3 FLOOD PROPAGATION

3.1 Reservoir routing

The most important equation to describe the water balance of a reservoir is the water balance:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = I - Q + A\left(P - E\right) \qquad \textit{Equation 3.1}$$

In finite differences form this equation can be written as:

$$S_1 = S_0 + \left(I - Q + A\left(P - E\right)\right) \cdot \left(t_1 - t_0\right) \qquad \textit{Equation 3.2}$$

where P is the rainfall, E is the evaporation, A is the surface area of the reservoir, S is the storage, I is the inflow and Q is the reservoir release (outflow). In Equation 3.2, the inflow, the rainfall and the evaporation are input data; the initial storage is an initial condition; the time is an independent variable. To determine the storage at a certain time t_1, the outflow and the surface area should be known. However, these depend on the water level in the reservoir, and thus on the storage to be computed. Equation 3.2, therefore, cannot be solved explicitly, but has to be solved iteratively.
For the solution of Equation 3.2, three extra equations are necessary to relate the outflow, the surface area and the storage to the water level. The following types of equations are widely applicable. They may have to be modified somewhat for application in a specific case.

$$A = A\left(H\right) \qquad \textit{Equation 3.3}$$

$$S = \int_{H_0}^{H} A\,\mathrm{d}H \qquad \textit{Equation 3.4}$$

$$Q = K\left(H - H_c\right)^c \qquad \textit{Equation 3.5}$$

Equation 3.3 is obtained from planimetering a topographical map. Often an exponential equation of the following type serves the purpose well:

$$A = A_0 \exp\left(b\left(H - H_0\right)\right) \qquad \textit{Equation 3.6}$$

where A_0 is the surface area at H_0. The equation plots a straight line on semi-logarithmic paper. But also a power function of the type:

$$A = A_0 + a\left(H - H_0\right)^b \qquad \textit{Equation 3.7}$$

can often be used. The equation plots a straight line on double logarithmic paper. Both Equations 3.6 and 3.7 are easily integrated to yield Equation 3.4.

Flood routing through a reservoir

In the case of a flood passing through the reservoir, the outflow hydrograph and the water levels in the reservoir can be computed. At the relative small time steps used for flood routing, the direct rainfall on the reservoir and the evaporation from the reservoir can be neglected.

The following procedure is commonly used in spillway design to determine the required dimensions of the spillway. Equation 3.5 is a spillway function. In the case of a free overflow spillway, the exponent c = 1.5 and the coefficient $K \approx 1.5*B$, where B is the spillway width; H_c is the crest level of the spillway. The equation can be modified to fit another spillway type, if required. The set of equations 3.2-3.5 can be solved iteratively:

1. Assume a certain spillway design by determining values for K, c and H_c. In most cases the simulation is started with a full reservoir:
$$H_0 = H_c$$

2. In a first approximation, Equation 3.2 is solved assuming that the outflow Q remains constant over the time step: $Q = Q(H_0)$ and that the effect of local rainfall and precipitation can be neglected in relation to the flood flows. The storage thus obtained is the first estimate of the storage S^*. The equation used is called the *predictor*:
$$S_1^* = S_0 + \left(I - Q\left(H_0\right)\right) \cdot \left(t_1 - t_0\right) \qquad [m^3]$$

3. On the basis of S^*_1, H^*_1 is computed using the inverse of Equation 3.4:
$$H_1^* = H\left(S_1^*\right)$$

4. With this first estimate of the waterlevel at t_1, a new estimated storage at t_1 can be made, using the *corrector*:
$$S_1 = S_0 + \left(I - Q^*\right) \cdot \left(t_1 - t_0\right)$$
with
$$Q^* = K\left(\frac{H_1^* + H_0}{2} - H_c\right)$$

5. The corresponding reservoir level follows from $H_1 = H(S_1)$.

6. If necessary steps 4 and 5 are repeated (substitutions H_1 for H^*_1) until no further significant change in S_1 occurs.

7. Subsequently, the procedure is repeated in step 2 for the following time step:
$$S_2^* = S_1 + \left(I - Q\left(H_1\right)\right) \cdot \left(t_2 - t_1\right)$$
until the full flood wave has been simulated.

