Workshop G-W-5: The beerkeg

Attention: Fictional case study, for education only

Question

Consider a **5** Liters Heineken® draft beer keg:

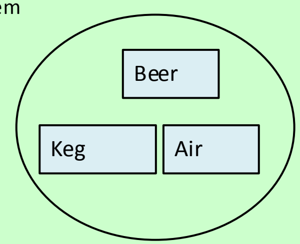
Question: *Quickly estimate* the time needed to cool the draft beer to **5**ºC when it is put in a refrigerator.

We know:

1. the keg is made of steel, the specific heat capacity of the steel is **460** J/(kg·K), the thermal conductivity of the steel is **43** W/(m·K), the thickness of the keg is **0.1** mm and the mass of the keg is **130.5** g. The pressure inside the keg is **2** bar;
2. the keg has a cylindrical shape and is filled with beer. The diameter of the keg is **16** cm and the height is **25** cm;
3. most of the bottom part is in contact with the air.
4. the density of the beer is **1060** kg/m^3, the specific heat of the beer is **4181** J/(kg·K), the initial temperature is **20**°C,
5. the temperature inside the refrigerator is **4**°C;
6. the heat transfer coefficient between the beer and the keg is **500** W/(m^2·K);
7. the heat transfer coefficient between the air and the keg is **10** W/(m^2·K)

Courtesy of www.heineken.com

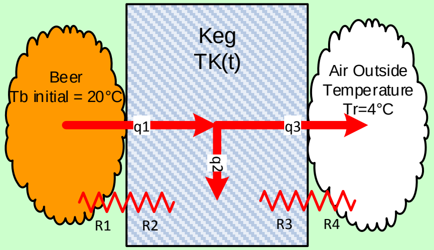
System



For a quick estimation, we can neglect q2, since

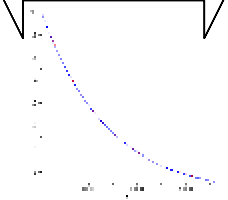
$$mbeer \cdot cbeer = 5.3 \cdot 4181 = 22159.3$$
$$mkeg \cdot ckeg = 0.1305 \cdot 460 = 60.03$$
$$mbeer \cdot cbeer >> mkeg \cdot ckeg$$

Sketch 1: The full model

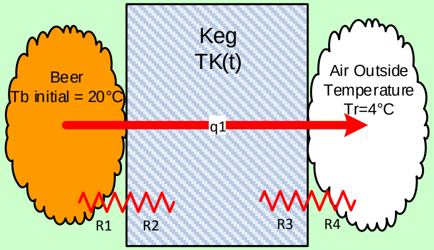


$$R_1 = \frac{1}{h_{in}A} \quad R_2 = R_3 = \frac{L/2}{KA} \quad R_4 = \frac{1}{h_{out}A}$$

Result compare:
full model – red line
Simplified model – blue line
Nearly same



Sketch 2: The new model



$$R_1 = \frac{1}{h_{in}A} \quad R_2 = R_3 = \frac{L/2}{KA} \quad R_4 = \frac{1}{h_{out}A}$$

The model

$$\frac{Tb(t) - Tr}{R_1 + R_2 + R_3 + R_4} = -mbeer \cdot cbeer \cdot \frac{dTb(t)}{dt}$$

Separation of variales

$$R = R_1 + R_2 + R_3 + R_4$$
$$\frac{Tb(t) - Tr}{R} = -mbeer \cdot cbeer \cdot \frac{dTb(t)}{dt}$$

$$\frac{Tb(t) - Tr}{-mbeer \cdot cbeer \cdot R} = \frac{dTb(t)}{dt}$$

$$\frac{dt}{-mbeer \cdot cbeer \cdot R} = \frac{dTb(t)}{Tb(t) - Tr}$$

$$\frac{t}{-mbeer \cdot cbeer \cdot R} + C = \ln(Tb(t) - Tr)$$

$$e^{\frac{t}{-mbeer \cdot cbeer \cdot R} + C} = Tb(t) - Tr$$

Initial condition
$$Tb(0) = 20$$

$$Tb(t) = 4 + 16e^{-0.00007334955107t}$$

How long it will take to cool the beer to 5 degree
$$5 = 4 + 16e^{-0.00007334955107t}$$

t = 37799.668 second
= 10.5 hours

Workshop G-W-5: The Splash Challenger

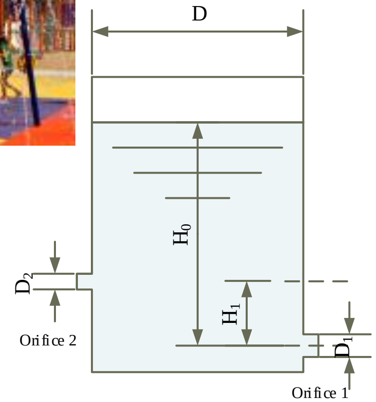
Question

With a big water tank, the splash challenger is able to spray water through several orifices to amuse children in the playground.

Questions:
Considering a two-orifice splash challenger as shown in the sketch:
A. Predict the remaining height of water 60 seconds after the start of the play;
B. Manufacturing errors lead to systematic variation of the diameters of orifices. This will affect the flow rate. Evaluate the sensitivity of the height of the water at 60th second with respect to the varying diameter of orifice 1.

