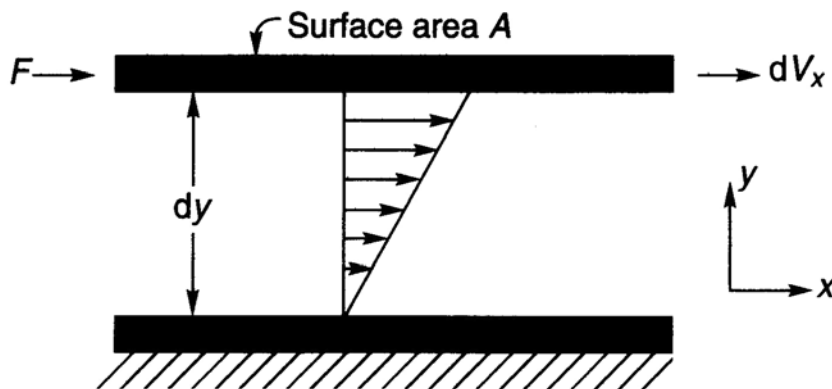


## Intermezzo II. RHEOLOGICAL PARAMETERS OF NON-SETTLING MIXTURES

### II.1 DEFINITIONS

An important property of a liquid or a fine homogeneous mixture (considered as continuum) is a law of viscosity. This quantifies a resistance of liquid/mixture to deformation of shearing. This is done by relating the shear stress,  $\tau$ , to shear rate  $dv_x/dy$  (sometimes called also strain rate or rate of shear strain). For Newtonian fluid the Newton's law of viscosity,  $\frac{F}{A} = \tau = \mu \frac{dv_x}{dy}$ , is valid (see Fig. II.1 and Chapter 1 for more details).



**Figure II.1.** Determination of viscosity (schematic).

Non-Newtonian fluids obey a rather more complicated relationship between the shear stress and the shear rate. This is determined experimentally in a viscometer. Plotted relationship  $\tau$  versus  $dv_x/dy$  is known as a rheogram (see Fig. II.1). A mathematical function that fits a rheogram is called a rheological model. Coefficients of this function are the rheological parameters.

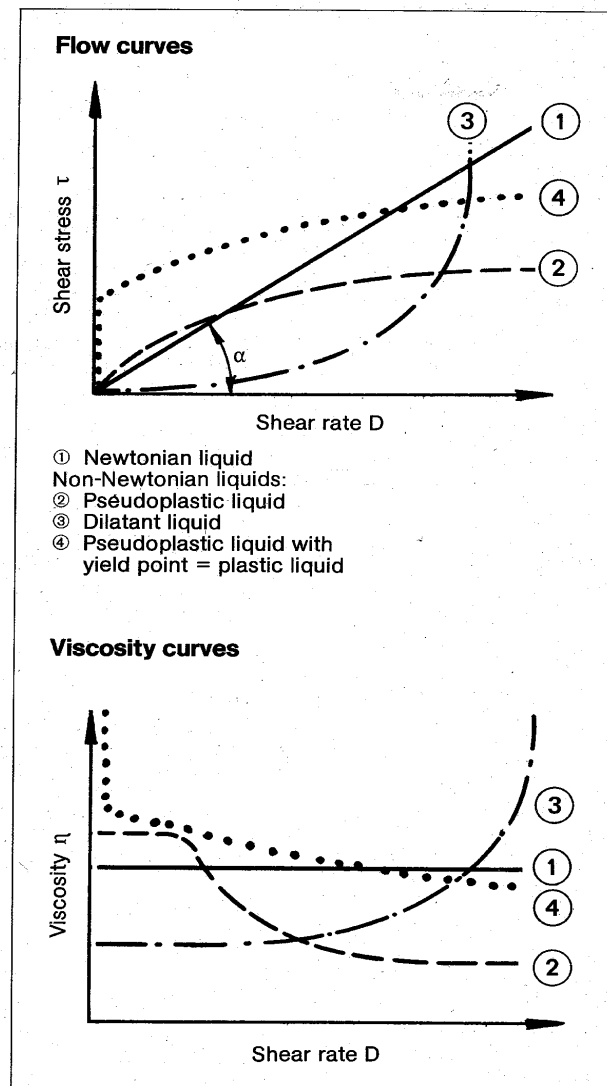
The rheogram of a Newtonian liquid is a straight line that passes through an origin and has a slope given by the value of dynamic viscosity of the liquid. The viscosity is independent of the shear rate. Rheograms of non-Newtonians do not always pass through the origin (the shear rate remains zero until a certain yield stress  $\tau_y$  is exceeded) and/or are not straight. This gives arise to different definitions of viscosity exploited from rheograms. The viscosity determined in the same way as in the Newtonian rheogram is called the secant viscosity (also apparent viscosity)  $\mu_a$ ,