8. At the end of the simulation the maximum reservoir level and the maximum discharge are obtained corresponding to the assumed spillway design.

The iterative procedure described, based on the set of Equations 3.2 through 3.5, is easy to perform in a spreadsheet. Figure 3.1 is an example of the output of the spreadsheet model RESSIMFL.

Two observations can be made from studying Figure 3.1. Firstly, the inflow and the outflow hydrographs intersect at the point of maximum outflow; and secondly, the volume enclosed by the two curves left of the intersection is equal to the volume enclosed to the right of the intersection (assuming the water level is at the spillway crest at the start of the inflow hydrograph). The former volume is the part of the inflow, which is temporarily stored in the reservoir above the crest of the spillway, the latter volume is the release of that same amount. Before the point of intersection the storage increases; after the point of intersection the storage decreases.

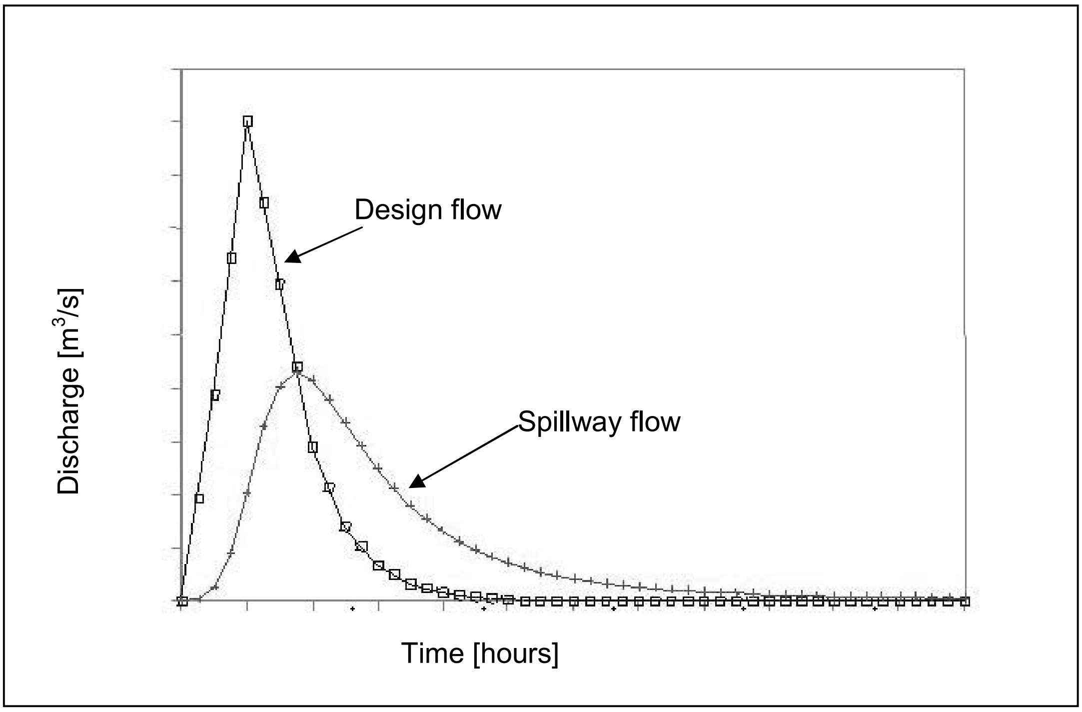

Figure 3.1: Inflow and outflow hydrograph of a reservoir

That the maximum outflow occurs at the point of intersection can be made clear by the following reasoning. It follows from Equation 3.1 that (neglecting the effect of rainfall and evaporation):

$$\frac{\mathrm{d}S}{\mathrm{d}t} = I - Q \qquad \text{Equation 3.8}$$

At the point of intersection this results in:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = 0$$

which because $S = S(H)$, and $\partial S / \partial H \neq 0$, results in:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0$$

Thus the maximum water level in the reservoir occurs when inflow equals outflow. Since the outflow $Q = Q(H)$, it follows that:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = 0$$

Hence, the maximum outflow occurs at the maximum water level.

Reservoir yield analysis

The previous paragraphs refer to the routing of a single flood wave, for which the process time-scale is in the order of hours to days, depending on the size of the catchment and the reservoir.

