

Design
brief

A unicycle is a human-powered, single-track vehicle with one wheel. Unicycles resemble bicycles, but are less complex.

In a market investigation, it was found that two groups of children often use unicycles. They are:

Group A: Age 6~7 and

Group B: Age 7~8;

Observation research indicates that among those children, **75%** are **Group A** children and **25%** are **Group B** children.

In designing a unicycle, defining the damping ratio is crucial regarding the comfort of the rider. The damping ratio of a unicycle can be specified as:

$$\zeta = \frac{c}{2\sqrt{mk}}$$

where **m** is the sum of the mass of the unicycle and the mass of the rider, **k** is the spring constant of the unicycle and **c** is the damping coefficient of the unicycle .

Question:

Find the optimal spring constant **k** and the optimal damping coefficient **c** of the unicycle to satisfy **as many children as possible** based on the following wish:

*Professor Jørgen Winkel, a senior ergonomics scientist, concluded that: “For **both** groups of children, the optimal damping ratio of a unicycle is about **0.3**”.*

In the optimization, we choose:

1. the mass of the unicycle to be **5 kg**;
2. the average mass of **Group A** children to be **20 kg**;
3. the average mass of **Group B** children to be **31 kg**;
4. to use gradient descent method in the optimization;
5. to use the values of the gradient directly in the gradient descent method (**NOT** the unit vector of the gradient);
6. the initial guess of **k** to be **14463 (N/m)** and **c** to be **1627 (N·s/m)**;
7. one step only in the optimization and the step size is **1·10⁵**.

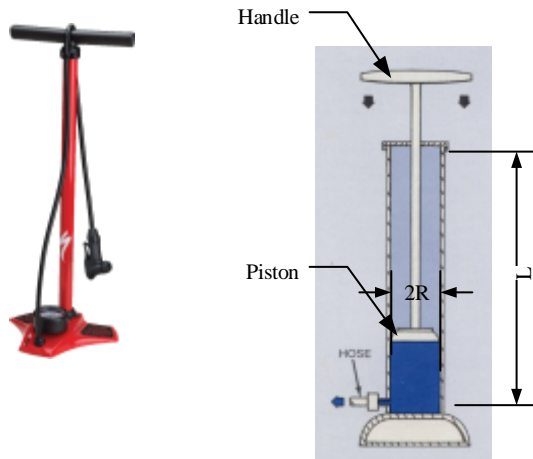


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Question	Question: Find the optimal initial temperature and the optimal amount of steam that should be produced in order to preheat either of the two types of cups as close as possible to 92°C. (based on workshop 10 questions) We choose: a. The initial guess is 3 g water and 110 °C, the step length is 1.3·10 ⁻⁹ ; b. 1 step ONLY in the optimization.	
Establish the objective function	Weight = 1 for every metric $Obj(m_{steam} = T_{steam\ initial}) = (T_{Hfinal}(m_{steam} = T_{steam\ initial}) - 92)^2 + (T_{Sfinal}(m_{steam} = T_{steam\ initial}) - 92)^2$	
the objective function	$Obj(m_{steam} = T_{steam\ initial}) = \left(\frac{8320m_{steam}T_{steam\ initial} + 9.8804 \cdot 10^6 m_{steam} + 15000}{16724m_{steam} + 750} - 92 \right)^2 + \left(\frac{8320m_{steam}T_{steam\ initial} + 9.8804 \cdot 10^6 m_{steam} + 13910}{16724m_{steam} + 695.5} - 92 \right)^2$	
The guess	The guess $m_{steam} = 0.003, T_{steam\ initial} = 110$	At this condition, for Hollandsche $T_{Hfinal}(m_{steam} = T_{steam\ initial}) _{0.003, 110} = 59.22^\circ C$ At this condition, for Standard $T_{Sfinal}(m_{steam} = T_{steam\ initial}) _{0.003, 110} = 62.08^\circ C$



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The objective function (to be minimized)

$$Obj(R, L) = (F - 300)^2 + (n - 0)^2 = (2 \cdot \pi \cdot 10^5 \cdot R^2 - 300)^2 + \left(\frac{6.6 \cdot 10^{-3}}{\pi R^2 L} \right)^2$$

Question

A bicycle pump functions via a hand-operated piston. During the up-stroke, this piston draws air through a one-way valve into the pump from the outside. During the down-stroke, the piston then displaces the air from the pump into the bicycle tyre.

Question:

Find the optimal radius R of the piston and the travel length L of the pump to satisfy the following wishes:

1. Kathleen Vandenbranden, a senior expert in ergonomics, said: "For pumping a bike, the maximum force should be around 300 N from ergonomics point of view";
2. Mark Broekhuis, a student cycling to university everyday, said: "I want to pump my tyres as quick as possible, e.g., with the least amount of full strokes.";

We choose:

1. the pump has a cylindrical shape;
2. the temperature of the pump, air and the tyre are constant during the process;
3. the tyre is torus-shaped, the volume of the tyre is 2.2 Liter and it should be pumped to 2 bar (relative pressure) over the outside pressure, which is 1 bar (100,000 Pa);
4. the tyre is flat in the beginning, e.g., there is NO air inside the tyre in the beginning;
5. to ignore all frictions and resistances;
6. to use gradient descent method in the optimization;
7. the initial guess of R is **0.016** meter and L is **0.45** meter;
8. one step only in the optimization and the step size is $1 \cdot 10^{-10}$.