5.
MODELING OF NON-STRATIFIED MIXTURE FLOWS (Pseudo-homogeneous flows)

Uniform (or almost uniform) distribution of transported solids across a pipeline cross section is characteristic of pseudo-homogeneous mixture flow. This flow occurs if the settling tendency of particles transported in a flowing liquid is weak in comparison with the tendency of a carrying liquid to keep particles suspended. Very fine particles are practically non-settling and the pseudo-homogeneous character of the mixture flow is maintained at all operational velocities in a pipeline. Coarser particles (as fine and medium sand) may form a fully-suspended mixture if the intensity of turbulence in a flow of a carrying liquid does not allow solid particles to settle. This is the case at high operational velocities in a pipeline.

The pseudo-homogeneous flow composed of very fine particles behaves differently than the pseudo-homogeneous flow composed of sand-size particles. This is due to rather different mechanisms of internal friction in these flows. The fluid-like (not mechanic) friction in the flowing matter is described the law of viscosity. Pseudo-homogeneous mixtures that obey Newton's law of viscosity (see Chapter 1) are called Newtonian mixtures. Pseudo-homogeneous mixtures obeying a more complex relationship between shear stress and strain rate than is that given by Newton's law of viscosity are called non-Newtonian mixtures.

Pseudo-homogeneous mixture flows experience no (or at least very weak) accumulation of solid particles near the bottom of a pipeline, thus the deposition-limit velocity is an irrelevant parameter to predict. A slip between phases plays also no role. Thus the attention is focused to predicting frictional head losses in pipelines.

5.1 NEWTONIAN FLOW OF AQUEOUS MIXTURE OF SAND OR GRAVEL

5.1.1 General model

Generally, the frictional head loss in the fully-suspended (pseudo-homogeneous) flow is predicted by Clift et al. (1982) as

$$\frac{I_m - I_f}{C_{vd}(S_s - 1)} = \frac{I_m - I_f}{S_m - 1} = A' I_f \qquad (5.1)$$

I_m	hydraulic gradient for pseudo-homogeneous mixture flow	[-]
I_f	hydraulic gradient for liquid (water) flow	[-]
C_{vd}	delivered volumetric solids concentration	[-]
S_s	relative density of solids	[-]
S_m	relative density of mixture	[-]
A’	empirical coefficient	[-].

An increase in the frictional head loss due to the presence of solid particles in a carrying liquid forming a Newtonian fully-suspended mixture is attributed to increased carrier friction at the pipeline wall. A measure of the slurry density effect on friction process is given by an empirical coefficient A'.

5.1.2 Equivalent-liquid model

The Eq. 5.1 gives the "equivalent liquid" model

$$I_m = S_m I_f \qquad \text{for} \qquad A' = 1 \qquad (5.2).$$

According to this model the pseudo-homogeneous slurry flow behaves as a flow of a single-phase liquid having the density of the slurry. The "equivalent liquid" has the density of the mixture but other properties (as viscosity) remain the same as in the liquid (water) alone. This model suggests that all suspended particles contribute to an increase in the mixture density. The increase in a mixture density is responsible for a proportional increase in the shear stress resisting the flow at the pipeline wall. The above equation is obtained in the same way as the Darcy-Weisbach equation (see Chapter 1) with the only one exception: the density of mixture is considered instead of the liquid density. Thus $-\frac{dP}{dx} = 4\frac{\tau_o}{D}$ and $\lambda_f = \frac{8\tau_o}{\rho_m V_m^2}$ provide $-\frac{dP}{dx} = \frac{\lambda_f}{D}\frac{\rho_m V_m^2}{2}$.

Rearrangements give $I_m = -\frac{dP}{dx\rho_f g} = \frac{\rho_m}{\rho_f}\frac{\lambda_f}{D}\frac{V_m^2}{2g} = S_m I_f$.

This model may be successful to predict flows of relatively fine particles (fine sand, coarse silt), particularly if solids concentration is relatively low so that the viscosity is not affected.