$$\mu_a = \frac{\tau}{\frac{dv_x}{dy}} \quad (\text{II.1}),$$

the viscosity determined as the tangent to the rheogram at a certain point is the tangent viscosity (also differential viscosity),  $\eta_t$ ,

$$\eta_t = \frac{d\tau}{d\left(\frac{dv_x}{dy}\right)} \quad (\text{II.2}),$$

(see Fig. II.2). The viscosity of non-Newtonian liquid is not constant, it is dependent on the shear rate.



**Figure II.2.** Rheograms (flow curves) and viscosity curves (tangent viscosity versus shear rate  $dv_x/dy$ ) of various liquids (schematic, after Schramm, 1981).

## II.2 RHEOGRAMS

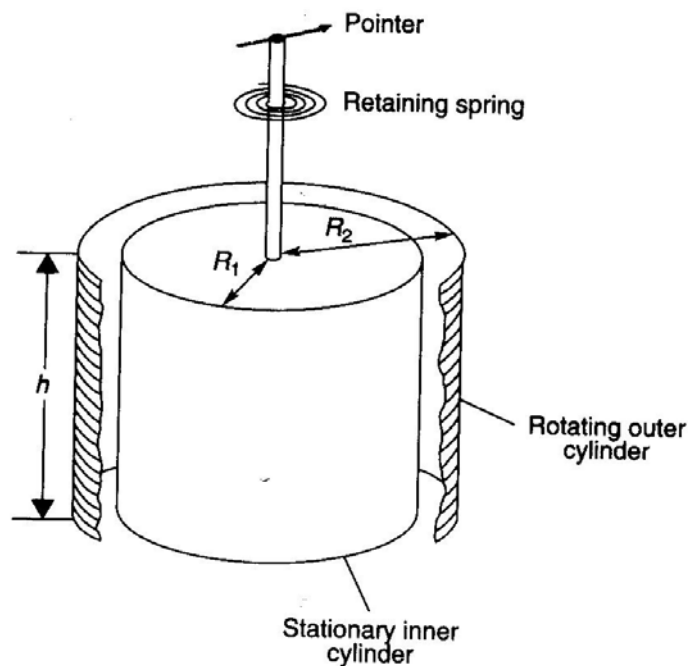
### II.2.1 Viscometry - a determination of rheograms

The basic condition for a correct determination of the rheological parameters is a laminar replacement (flow) of a sheared matter in a viscometer. Basically, there are two types of instruments appropriate to obtain data required to plot the rheograms - a rotational viscometer and a tube viscometer (Figs. II.3a and b).

#### *Rotational viscometer*

Rotational viscometers are widely available and require only a small sample of mixture for testing, but they introduce much larger uncertainties than do tube viscometers.

The basic concept is that fluid is sheared in a narrow zone between the rotating part and a stationary part of the instrument. The shear rate  $dv_x/dy$  is determined from the angular velocity of the rotating part and the shear stress  $\tau$  from the torque of the rotating part. Calibration formulas are required to transform the torque and speed readings to data plotting the rheograms. These formulas themselves are dependent on the rheological properties of the mixtures, and are thus subject to considerable uncertainties.



