1.1 Hertzian Contact Problem

In 1880, Heinrich Hertz analysed the contact problem of two elastic bodies. He developed a theory for the computation of the contact area and the contact pressure between the surfaces of two bodies. Based on his model, the induced stresses to the bodies can be predicted. In this homework, the solution quality of a numerical prediction of the contact phenomena shall be analyzed based on the Hertz theory.

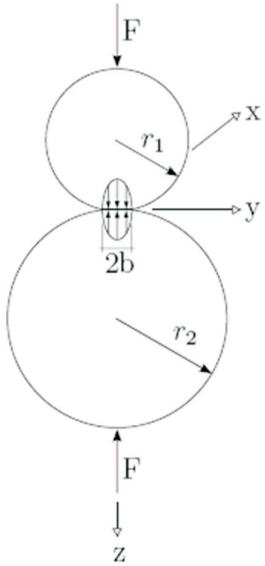

Figure 1.1: Hertz contact model for two infinitely long cylinders.

Solve the problem using the following steps:

1. study relevant literature on the Hertz theory and list the assumptions of the Hertz contact model. Explain, briefly, the consequences for the numerical modeling and simulation.
2. show the analytical solution of the problem found by Hertz.
3. propose a numerical model and comment on the chosen geometric properties, the type of loading and the essential boundary conditions. Give a sketch of your model with all relevant data for a complete model description. Exploit symmetry to reduce the model size.
4. model the problem with Abaqus and run simulations for three different mesh refinements. Report the solution results in comparison to the analytical Hertz solution and provide adequate error measures (relative error in energy norm (alternative: L2-norm), contact pressure, contact area). Provide the time effort of the analysis and comment on your results.
5. show the contact pressure versus the contact witdh in a diagram and compare the numerical prediction to the analytical solution. Comment on your results.
6. show a plot of the distribution of the relevant stress component. Comment on the results.