5.1.3 Liquid model

If A' = 0 in the Eq. 5.1, it is assumed that solids present in a flowing liquid do not affect a flow friction at all,

$$I_m = I_f \qquad \text{for} \qquad A' = 0 \qquad (5.3).$$

Such a behavior in fast-flowing horizontal fully-suspended sand mixtures was reported by Carstens & Addie (1981). This behavior can be explained by an assumption that relatively coarse particles suspended in fast-flowing mixture are repelled from the pipeline wall due to large velocity gradient near the pipeline wall (a possible effect of lift forces discussed in Chapter 1). Since solid particles are not present in the region nearest the pipeline wall they also do not affect the wall shear stress that is decisive for the friction process.

5.1.4 Experimental observations

Pipeline tests in the Laboratory of Dredging Technology have revealed that the liquid model is not applicable to fast-flowing fully-suspended mixtures. The equivalent-liquid model tends to overestimate slightly the frictional losses. The parameter A' of the general model seems to be dependent on flow conditions.

5.2 NON-NEWTONIAN FLOW OF AQUEOUS MIXTURE OF SILT OR CLAY

Very fine particles (of silt size and finer, $d < 40$ μm approximately) are practically non-settling in a flow of carrying liquid. They interfere with the carrying liquid to increase its density and viscosity. In a mixture flow containing these fine particles the viscosity of the pseudo-homogeneous mixture grows with the increasing fraction of solids in the mixture.

The suspension does not obey Newton's law of viscosity and its constitutive rheological equation (the equation relating the shear stress, τ, with the shear rate, dv_X/dy) has to be determined experimentally to give a basis for friction-loss predictive models.

Modeling of non-Newtonian mixtures is even more complex than the modeling of Newtonian mixtures in pipelines. A constitutive rheological equation of a non-Newtonian mixture is rather sensitive to too many factors. Actually, each particular mixture obeys its own law of viscosity. Therefore it is necessary to know a rheogram [a relationship between shear stress, τ, and strain rate, dv_X/dy] for each particular mixture handled in a pipeline. A tube viscometer is a preferable instrument to determine a rheogram of a mixture. Conditions within the tube viscometer are geometrically similar to those in prototype pipes, assuring the similarity in the stress distributions. Data from a tube viscometer can be successfully used to scale up the frictional head loss to larger pipes or to determine a mixture rheological model.

A general procedure for a determination of the frictional head losses in a pipeline flow of a non-Newtonian mixture is composed of the following steps:

1. the rheological parameters of a mixture
2. the mixture flow regime (laminar or turbulent)
3. the losses using a scale-up method or an appropriate friction model.

5.2.1 Rheological parameters of a mixture

The rheology of a mixture is determined from laminar-flow data obtained from measurements in either a tube viscometer or a rotational viscometer. The rheological constants are determined from measured values of parameters V_m, $\Delta P/L$, D etc. according to a method discussed in *Intermezzo II*.

5.2.2 Mixture flow regime (laminar or turbulent)

An information whether pipe flow is laminar or turbulent is important because it determines a method for the friction-loss prediction. A laminar regime in non-Newtonian mixtures holds to higher velocities than for Newtonian mixtures in a pipeline of the same diameter. The laminar regime may occur in a dredging pipeline if highly viscous mixtures are transported.

The most accurate method of a determination of a transitional velocity, V_T, between the laminar and the turbulent regime is to find experimentally (in a laboratory pipe) an intercept between I_m-V_m curves for laminar and turbulent regimes of the mixture flow. Then the transition can be scaled up with the resistance curves to pipes of larger sizes.