**Figure II.3a.** Rotational viscometers

### Tube viscometer

The pressure gradient  $\Delta P/L$  and mean mixture velocity  $V_m$  are measured in the tube viscometer of diameter  $D$ . These data provide the pseudo-rheogram (Fig. II.4), that relates  $\tau_0 = \frac{D\Delta P}{4L}$  with  $\frac{8V_m}{D}$  in place of  $\tau$  and  $dv_x/dy$  given in the normal rheogram. For laminar flows of non-Newtonians the pseudo-rheogram can be transformed to the normal rheogram by using the Rabinowitsch-Mooney transformation described in the paragraph II.3.

#### Couette system

Coaxial cylinder sensor systems

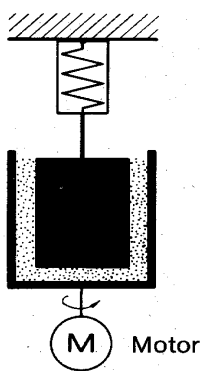
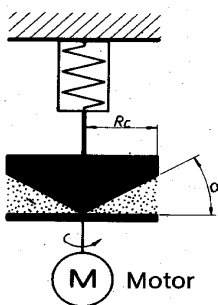
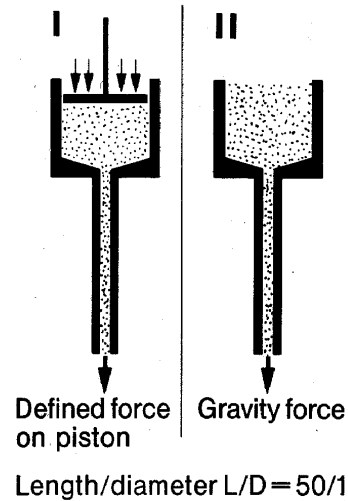


Plate and cone sensor systems



(a)

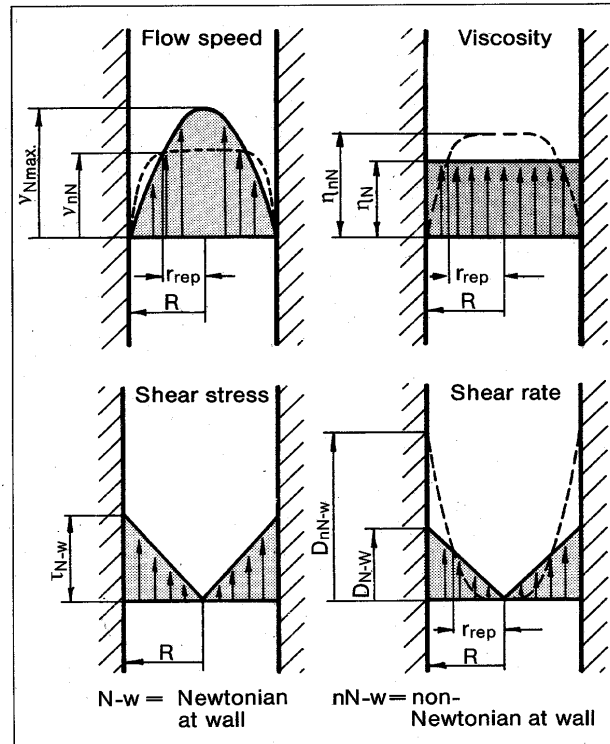
#### Absolute capillary viscometer



(b)

**Figure II.3b.** Rotational viscometers of the Couette type (a) and capillary viscometers (b) (after Schramm,1981).

Capillary viscometers use vertical capillaries (tubes of a very small diameter). The small diameter (usually until about 30 mm) helps to maintain the laminar regime in a wide range of flow rates through the tube (remember  $Re \approx 2300$  at the threshold between laminar and turbulent flow). Of course, there is a big difference in scale between capillaries and dredging pipelines. However, the conditions within a tube viscometer (Fig. II.4) must be geometrically similar to those in a dredging pipeline. This assures the similarity in the stress distribution across the pipe cross section.