For reservoir yield analysis flood waves are also crucial, as they contain most of the water that a reservoir is supposed to store for later use. However the process time-scale of reservoir yields is much longer than that of individual flood waves and is in the order of month to a year. Hence the time step in reservoir yield analysis generally varies from a week to month. Within such a month various smaller floods may have occurred. However at this process scale these variations are not relevant.

In reservoir yield analysis, the same equation (Equation 3.2) is used as for flood routing. In this case however, the effect of rainfall and evaporation can no longer be disregarded.

In yield analysis, the time series of *P*, *E* and *I* are known values. The variation of the storage *S* over time and the reservoir outflow, or release, *Q* are the unknown parameters. The reservoir release is composed of the draft, *D*, being the planned or envisioned release, and the spill over the spillway, *L*.

$$Q = D + L \qquad \text{Equation 3.9}$$

The way the yield analysis is approached is by assuming a certain draft, possibly as a function of time, *D(t)*, on the basis of which the reservoir simulation is made. The spill, *L(t)*, follows from the reservoir operation.

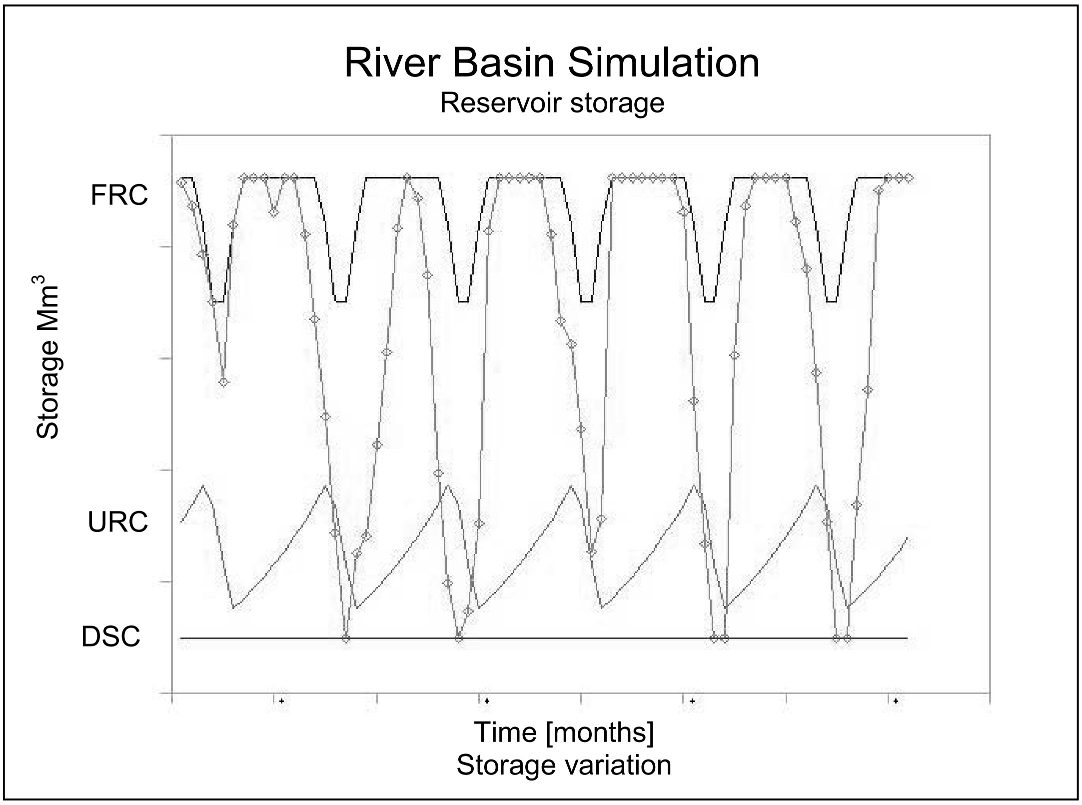

Figure 3.2: Reservoir operating rules (rule curves) and simulating storage variation by the model WAFLEX

The solution of Equation 3.2 is only possible if operating rules are used, that determine the release as a function of storage and a set of rule curves that can be functions of time:

$$Q = Q\left(S, RC_1\left(t\right)\right), RC_2\left(t\right), \ldots\ldots.. RC_n \qquad \textit{Equation 3.10}$$