Theoretical models for a regime transition are also available. These are often the by-products of the flow models for laminar and turbulent flows of non-Newtonians. Thomas (1963) proposed for Bingham plastic flow the following equation that is often used in practice

$$V_T = \frac{2100\eta_B}{\rho_m D}\left[1 + \frac{\tau_y D}{6\eta_B V_T}\right] \quad (5.5).$$

The simpler equation for the transition velocity in a Bingham plastic flow is obtained if the Bingham Reynolds number

$$Re_B = \frac{\rho_m V_m D}{\eta_B\left(1 + \frac{\tau_y D}{6\eta_B V_m}\right)} \quad (5.6)$$

V_T	value of V_m at the transition between laminar and turbulent regime of non-Newtonian mixture flow	[m/s]
V_m	mean mixture velocity in a pipe	[m/s]
D	pipe diameter	[m]
ρ_m	density of mixture	[kg/m^3]
η_B	tangential viscosity of Bingham plastic mixture	[Pa.s]
τ_y	yield stress of Bingham plastic mixture	[Pa]
Re_B	Reynolds number of flow of Bingham plastic mixture	[-]

is taken as equal to 2100. This threshold value of Reynolds number is identical with that for a Newtonian flow. Since $\frac{\tau_y D}{6\eta_B V_m} >> 1$ the equation $Re_B = 2100$ gives for flow of Bingham plastic mixture

$$V_T \approx 19\sqrt{\frac{\tau_y}{\rho_m}} \quad (5.7)$$

5.2.3 Friction losses using a scale-up method or an appropriate friction model

Friction-loss predictions based on the flow modeling are generally less accurate than those based on the scaling up of tube viscometer data.

5.2.3.1 Scale-up methods

The I_m-V_m results from tube viscometers can be scaled up to prototype pipes without an intermediary of rheological model. The principle of the scaling-up technique is that in non-Newtonian flows the wall shear stress is unchanged in pipes of different sizes D. The wall shear stress fully determines the stress distribution within the pipe.

Scale-up techniques are different in laminar and turbulent flow. Scaling up between two different pipeline sizes [e.g. between a tube viscometer (index 1) and a prototype pipeline (index 2)] is carried out as follows.

Laminar flow:
the *Rabinowitsch-Mooney transformation* applies:

$$\tau_0 = \frac{D.\Delta P}{4L} = \text{idem}, \quad \frac{8V_m}{D} = \text{idem};$$

this says that if a τ_o versus $\frac{8V_m}{D}$ relationship for a laminar flow of a certain mixture is determined (experimentally) in one pipe it is valid also for pipes of all different sizes.

Thus

$$I_{m2} = I_{m1}\frac{D_1}{D_2} \quad (5.8)$$

$$V_{m2} = V_{m1}\frac{D_2}{D_1} \quad (5.9).$$

Turbulent flow:

$$\tau_0 = \frac{D.\Delta P}{4L} = \text{idem but } \frac{8V_m}{D} \neq idem$$

because the near-wall velocity gradient is not described by $8V_m/D$ (see Chapter 1, p. 1.5); instead the friction-law is sought relating friction coefficient λ_f with mean velocity V_m.

Thus the turbulent-flow data I_m, V_m from pipeline (1) can be scaled to pipeline (2) using

$$I_{m2} = I_{m1}\frac{D_1}{D_2} \qquad (5.10)$$

$$V_{m2} = V_{m1}\left[1 + 2.5\sqrt{\frac{\lambda_f}{8}}\ln\left(\frac{D_2}{D_1}\right)\right] \qquad (5.11)$$

I_m	hydraulic gradient for pseudo-homogeneous mixture flow	[-]
V_m	mean mixture velocity in a pipe	[m/s]
D	pipe diameter	[m]
λ_f	Darcy-Weisbach friction coefficient for fluid flow	[-].

In Eq. 5.11 the final term within the brackets determines an effect of the equivalent turbulent-flow viscosity on wall shear stress. This term is usually not greatly different from zero.

