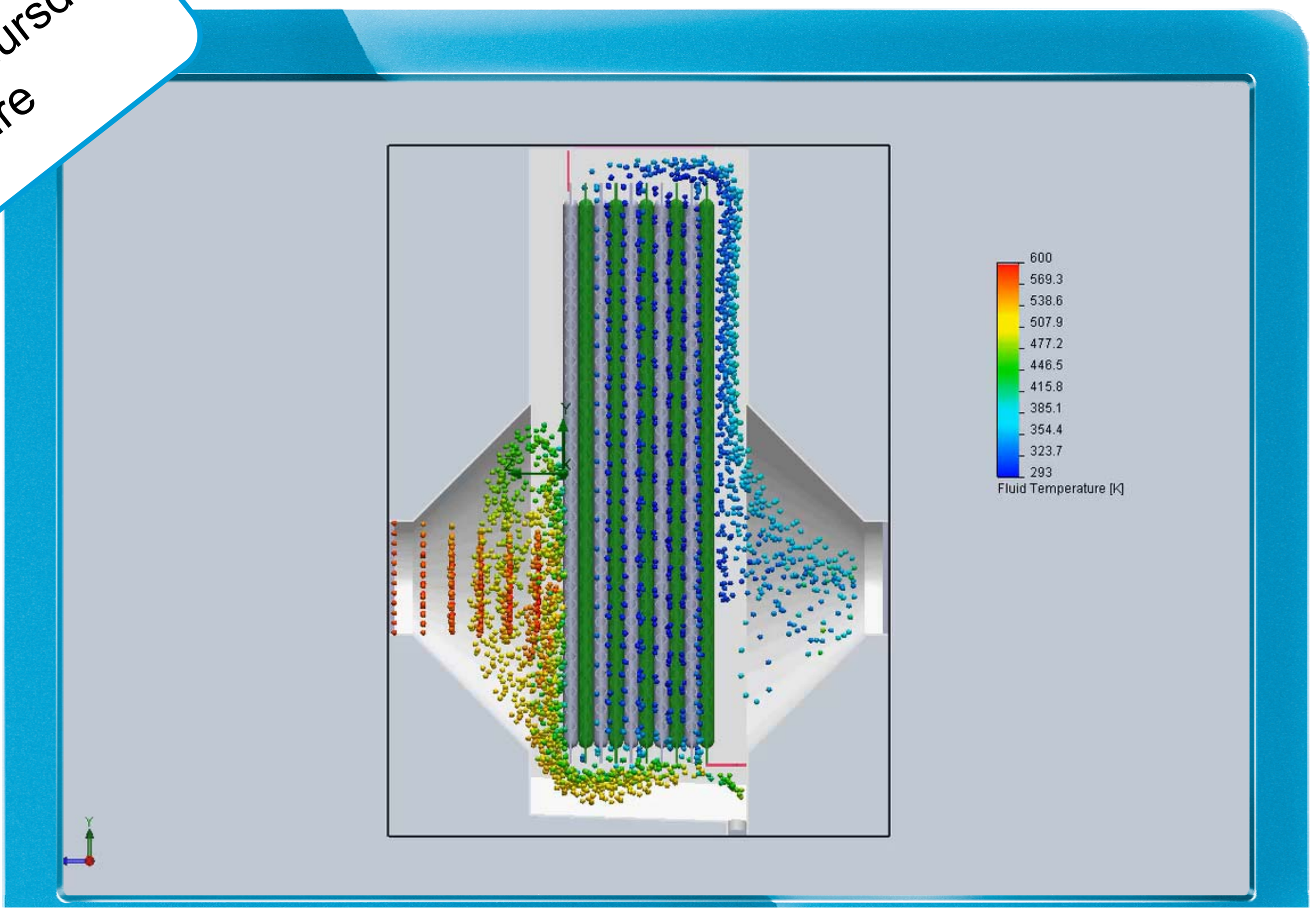
computers
are tools

experience
can be harnessed

hard work
is the way to
success

My problem is complicated?

week Thursday
lecture



hank You!

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