**Figure II.4.** Distribution of laminar flow of Newtonian (N) and non-Newtonian (nN) liquid in a pipeline (after Schramm,1981).

As shown in Chapter 1 the shear stress varies linearly from zero in the tube centre to  $\tau_0$  at the wall. This is valid for tube flow of both Newtonian and non-Newtonian fluids/mixtures. Thus the stress distribution is given by the value of wall shear stress  $\tau_0$ . According to the known force balance (see Chapter 1)  $\frac{\Delta P}{L} A = \tau_0 O$  , i.e.

$\frac{\Delta P}{L} = \frac{4\tau_0}{D}$  , there is a direct relationship between  $\Delta P/L$  (measured in a tube viscometer) and  $\tau_0$ . The value of the shear rate  $dv_x/dy$  at the tube wall is related to  $V_m$  (also measured in a tube viscometer). The relationship between  $dv_x/dy$  at the pipeline wall and  $V_m$  requires an analysis. For laminar flows of Newtonian liquids this is given by  $\frac{8V_m}{D}$  (see Eq. 1.13 in Chapter 1). For non-Newtonians the value of

$dv_x/dy$  is no longer equal to  $\frac{8V_m}{D}$  , but it is proportional to that quantity, as determined by Rabinowitsch and Mooney.

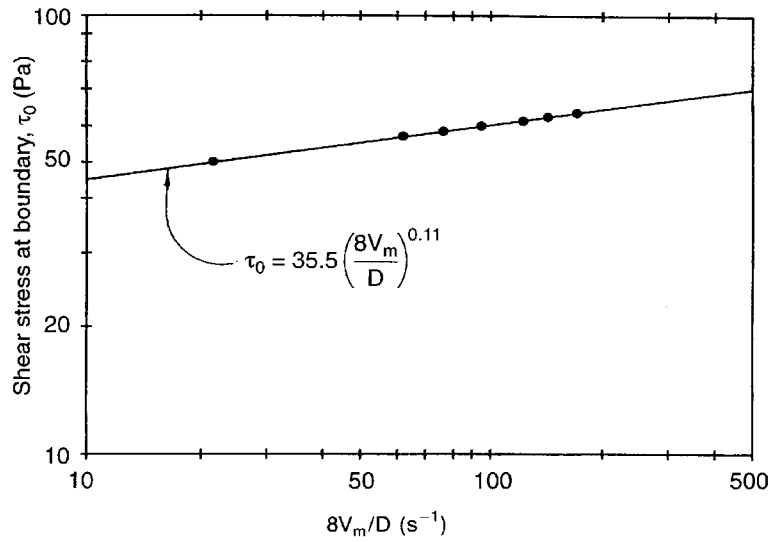


Figure II.5. Logarithmic plot of  $\tau_0$  versus  $8V_m/D$  for data from a tube viscometer, (after Wilson et al., 1997).