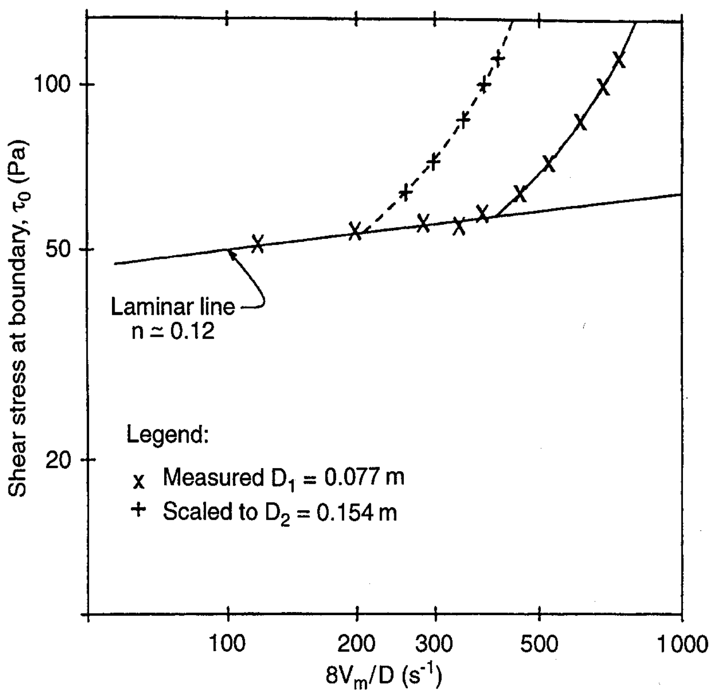

Figure 5.1. Scaling to a larger pipeline.

5.2.3.2 Flow-friction models

The scale-up technique is certainly a preferable predictive method. However, sometimes the tube test data are not available because a test instrument or a mixture sample are not available. Test data for turbulent flow regime might be unavailable even if tube viscometer tests are carried out. This might be, for instance, due to a small diameter of a viscometer tube that causes that the turbulent flow regime is not reached even at the highest velocities in the tube. If data, and thus a rheogram, are not available the rheological parameters employed in theoretical rheological models (constitutive equations) have to be used to derive a I-V relationship for mixture flow.

Laminar flow:

For a **laminar flow** a chosen constitutive equation is integrated over a pipe cross section and hence velocity distribution obtained. This gives a relation between pressure gradient and mean velocity (the same procedure as described in Chapter 1 for Newtonian liquid flow) in a homogeneous flow of mixture.
Integrating of the yield pseudo-plastic rheological model

$$\tau = \tau_y + K\left(\frac{dv_x}{dy}\right)^n$$

over a pipe cross section $A = \frac{\pi D^2}{4}$ gives

$$\frac{8V_m}{D} = \frac{4}{K^{\frac{1}{n}}\tau_o^3}\left(\tau_o - \tau_y\right)^{\frac{1+n}{n}}\left[\frac{\left(\tau_o - \tau_y\right)^2}{1+3n} + \frac{2\tau_y\left(\tau_o - \tau_y\right)}{1+2n} + \frac{\tau_y^2}{1+n}\right] \quad (5.12)$$

where the wall shear stress $\tau_o = \frac{D.\Delta P}{4L}$.

For a Bingham mixture integrating of a constitutive equation provides

$$\frac{8V_m}{D} = \frac{\tau_o}{\eta_B}\left[1 - \frac{4\tau_y}{3\tau_o} + \frac{\tau_y^4}{3\tau_o^4}\right] \quad (5.13).$$

Eqs. 5.12 and 5.13 give a relationship between a frictional head loss and mean mixture velocity in a pipeline as a function of rheological parameters of a mixture. This relationship can be rewritten to the standard friction-loss equation

$$I_m = \frac{\Delta P}{L\rho_f g} = \frac{\lambda_{nN}}{D}\frac{V_m^2}{2g} \quad (5.14)$$

in which, for a laminar flow,

$$\lambda_{nN} = \frac{64}{Re_{nN}} \quad (5.15)$$

if the equation for Reynolds number Re_{nN} for a non-Newtonian flow gets a modified form given by a rheological type of a flowing mixture. For the Bingham plastic mixture (n = 1, K = η_B) combining of Eq. 5.13 and Eq. 5.14 provides

$$\lambda_B = \frac{64\eta_B}{DV_m\rho_m}\left[1-\frac{4\tau_y}{3\tau_o}+\frac{\tau_y^4}{3\tau_o^4}\right]^{-1} = \frac{64}{Re_B} \qquad (5.16)$$

and thus the modified Reynolds number, by neglecting the fourth-power term in the above equation

$$Re_B = \frac{\rho_m V_m D}{\eta_B\left(1+\frac{\tau_y D}{6\eta_B V_m}\right)} \qquad (5.6).$$

