<u>Hand Out AE1103 Statics: "Properties of Thin-Walled Cross-Sections"</u>

Thin-walled Structures

In the aerospace industry most structures have relatively thin cross-sections in terms of wall thicma or cross-section can be considered thin-walled if the wall thickness t is much smaller than the height h and width b or radius R. Or in formulas:

$$t << b, h, R \qquad (1)$$

The order of magnitude of much smaller is typically 100 or more.

Consequences for the calculation of cross-sectional properties

As a result of this definition of thin-walled structures certain assumptions may be made with regards to the calculation of cross-sectional properties such as the centroid and the area moments of inertia.

1. First of all it is assumed that all dimensions are given with respect to the centre line of a cross-section (see figure below) and that you may neglect the distance from the centre line to the edge of the cross-section (i.e. ½t).

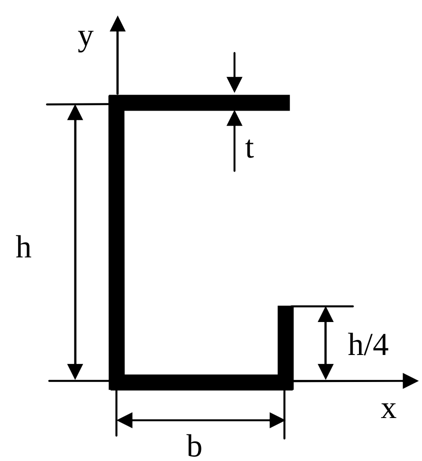

Figure 1

2. Secondly when calculating centroids and area moments of inertia you neglect all higher order terms of t (i.e. t^2 and higher). See worked out example on the next page.

Example:
Of the thin-walled structure in figure 1 calculate:

1. The location of the centroid in y-direction, y_{cg}.

Answer:

$$\bar{y}_{cg} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} \tag{2}$$

With:

$$\sum_{i=1}^{4} A_i = bt + ht + bt + \frac{h}{4}t = 2bt + \frac{5}{4}ht \tag{3}$$

And:

$$\sum_{i=1}^{4} y_i A_i = hbt + ht\frac{h}{2} + bt\frac{t}{2} + \frac{h}{4}t\frac{h}{8} \tag{4}$$

Neglecting terms of t^2 and simplifying gives:

$$\sum_{i=1}^{4} y_i A_i = hbt + \frac{17}{32}h^2 t \tag{5}$$

The location of y_{cg} now becomes:

$$\bar{y}_{cg} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} = \frac{hbt + \frac{17}{32}h^2 t}{2bt + \frac{5}{4}ht} = \frac{hb + \frac{17}{32}h^2}{2b + \frac{5}{4}h} \tag{6}$$

2. The principle moment of inertia about the principle x-axis through the centroid I_x.

Answer:

$$I_x = \sum_{i=1}^{4}\left\{ I_{x_i} + A_i d_i^2 \right\}$$

$$= \left\{ \frac{bt^3}{12} + bt\left(h - \bar{y}_{cg}\right)^2 + \frac{h^3 t}{12} + ht\left(\frac{h}{2} - \bar{y}_{cg}\right)^2 + \frac{bt^3}{12} + bt\bar{y}_{cg}^2 + \frac{\left(\frac{h}{4}\right)^3 t}{12} + \frac{ht}{4}\left(\bar{y}_{cg} - \frac{h}{8}\right)^2 \right\} \tag{7}$$

Neglecting higher order terms of t gives:

$$I_x = \left\{ bt\left(h - \bar{y}_{cg}\right)^2 + \frac{h^3 t}{12} + ht\left(\frac{h}{2} - \bar{y}_{cg}\right)^2 + bt\bar{y}_{cg}^2 + \frac{h^3 t}{768} + \frac{ht}{4}\left(\bar{y}_{cg} - \frac{h}{8}\right)^2 \right\} \tag{8}$$

Thin-walled circular cross-sections

For thin-walled circular cross-sections similar assumptions may be made.
This results that for a thin-walled circle of radius R and thickness t the following cross-sectional constants may be calculated.

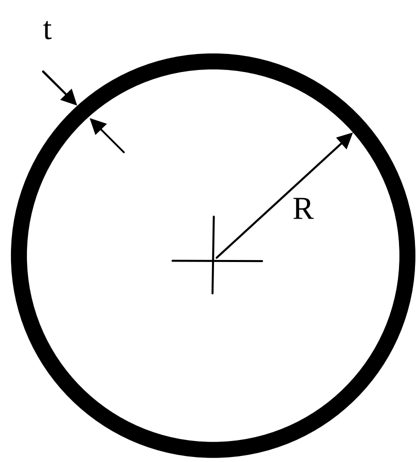

Figure 2

For the cross-sectional area A:

$$A = 2\pi Rt \tag{9}$$

The polar moment of inertia J_0 for a thin-walled ring can be derived by considering a small area dA of the ring (see figure 3):

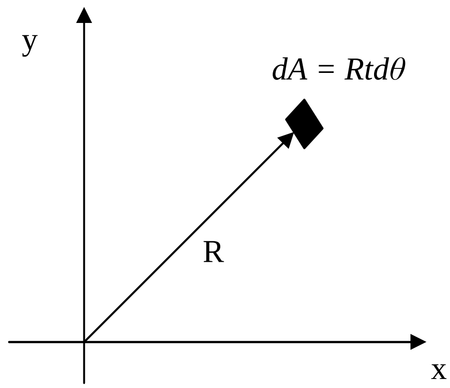

Figure 3

With:

$$J_0 = \int_A r^2 dA = 2\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} R^2 dA = 2R^3 t \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta = 2\pi R^3 t \qquad (10)$$

And finally, for the area moment of inertia with respect to the (principal) x- and y- axis, I_x and I_y we find:

$$I_x = I_y = \frac{1}{2} J_0 = \pi R^3 t \qquad (11)$$

GNS, September 2009