We know:
1. the shape of the tank is cylindrical and its diameter (D) is 1 meter;
2. the height difference (H1) between the two orifices is 30 cm;
3. the initial height of water (H0) in the tank is 1 m regarding orifice 1;
4. the shapes of both orifices are circular; the diameter of orifice 1 (D1) is 3 cm and the diameter of orifice 2 (D2) is 2 cm; the discharge coefficient is 0.6 for both orifices.

We choose:
1. to use the Euler method to solve the questions (2 steps).



The model

Mode 1 $h(t)>H_1$

$$-\rho\pi\left(\frac{D}{2}\right)^2\frac{dh(t)}{dt}=\rho\pi\left(\frac{D_1}{2}\right)^2 c_d\sqrt{2gh(t)}+\rho\pi\left(\frac{D_2}{2}\right)^2 c_d\sqrt{2g(h(t)-H_1)}$$

Model 2 $0\le h(t)\le H_1$

$$-\rho\pi\left(\frac{D}{2}\right)^2\frac{dh(t)}{dt}=\rho\pi\left(\frac{D_1}{2}\right)^2 c_d\sqrt{2gh(t)}$$

Euler method get the water height at 60th second, using model 1

$$\frac{dh(t)}{dt}=-\frac{c_d\sqrt{2g}}{D^2}\left(D_1^2\sqrt{h(t)}+D_2^2\sqrt{h(t)-H_1}\right)=-2.656*10^{-4}(9*\sqrt{h(t)}+4\sqrt{h(t)-0.3})$$

$$h(30)=h(0)+30\cdot\left.\frac{dh(t)}{dt}\right|_{t=0}=1+30\cdot 2.656\cdot 10^{-4}\cdot(9\cdot\sqrt{1}+4\cdot\sqrt{0.7})=0.9016$$

$$h(60)=h(30)+30\cdot\left.\frac{dh(t)}{dt}\right|_{t=30}=0.9016+30\cdot 2.656\cdot 10^{-4}\cdot(9\sqrt{0.9016}+4\cdot\sqrt{0.6016})=0.80862$$

Water level higher than 0.3, thus model 1 works

Make diameter 1% larger as 0.0303

Euler method get the water height at 60th second, using model 1

$$\frac{dh(t)}{dt}=-2.656*10^{-4}(9.1809*\sqrt{h(t)}+4\sqrt{h(t)-0.3})$$

$$h(30)=h(0)+30\cdot\left.\frac{dh(t)}{dt}\right|_{t=0}$$

$$=1+30\cdot 2.656\cdot 10^{-4}\cdot(9.1809\cdot\sqrt{1}+4\cdot\sqrt{0.7})=0.9002$$

$$h(60)=h(30)+30\cdot\left.\frac{dh(t)}{dt}\right|_{t=30}$$

$$=0.9002+30\cdot 10^{-4}\cdot 2.656(9.1809\sqrt{0.9002}+4\cdot\sqrt{0.6002})=0.8061$$

Sensitivity (a more accurate answer should be -9, depending on the digits)

$$senstivity=\frac{h1(60)-h(60)}{0.0303-0.03}=\frac{(0.8061-0.80862)}{0.0003}=-8$$

Question

A **1.5** liter bottle of Spa mineral water was taken out from the refrigerator for **half hour**.

Question:
Make a *quick estimation* of the temperature of the water at this moment based on the following *information*:

1. The bottle is made of PolyEthylene Terephthalate (PET), the specific heat capacity of PET is **1200** J/(kg·K), the thermal conductivity of PET is **0.15** W/(m·K), the thickness of the bottle is **0.1** mm and the weight of the bottle is **12** g;
2. The bottle is filled with mineral water;
3. The contact area between the mineral water and the bottle and the contact area between the bottle and the air are approximately the same, equal to **0.08** m^2.
4. The mineral water density is **1000** kg/m^3, the specific heat of the mineral water is **4181** J/(kg·K), the initial temperature is **3**°C;
5. The environment (air) temperature is **25**°C;
6. The heat transfer coefficient between the mineral water and the bottle is **500** $W/(m^2 \cdot K)$;
7. The heat transfer coefficient between air and the bottle is **50** $W/(m^2 \cdot K)$.

Courtesy of www.spa.com

Answer

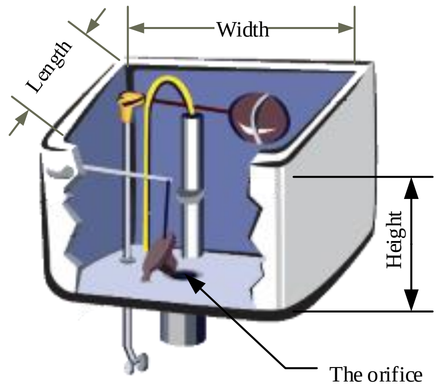
Referring to the beerkeg. The estimated temperature is 17ºC

Question

Consider flushing a toilet:

Questions:
1. How much water is used in the **first second** of flushing?
2. In time, gradually built-up of dirt will cause the orifice to become smaller. This will affect the flow rate. Evaluate the sensitivity of the quantity of the water flushed in the first second to the change of the radius.

We choose:
1. The shape of the water tank is a box, the inner width is **0.26** m, the length is **0.16** m and the initial water height is **0.28** m;
2. The shape of the orifice is a circle, the radius is **3** cm, the discharge coefficient is **0.6**;
3. To use the Euler method to solve the question (**3 steps**).



Answer

Referring to the bathtub. Water flushed in the first second is about 3.7kg, sensitivity of the radius is about 208 (it may vary a little bit, depending on your steps)