I_m	hydraulic gradient for mixture flow	[-]
V_m	mean mixture velocity in a pipe	[m/s]
D	pipe diameter	[m]
λ_{nN}	Darcy-Weisbach friction coefficient for non-Newtonian flow	[-]
g	gravitational acceleration	[m/s^2]
Re_B	Reynolds number of flow of Bingham plastic mixture	[-]
ρ_m	density of mixture	[kg/m^3]
η_B	tangential viscosity of Bingham plastic mixture	[Pa.s]
τ_y	yield stress of Bingham plastic mixture	[Pa]

A solution of the integral equation for a Bingham plastic flow (Eq. 5.13) can be accomplished using the *Hedström nomograph*. The nomograph (Fig. 5.2) gives the friction coefficient λ_B as a function of two dimensionless groups:

Hedström number, He,

$$He = \frac{\tau_y D^2 \rho_m}{\eta_B^2} \qquad (5.17)$$

and Reynolds number, Re_b,

$$Re_b = \frac{\rho_m V_m D}{\eta_B} \qquad (5.18).$$

Steep lines for different He values give the friction coefficient for a laminar flow regime. The less steep line valid for all He values gives the friction coefficient in a turbulent regime of mixture flow. It is suggested that there is no effect of the yield stress on pipeline friction if the flow is turbulent.

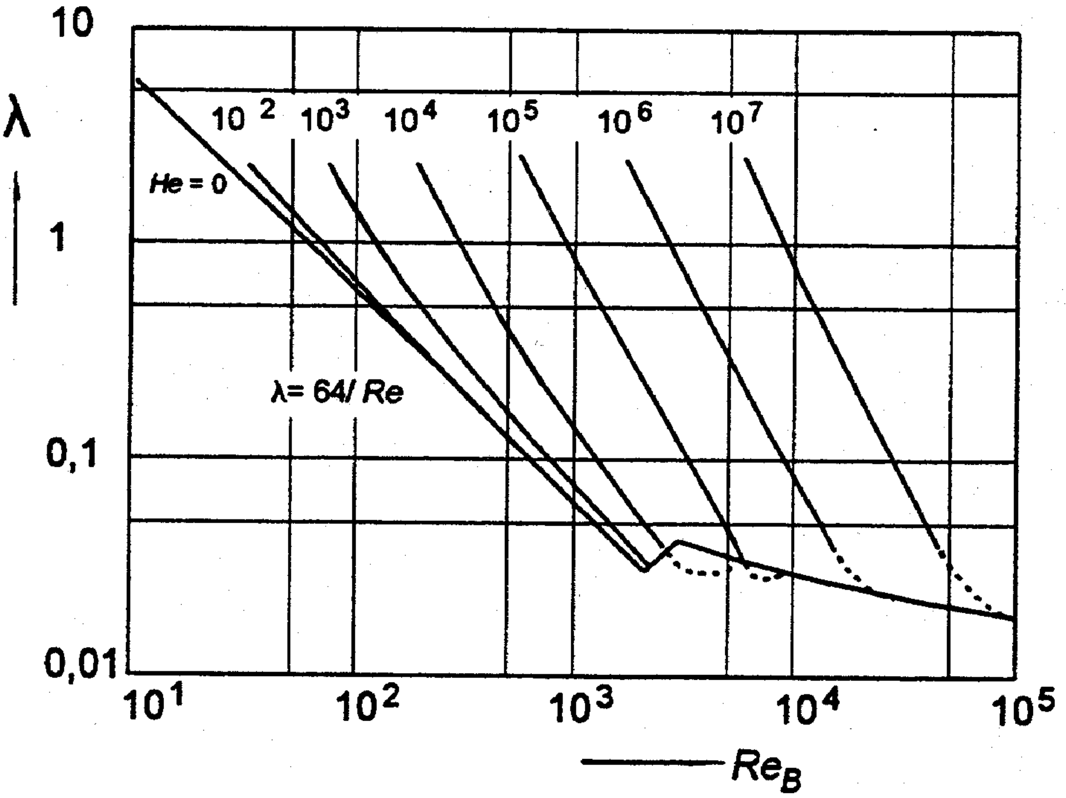

Figure 5.2. Friction coefficient λ_B as a function of He number (Eq. 5.17) and Re_b number (Eq. 5.18).