Although, in principle, many different operating rules can be used, most reservoirs follow the basic operating rules of the following example. Figure 3.2 shows three operating rules:

- The Flood Rule Curve (FRC), which is a hard boundary (meaning it may not be crossed[1]). The FRC represents storage levels, which are a function of time, FRC(*t*). If the storage is more than FRC, all additional water is spilled (L*dt=S-FRC):

  If *S* > FRC, then *Q* = *D* + (*S* - FRC)/dt and *S* = FRC

- The Utility Rule Curve, URC(*t*), which is a soft boundary (it may be crossed). If the storage reaches, or crosses, the URC, the release from the reservoir is reduced by a certain rationing percentage *r*:

  If *S* < URC, then *Q* = *r* * *D* and the water balance is redone with *Q* = *r* * *D*

- The Dead Storage Curve, DSC(*t*), which is a hard boundary. The storage may never drop below this level as a result of releases, only due to evaporation. The dead storage requirement is often for environmental or ecological reasons. If, as a result of the draft, the storage drops below the DSC, then the release is reduced in the following way:

  If *S* < DSC, then *Q* = Max (0, D - (DSC - S)/dt) and redo water balance with this release.

  As a result of evaporation it is possible that D - (DSC - S)/dt < 0. In that case the release is zero and the water balance results in storage below DSC.

The areas between the curves are generally called zones, and the drawing of rule curves is also called zoning of the reservoir storage. In Figure 3.2, zone 1 is the area under DSC; zone 2 is the area between DSC and URC; zone 3 is the area between URC and FRC; and zone 4 is the area above FRC. The line indicated by the symbols is the storage variation as simulated by the spreadsheet model WAFLEX (Savenije, 1995b).

[1] The FRC is only a hard boundary at the time scale used for reservoir simulation (a time step of a week, decade or month). During day-to-day operation, the FRC can be crossed temporarily during the spilling operation.

During the simulation, the following steps are followed:

1. Establish a draft pattern *D(t)* and assume that the release *Q=D(t)*. The inflow *I* is taken as the average over the time step $dt=(t_1-t_0)$.
2. Solve the water balance equation 3.2 in numerical form:

$$S_1 = S_0 + (I - Q)(t_1 - t_0) + (P - E)(t_1 - t_0)A(H_0) \qquad \text{Equation 3.11}$$

   where $S_1 = S(t_1)$ and $A(H_0)$ is the inundated area at water level $H(t_0)$. Although it would be more correct to use the inundated area as a function of the average water level between t_1 and t_0, which would require an extra iteration in the computation, such a procedure is generally not necessary as the error made by the rainfall and evaporation term is expected to be small.
3. Check the operating rules. If necessary the release *Q* and the storage S_1 should be adjusted according to the operating rules.
4. Now that S_1 and *Q* are known, the computation for the next time step can be started similar to step 2:

$$S_2 = S_1 + (I - Q)(t_2 - t_1) + (P - E)(t_2 - t_1)A(H_1)$$

5. The above steps are repeated until the end of the data series is reached. At the end of the simulation the shortage of water is computed, as well as the amount of water spilled. On the basis of this information a decision can be taken to adjust the release pattern, *D(t)*, or to use other rule curves.

The order to be able to draw quantitative conclusions from the reservoir yield analysis, the starting value of the storage, S_0, should be equal to the end value, S_n. This can generally be achieved by one iteration where "lay" S_n is used as S_0, provided the reservoir volume is not too large in relation to the inflow.

3.2 Flood routing in natural channels

The volume of water in a channel at any instant is called channel storage *S*. The most direct determination of *S* is by measurement of channel volume from topographic maps. However lack of adequately detailed maps plus the need to assume or compute a water-surface profile for each possible condition or flow in the channel makes this approach generally unsatisfactory. Since Equation 3.2 involves only *S* ($S = S_1 - S_0$), absolute values of storage need not be known. Values of S can be found by solving Equation 3.2, using actual values of inflow and outflow (Figure 3.3). For flood routing, the effect of rainfall and evaporation on the storage in the channel reach can be disregarded. The hydrographs of inflow and outflow for the reach are divided into short time intervals, average values of *I* and Q are determined for each period, and values of *S* computed by subtracting Q from *I*.