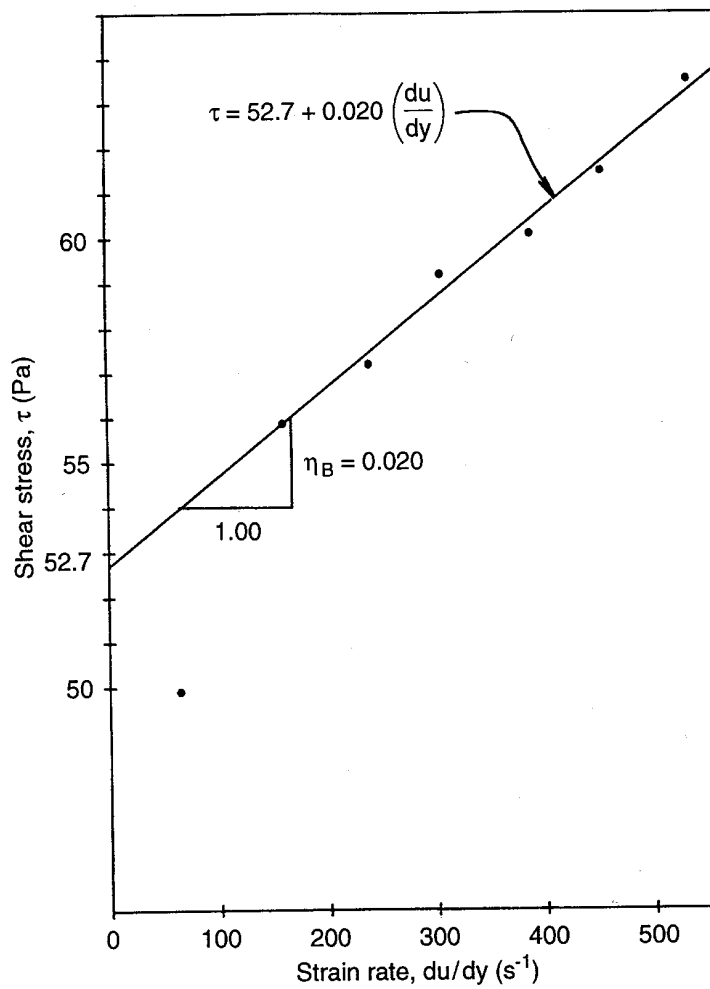


Figure II.6. Rheogram obtained for data of Fig. II.4 using Mooney-Rabinowitsch technique (after Wilson et al., 1997).

**II.2.2 Rabinowitsch-Mooney transformation**

The Rabinowitsch-Mooney transformation transforms the pseudo-rheogram (the rheogram based on the pipe-wall values of shear stress and shear rate, Fig. II.5) to the normal rheogram (based on local values of shear stress and shear rate at an arbitrary position within a pipe) for a laminar flow of non-Newtonians (see Fig. II.6). The transformation is equally applicable in the opposite direction.

According to the transformation

the pseudo-rheogram data points  $[\tau_0, \frac{8V_m}{D}]$  are transformed to

the normal rheogram points  $[\tau, \frac{dv_x}{dy}]$  using the following rules:

$\tau = \tau_0 \qquad \text{and} \qquad \frac{dv_x}{dy} = \left( \frac{3n'+1}{4n'} \right) \frac{8V_m}{D}.$
---

This means that for each data point the x-ordinate shifts from  $\frac{8V_m}{D}$  to  $\left( \frac{3n'+1}{4n'} \right) \frac{8V_m}{D}$  and the y-ordinate remains unchanged. The parameter  $n'$  is the local tangent of the pseudo-rheogram in the logarithmic plot, i.e.  $n' = \frac{d(\ln \tau_0)}{d\left(\ln \frac{8V_m}{D}\right)}$ .

On Fig. II.4 the measured points are fitted by the pseudo-rheogram that gives  $n' = 0.11$  and thus  $\left( \frac{3n'+1}{4n'} \right) = 3.023$ . The measured points of the pseudo-rheogram are transformed into the points of the normal rheograms (Fig. II.5) using  $\tau = \tau_0$  and  $du/dy = dv_x/dy = 3.023 \frac{8V_m}{D}$ . The normal rheogram is a line that fits the points in the  $\tau$  versus  $du/dy$  (strain rate) plot.

## II.3 RHEOLOGICAL MODELS

The simplest rheological model is the Newtonian fluid, represented by equation

$$\tau = \mu \frac{dv_x}{dy} \quad (\text{II.3})$$

with a single rheological parameter,  $\mu$ . It is given by flow curve (1) in Fig. II.1.

### II.3.1 Two-parameter models

Bingham-plastic model contains rheological parameters  $\tau_y$ ,  $\eta_B$ . The rheogram of Bingham plastic mixtures is linear and passes through  $\tau_y$  (yield stress) at  $dv_x/dy = 0$  (i.e. the imposed shear stress must be higher than  $\tau_y$  to cause flow). The rheogram has a slope  $\eta_B$  (called plastic viscosity or tangent viscosity)

$$\tau = \tau_y + \eta_B \frac{dv_x}{dy} \quad (\text{II.4}).$$

This behavior is the most usual for concentrated homogeneous mixtures that are dredged (aqueous mixtures of non-cohesive clay etc.).