Turbulent flow:

In a **turbulent flow** the rheological models are again the basis for friction models. However, an integration of a rheological model, and thus a direct determination of velocity profile, is not possible in turbulent flow (see Chapter 1). A friction law is required that relates the friction coefficient, λ_{nN}, with the flow Reynolds number, Re, and the pipe-wall roughness factor.

The *Slatter model* (Slatter, 1995) was tested by data from a number of non-Newtonian mixtures (kaolin etc.) in various viscometric tubes. It suggests the following equations to describe a friction law for a yield pseudo-plastic mixture in different turbulent flow regions delimited by a value of the roughness Reynolds number

$$Re_r = \frac{8\rho_m V_*^2}{\tau_y + K\left(\frac{8V_*}{d_{85}}\right)^n} \qquad (5.19),$$

- smooth wall turbulent flow ($Re_r \le 3.32$)

$$\sqrt{\frac{8}{\lambda_{nN}}} = 2.5 \ln\left(\frac{D}{2d_{85}}\right) + 2.5 \ln Re_r + 1.75 \qquad (5.20)$$

- fully developed rough wall turbulent flow ($Re_r > 3.32$)

$$\sqrt{\frac{8}{\lambda_{nN}}} = 2.5 \ln\left(\frac{D}{2d_{85}}\right) + 4.75 \qquad (5.21)$$

Re_r	roughness Reynolds number for non-Newtonian flow	[-]
V_*	shear velocity, $V_* = V_m(\lambda_{nN}/8)^{0.5}$	[m/s]
d_{85}	characteristic particle size	[m]

The frictional head loss is again given by Eq. 5.14

$$I_m = \frac{\Delta P}{L\rho_f g} = \frac{\lambda_{nN}}{D}\frac{V_m^2}{2g} \quad .$$

5.2.4 Prediction of frictional losses during transportation of silt mixture in a dredging pipeline

The figures 5.3-5.5 show measured rheological characteristics of the aqueous mixture of silt dredged from Caland Kanaal in the Europort entrance (taken from v.d. Berg, 1998). The mixture behaves like Bingham plastic liquid.

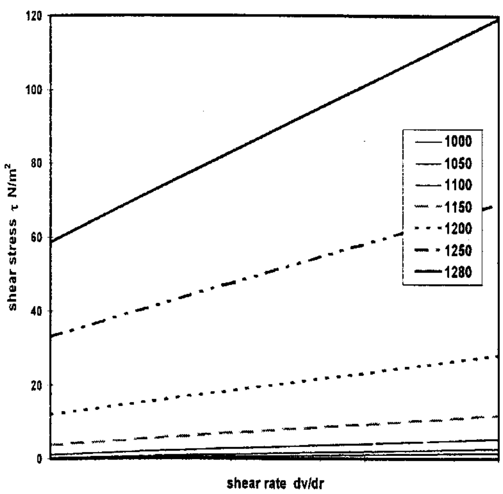

Figure 5.3. Rheogram of the “Caland”silt mixture measured for different mixture densities (measurements: rotoviscometer).

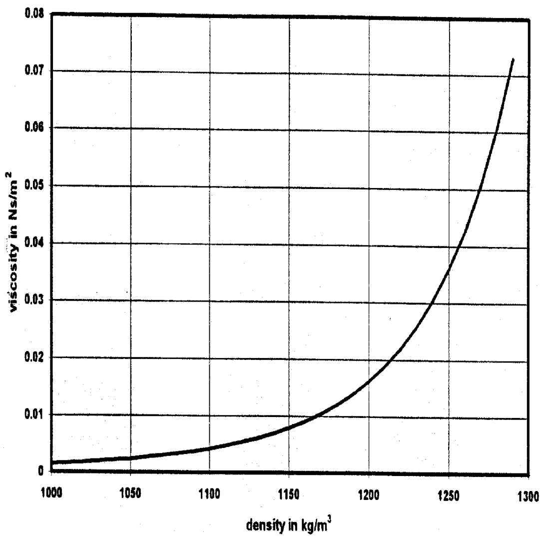

Figure 5.4.