Storage volumes are computed by summing the increments of storage from any arbitrary zero point.

When values of *S* computed as just described are plotted against simultaneous outflow (Figure 3.4), it usually appears that storage is relatively higher during rising stages than during falling stages. As a wave front passes through a reach, some storage increase occurs before any increase in outflow. After the crest of the wave has entered the reach, storage may begin to decrease although the outflow is still increasing. Nearly all methods of routing stream flow relate storage to both inflow and outflow in order to allow for these variations.

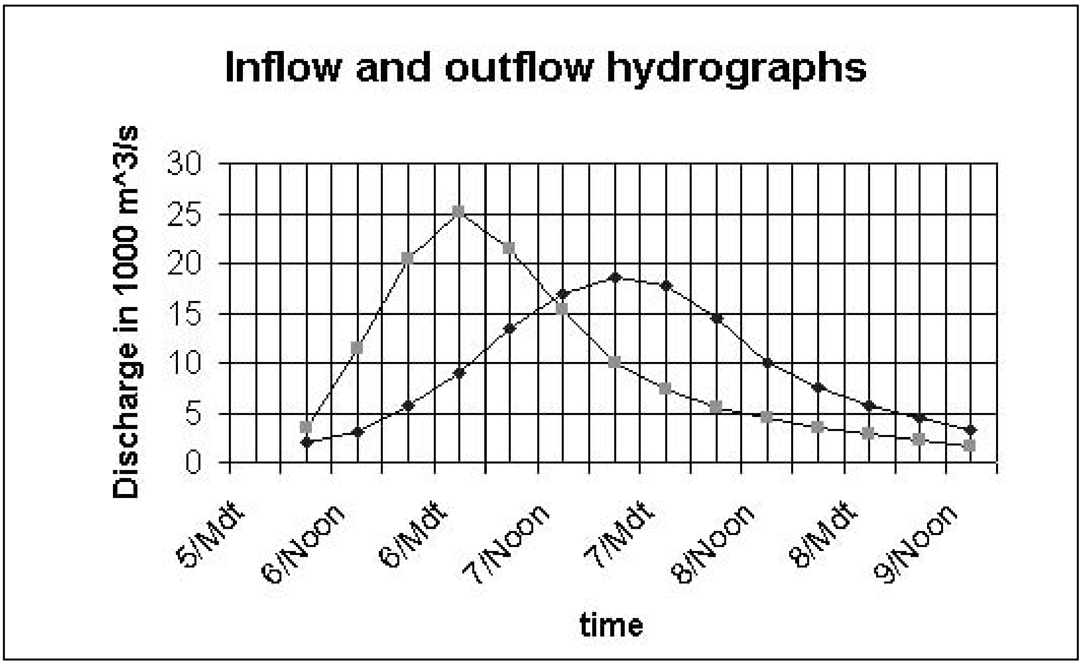

Figure 3.3: Inflow and outflow hydrographs for a reach of river, showing method of calculating channel storage