Power-law model approximates rheograms which pass through origin (i.e. any imposed shear stress causes flow) but they are not a straight line [flow curves (2) and (3) in Fig. II.1]. The model has parameters  $K$  and  $n$ ,

$$\tau = K \left( \frac{dv_x}{dy} \right)^n \quad (\text{II.5}).$$

Different types of fluid behavior may occur:

$0 < n < 1$ : pseudo plastic (“shear-thinning”) fluid (becomes *thinner* under the increasing shear rate)

$n > 1$ : dilatant (“shear-thickening”) fluid (becomes *thicker* under the increasing shear rate).

### II.3.2 Three-parameter model

Yield-power-law model (Buckley-Herschel) with parameters  $\tau_y$ ,  $K$ ,  $n$

$$\tau = \tau_y + K \left( \frac{dv_x}{dy} \right)^n \quad (\text{II.6}).$$

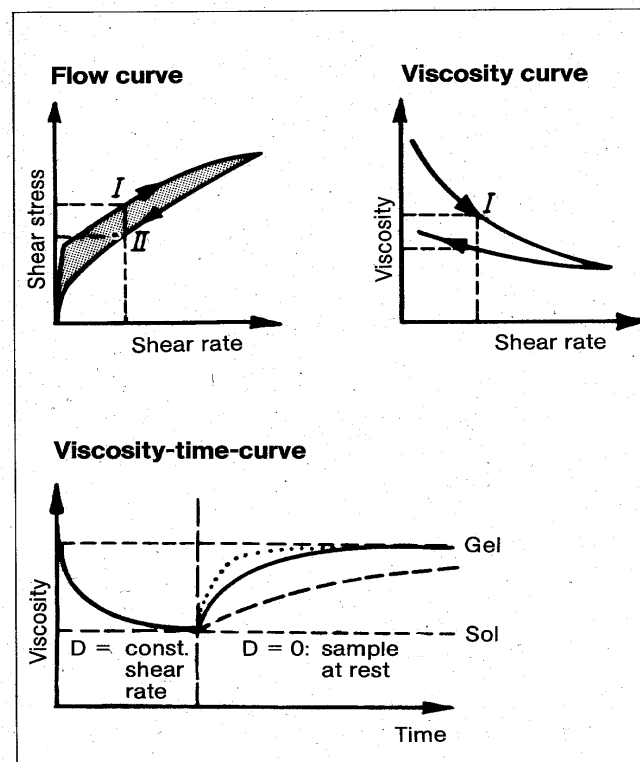
It is given by flow curve (4) in Fig. II.1.



**REMARK:**

The above considered rheological models are the time-independent models. However, a full description of rheological behavior also includes variation with time, because many homogeneous mixtures exhibit time-dependent behaviour. The mixtures can exhibit either the thixotropic behaviour or the rheopectic behaviour.

The mixture is thixotropic if the viscosity of the mixture subjected to shearing under the constant shear rate  $dv_x/dy$  (e.g. in the rotational viscometer) diminishes in time. This is shown schematically on Fig. II.7. The rheopectic mixtures become gradually more viscous if subjected to shearing of constant value for some time.



**Figure II.7.** Behaviour of a thixotropic mixture (after Schramm, 1981).  
(Legend: Viscosity = apparent viscosity,  $D = dv_x/dy =$  shear rate, Sol = solution)

**II.4 REFERENCES**

- Schramm, G. (1981). *Introduction to Practical Viscometry*. HAAKE Publications.  
Wilson, K.C., Addie G.R., Sellgren, A. & Clift, R. (1997). *Slurry Transport Using Centrifugal Pumps*. Blackie Academic & Professional.