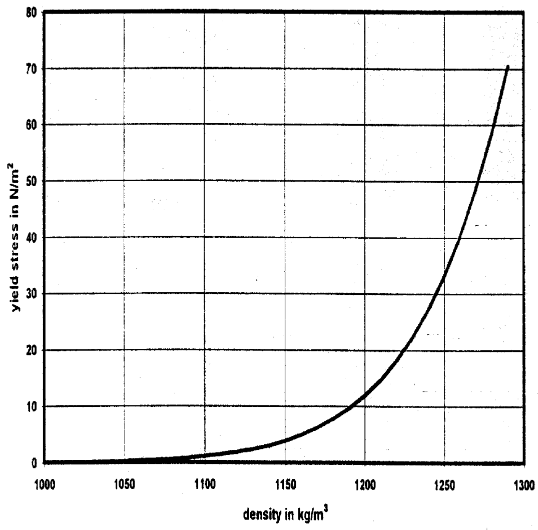

Figure 5.5.

Figure 5.4. Relationship between viscosity and density of the silt mixture.

Figure 5.5. Relationship between yield stress and density of the silt mixture.

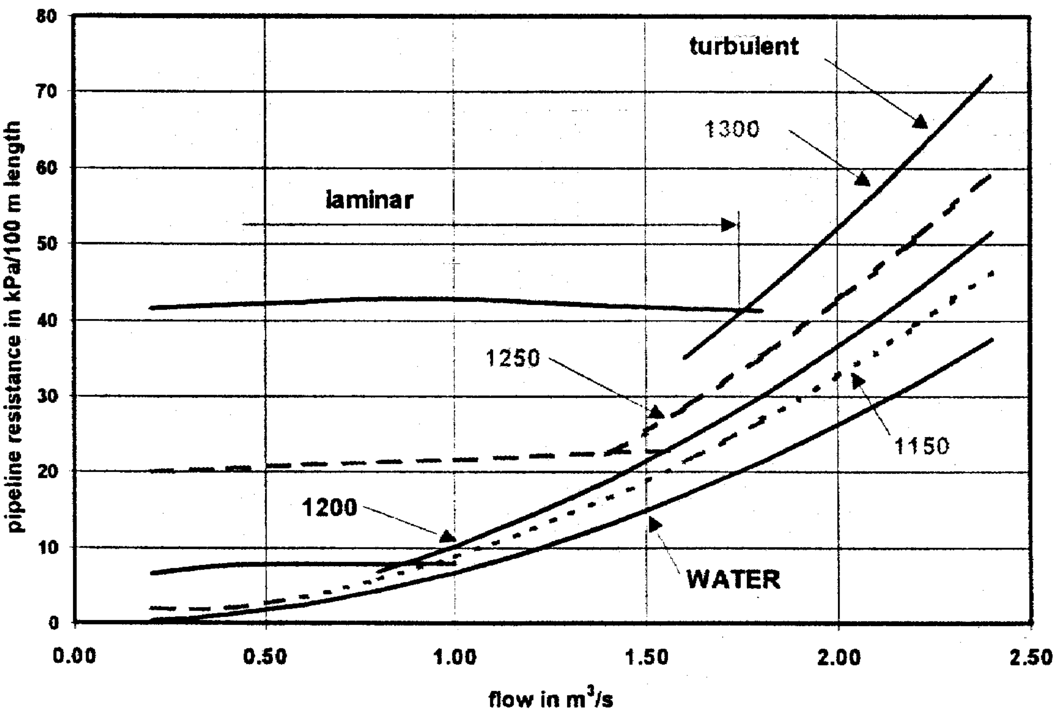

Figure 5.6. Prediction of pipeline resistance of the silt mixture using a friction model (lines for various mixture densities in a 700 mm pipeline) (from v.d. Berg, 1998).