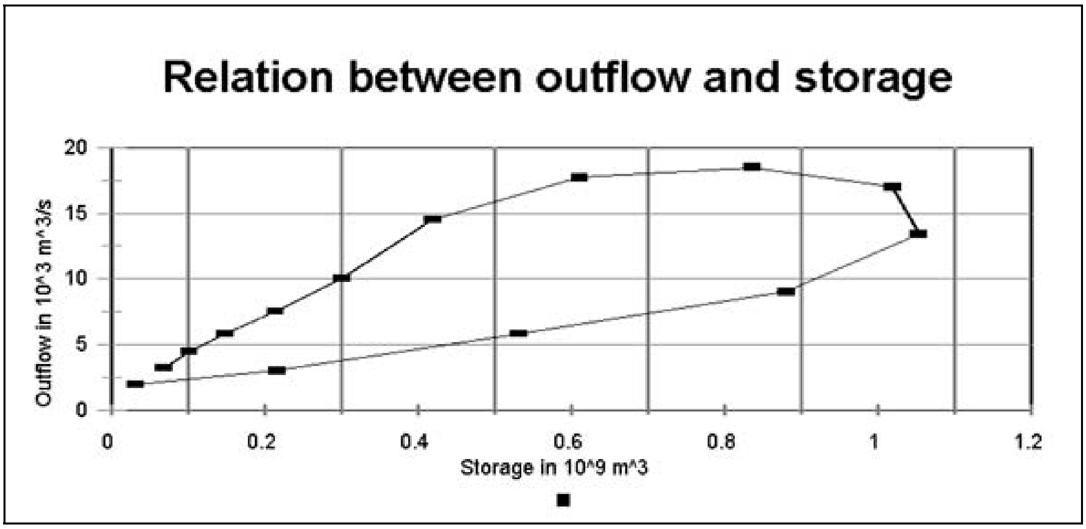

Figure 3.4: Relation between outflow and storage for the data of Figure 3.3

A very widely utilized assumption is that storage is a function of weighted inflow and outflow, which yields the Muskingum equation:

$$S = K\left[xI + (1-x)Q\right] \qquad \textit{Equation 3.12}$$

where *S*, *I*, and *Q* are simultaneous values of storage, inflow, and outflow over a reach Δx. The dimensionless constant *x* indicates the relative importance of *I* and *Q* in determining storage, and *K* is a storage constant with the dimension of time. The value of *K* approximates the time of travel of the wave through the reach. If the celerity of propagation of the wave is *c*, and the length of the reach considered is Δx , then $K = \Delta x/c$. A flood wave in a river behaves as a mass wave with the equation:

$$c = \frac{\mathrm{d}q}{\mathrm{d}h} \approx 1.67v \qquad \textit{Equation 3.13}$$

where q is the discharge per unit width, v is the cross-sectional average flow velocity and h is the cross-sectional average depth of flow. If the discharge obeys Mannings formula in a rectangular cross-section, then the wave celerity is about 60-70% higher than the average flow velocity v. Hence the wave celerity depends on the flow velocity and K is not constant: it is larger for larger floods.

determination of x

In theory, the constant x varies from 0 to 0.5. Cunge (1969) showed that x can be related to physical parameters:

$$x = 0.5\left(1 - \frac{q}{S_b cL}\right) \qquad \textit{Equation 3.14}$$

where S_b is the bottom slope.

Since d*S*/dt = *I*-*Q*, differentiating Equation 3.12 yields

$$I - Q = \frac{dS}{dt} = K\left[x\frac{dI}{dt} + (1-x)\frac{dQ}{dt}\right] \qquad \textit{Equation 3.15}$$

If I = Q, then at the point of intersection:

$$x = \frac{dQ/dt}{dQ/dt - dI/dt} \qquad \textit{Equation 3.16}$$

which permits estimating x from concurrent inflow and outflow records. For a reservoir where *Q* = f(S), d*S*/dt and d*Q*/dt must be zero when *I* = *Q*. Therefore *x* for this case is zero. A value of zero indicates that the outflow alone determines storage (as in a reservoir). When *x* = 0.5, inflow and outflow have equal influence on storage. In natural channels *x* usually varies between 0.1 and 0.3.

Values of *K* and *x* for a reach are usually determined by trial. Values of *x* are assumed, and storage is plotted against $xI + (1-x)Q$. The value of *x* which results in the data conforming most closely to a straight line is selected (Figure 3.5). The travel time *K* is the slope of the line relating *S* to $xI+(1-x)Q$.