Fig. 5.6 shows the resistance curves for the Caland silt mixture flow in a 700 mm dredging pipeline predicted by the Hedström method. The intercepts between laminar curves and turbulent curves for a certain chosen mixture density determines the transition velocity V_T. A laminar regime holds to the mean mixture velocity 1.7 m/s if silt-water mixture has density of 1300 kg/m^3.

5.3 REFERENCES

van den Berg, C.H. (1998). *Pipelines as Transportation Systems.* European Mining Course Proceedings, MTI.

Carstens, M.R. & Addie, G.R. (1981). A sand-water slurry experiment, *Journal of the Hydraulic Division*, ASCE, **107**(HY4), 501-7.

Clift, R., Wilson, K.C., Addie G.R. & Carstens, M.R. (1982). A mechanistically-based method for scaling pipeline tests for settling slurries. *Proc. Hydrotransport 8*, BHRA, Cranfield, UK, pp. 91-101.

Slatter, P. (1995). The turbulent flow of non-Newtonian slurries in pipes, *Proc. 8th Int. Conf. on Transport and Sedimentation of Solid Particles*, CTU Prague.

Thomas, D.G. (1963). *Ind. Eng. Chem.*, **55**, p. 27.

5.4 RECOMMENDED LITERATURE

Govier, G.W. & Aziz, K. (1972). *The Flow of Complex Mixtures in Pipes.* Van Nostrand Reinhold Company.

Wilson, K.C., Addie G.R., Sellgren, A. & Clift, R. (1997). *Slurry Transport Using Centrifugal Pumps.* Blackie Academic & Professional.

CASE STUDY 5

Frictional head loss in flow of non-Newtonian mixture through a horizontal pipeline

The silt dredged from Calandkanaal of the Rotterdam harbour is transported hydraulically at the mixture density 1250 kg/m^3 through a 500-metre long horizontal pipeline of the diameter 700 mm. The rheometrical test of the mixture sample in a viscometer has shown that the silt mixture behaves as a Bingham liquid with the yield stress 33 Pa and the plastic viscosity 36 mPa.s.

Determine the pressure drop due to friction over the entire pipeline length for two mixture flow rates: 1.0 m^3/s and 2.0 m^3/s.

Inputs:

ρ_m = 1250 kg/m^3
τ_y = 33 Pa
η_B = 0.036 Pa.s
L = 500 m
D = 700 mm
Q_m = 1.0 and 2.0 m^3/s

Solution:

a. Mean mixture velocity and laminar-turbulent threshold

$V_m = 4Q_m/(\pi D^2)$, i.e. 2.60 m/s for Q_m=1.0 m^3/s and 5.20 m/s for Q_m=2.0 m^3/s.
V_T = 3.09 m/s (Eq. 5.7).
Thus the flow is laminar for Q_m=1.0 m^3/s and turbulent for Q_m=2.0 m^3/s.

b. Frictional pressure drop in the laminar flow

Re_B = 1500 (Eq. 5.6) for V_m = 2.60 m/s
$\lambda_B = 64/ Re_B = 0.043$
I_m = 0.021 (Eq. 5.14), thus ΔP = 0.021 x 500 x 9810 = 103 814 Pa.

The total pressure drop due to friction is 104 kPa (approximately 1 bar) at the flow rate 1.0 m^3/s of the silt mixture through a 700-mm pipeline that is 500 meter long.

c. Frictional pressure drop in the turbulent flow

Re_b = 126 389 ≈ 1.3 x 10^5 (Eq. 5.18) for V_m = 5.20 m/s
λ_B ≈ 0.021 (Fig. 5.2) for Re_b ≈ 1.3 x 10^5 (He number value is not important)
I_m = 0.041 (Eq. 5.14), thus ΔP = 0.041 x 500 x 9810 = 202 800 Pa.

The total pressure drop due to friction is 203 kPa (approximately 2 bar) at the flow rate 2.0 m^3/s of the silt mixture through a 700-mm pipeline that is 500 meter long.