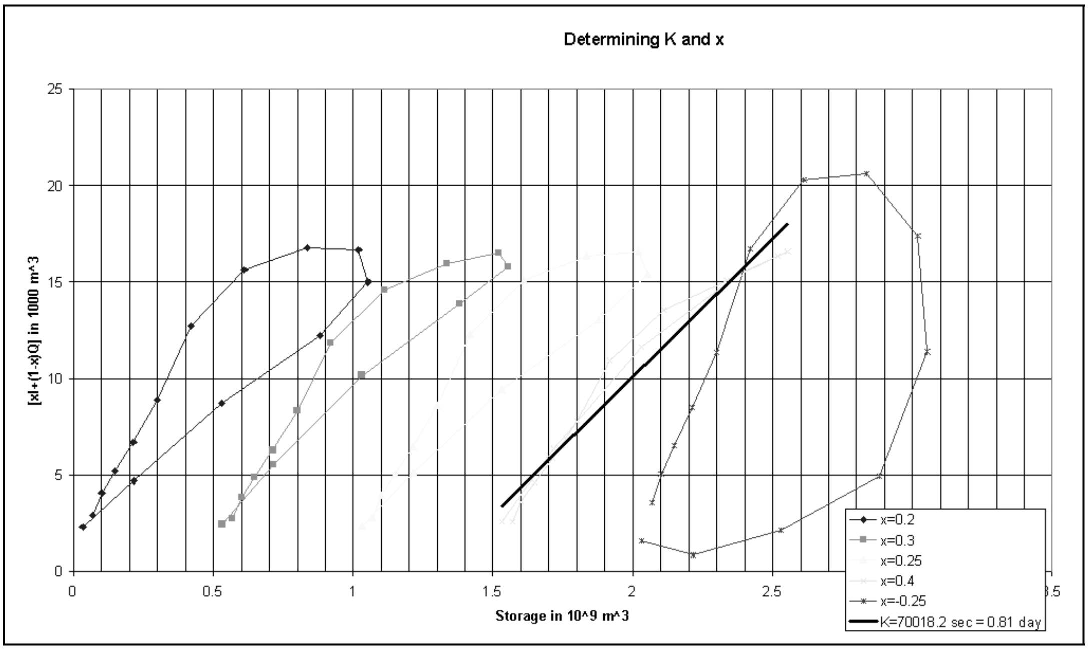

Figure 3.5: Method of determining K and x for the Muskingum method of routing

The Muskingum routing equation is found by substituting Equation 3.11 and solving for Q_2,

$$Q_2 = c_0 I_2 + c_1 I_1 + c_2 Q_1$$ *Equation 3.17*

$$c_0 = \frac{-2x + \Delta t / K}{2(1-x) + \Delta t / K}$$ *Equation 3.18a*

$$c_1 = \frac{2x + \Delta t / K}{2(1-x) + \Delta t / K}$$ *Equation 3.18b*

$$c_2 = \frac{2(1-x) - \Delta t / K}{2(1-x) + \Delta t / K}$$ E*quation 3.18c*

$$c_0 + c_1 + c_2 = 1$$ *Equation 3.18d*

The significance of Equation 3.18d may be seen if it is noted that, for steady flow ($I_1 = I_2 = Q_1 = Q_2$), Equation 3.17 can be correct only when the sum of the constants is unity. It is important that K and t be in the same units when used in Equations 3.18. When *Q* and *I* are in m^3/s and storage is computed in cubic meters, the units of *K* and *t* are seconds.

Table 3.1: Application of the Muskingum method

Date	Hour	I, m^3/sec	C_0I_2	C_1I_1	C_2Q_1	Q, m^3/sec
4/9	6 a.m.	1000	…	…	…	1000
	Noon	2400	*-408*	*530*	*640*	*762**
	6 p.m.	3900	*-663*	*1272*	*488*	*1097*
	Midnight	5000	*-850*	*2067*	*702*	*1919*
4/10	6 a.m.	4900	*-833*	*2650*	*1228*	*3045*
	Noon	4000	*-680*	*2597*	*1949*	*3866*

Note: Computed values are in Italic. Coefficients used are c_0 = -0.17, c_1 = 0.53 and c_2 = 0.64.
* The first computed outflow often drops when inflow increases sharply. This is simply disregarded.

The application of the Muskingum method is illustrated in Table 3.1. Values of c_0, c_1 and c_2 were computed by substituting K = 0.82 day, x= 0.3 (Figure 3.5), and Δt = 6 hr in Equation 3.16a, b, c. Values of I are tabulated, and the products c_0I_2 and c_1I_1 are computed. With an initial value of Q_1 given or estimated, the product c_2Q_1 is calculated and the three products added to obtain Q_2. The computed value of Q_2 becomes Q_1 for the next routing period, and another value of Q_2 can be determined. The process continues as long as values of I are known. This routing is easily performed in a spreadsheet.

Storage in a river reach actually depends on water depths. The assumption that storage is correlated with rates of flow is a valid approximation only when stage and discharge relations are closely correlated. Because of the hysteresis effect, this is not completely correct. In streams, with a complex slope-stage-discharge relation, more complex routing methods (and more data, particularly on geometry) are required to obtain satisfactory accuracy. The methods described in the preceding sections assume that the longitudinal profile of the water surface in a reach is the same every time a given combination of inflow and outflow occurs. This is also an approximation but is usually sufficiently precise if the reach is not excessively long. In general, the methods, which have been described, are satisfactory on the great majority of streams.

In making a Muskingum routing, one has to take into account the value of the Courant number: $N_{Cr}=c\Delta t/\Delta x$, which should always be less than 1. If the flood wave can travel through the reach Δx in a time less than the time step Δt, then computational instabilities may occur. Hence:

$N_{Cr} = \frac{c\Delta t}{\Delta x} \leq 1$, or with $K=\Delta x/c$:

$$\Delta t \leq \frac{\Delta x}{c} \leq K$$

Kinematic routing

Kinematic routing involves the simultaneous solution of the continuity equation:

$$Q = I - L\frac{\Delta A}{\Delta t} \qquad \text{Equation 3.19}$$

and a flow equation such as the Manning equation:

$$Q = KAR^{2/3}S^{1/2} \qquad \text{Equation 3.20}$$

where A is the cross-sectional area, L the length of the reach, and hence $L\Delta A$ is the change in storage. In kinematic routing the energy slope S is taken as the bed slope S_b and an iterative solution is used until both equations yield consistent values of Q. A mean cross section of the reach is a required input. Kinematic routing is typically performed on a computer.

In the form described above, kinematic routing is subject to all of the assumptions of hydrologic routing and its principal advantage is an ability to deal with non-linear storage-stage relations on the basis of a measured cross section. The reliability of kinematic and hydrologic routing are roughly the same. Neither method works well on very flat slopes where second-order terms in the energy equation may exceed the bed slope, nor on very steep slopes where supercritical flow occurs.

The rate of convergence of the solution depends on how well Q_2 is estimated for the first trial. Many assumptions are possible such as $Q_2 = Q_1$ or $Q_2 = Q_1 + (Q_1 - Q_0)$. Another possibility is to use a very short routing period such that Q is small and eliminate the iteration.

Local inflow

The previous discussion has considered the routing of inflow entering at the head of a reach. In almost all streams there is additional inflow from tributaries, which enter the main stream between the inflow and outflow points of the reach. Occasionally this *local inflow* is small enough to be neglected, but often it must be considered. The conventional procedures are (1) add the local inflow to the mainstream inflow, and consider the total as I in the routing operation, or (2) route the main-stream inflow through the reach, and add the estimated local inflow to the computed outflow. The first method is used when the local inflow enters the reach near its upstream end, while the second method is preferred if the greater portion of the tributary flow joins the main stream near the lower end of the reach. The local inflow might also be divided into two portions, one part combined with the mainstream inflow and the remainder added to the computed outflow.

If the lateral inflow is known, one can use the four-point Muskingum method where there is a c_3 to be multiplied with the lateral inflow Q_L (see Ponce, 1979).

$$Q_2 = c_0 I_2 + c_1 I_1 + c_2 Q_1 + c_3 Q_L \qquad \textit{Equation 3.21}$$

$$c_3 = \frac{2\Delta t / K}{2(1-x) + \Delta t / K} \qquad \textit{Equation 3.22}$$

The hydrograph of lateral inflow may be estimated by comparison with stream flow records on tributary streams or by use of rainfall-runoff relations and unit hydrographs. In working with past data, the total volume of local inflow should be adjusted to equal the difference between the reach inflow and outflow, with proper allowance for any change in channel storage during the computation period. Since local inflow may be a small difference between two large figures, slight errors in the stream flow record may result in large errors in local inflow, even to the extreme of indicating negative local inflows.

There also is a so-called three parameter Muskingum method which allows for lateral inflow or lateral seepage loss as a percentage of the flow (O'Donnell, 1985):

$$\frac{\mathrm{d}S}{\mathrm{d}t} = I(1+\alpha) - Q \qquad \textit{Equation 3.23}$$

$$S = K\left[x(1+\alpha)I + (1-x)Q\right] \qquad \textit{Equation 3.24}$$