# SPECIAL FLOW CONDITIONS IN 

 DREDGING PIPELINES
### 6.1 INCLINED FLOWS

Solids, dredged from a bottom of a waterway or in a borrowing pit, are transported hydraulically to the board of a dredge by using an inclined pipeline. A pipeline slope varies from rather flat to almost vertical according to the depth from which the material is dredged. The largest modern THSD's are capable of dredging from water deeper than 100 meter. This gives an imagination of how long inclined pipelines might be. Mixture flows, particularly that exerting a certain degree of stratification, are very sensitive to changes in a pipeline inclination. In this chapter the effects are discussed of the pipeline inclination on the hydraulic gradient and the deposition-limit velocity in mixture flow. Further the effects of inclination on a flow pattern are demonstrated and a physical description is given of these inclination effects.

### 6.1.1 Static head and friction head

Consider a horizontal pipeline of a certain length occupied by a flowing mixture. A total pressure drop in mixture flow over the pipeline section is equal to the pressure drop due to internal friction in flowing mixture if there are no additional minor losses from the local sources of energy dissipation as are pipe joints, bends etc. The frictional head loss in the mixture flow is due to both the frictional losses in a carrying liquid and the additional losses due to a presence of solids in a carrying liquid as described in details in Chapters 4 and 5.

If the pipeline section is inclined the total pressure drop over the section changes considerably. The differential pressure transmitter measuring the pressure difference over the pipeline section senses a considerable increase (in case of the ascending pipeline section) or a considerable decrease (in case of the descending pipeline section) of the total pressure differential. This is due to a hydrostatic pressure differential emerging as a result of a change of a geodetic position of the end of the pipeline section regarding to its begin. A change of a pipeline elevation gives to arise to the static pressure differential caused by a pressure exerted by a mixture column of a height given a vertical distance between the begin and the end of a pipeline section. The hydrostatic column increases pressure at the beginning of a pipeline section in the case of an ascending pipeline or at the end of a pipeline section in the case of a descending pipeline. Considering a pipeline section of a length $L$ inclined to an angle $\omega$ from a horizontal position, the height of a vertical column is $\mathrm{H}_{\mathrm{pipe}}=\mathrm{L} . \sin \omega$.

Thus the total pressure gradient $\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right) / \mathrm{L}=\Delta \mathrm{P} / \mathrm{L}$ over a pipeline section of the length $L$ (see Fig. 6.1) is composed of

- the static pressure gradient $\left(\rho \mathrm{gH}_{\text {pipe }}\right) / \mathrm{L}$, giving the potentially reversible effect of elevation change on the total pressure gradient in a mixture flow of the density $\rho$ gaining the height $\mathrm{H}_{\text {pipe }}$ and
the pressure gradient due to friction $\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right) / \mathrm{L}-\left(\rho \mathrm{gH}_{\mathrm{pipe}}\right) / \mathrm{L}$ that is the irrecoverable energy loss due to friction in inclined mixture flow over the pipe length L .


Figure 6.1. Schematic length-section of inclined pipe.

The pressure differential between two pipeline cross sections 1 and 2 separated from each other by the pipeline length L is measured usually as a manometric pressure differential $\mathrm{P}_{1}+\mathrm{H}_{\text {pipe. }} \rho_{\mathrm{f}} \mathrm{g}-\mathrm{P}_{2}$, i.e. $\Delta \mathrm{P}+\mathrm{H}_{\text {pipe }} \cdot \rho_{\mathrm{f}} \mathrm{g}$ (Fig. 6.1). The total pressure differential $\Delta \mathrm{P}$ is obtained by eliminating the static pressure differential due to the water column in the hoses of the differential pressure transmitter (manometer). The pressure differential due to friction is obtained from the total pressure differential $\Delta \mathrm{P}$ by subtracting (or adding) the static pressure differential $\mathrm{H}_{\text {pipe }}$. $\rho . g$ due to a mixture column in the measuring pipe section from a pipe elevation.

The static pressure gradient $H_{\text {pipe }} \rho . \mathrm{g} / \mathrm{L}$ is produced by a mixture column of the height $H_{\text {pipe }}$ in the pipeline section $L$. The density of the column, $\rho$, is determined from the concentration of solid particles in the section $L$ which contribute to the weight of the slurry column. Empirical models for the prediction of friction losses in
inclined pipelines assume that the slurry of the column has the density $\rho_{\mathrm{m}}=\rho_{\mathrm{f}}+$ $\mathrm{C}_{\mathrm{vd}}\left(\rho_{\mathrm{S}}-\rho_{\mathrm{f}}\right)$.

### 6.1.2 Deposition-limit velocity

Deposition-limit velocity tends to increase with a pipe slope and reaches its maximum at an angle of about $25-35 \mathrm{deg}$ in an ascending pipeline. A further increase in an angle of an ascending pipe inclination causes a gradual decrease of the $\mathrm{V}_{\mathrm{dl}}$ value until zero at the limit inclination angle 90 deg . At angles higher than approximately 45 deg a bed is gradually disintegrated owing to a continuously diminishing cross-pipe component of solid particle weight - the force component usually responsible for the formation of a bed. In a descending pipeline the deposition-limit velocity gradually decreases when the pipeline is inclined gradually from 0 deg to -90 deg.

### 6.2 EMPIRICAL MODELING OF INCLINED FLOWS

Typical models for inclined flows are extensions of models for flows at limit inclinations: in horizontal and vertical pipes.

### 6.2.1 Vertical-flow model

Uniform distribution of solids across a pipeline cross section is characteristic of mixture flow in a vertical pipeline. The homogeneous character of mixture makes prediction of vertical flows easier than prediction of horizontal and inclined flows. Coulson et al. (1996) summarized the simplest conclusions for the prediction of frictional head loss in a vertical mixture pipeline as follows:

- for non-settling suspensions the standard equation for a single phase fluid is used with the physical properties of the suspension in place of those of the liquid (i.e. transported particles do not affect the friction process in coarse-particle mixture flow in a vertical pipeline)
- for a suspension of coarse particles the value calculated for the carrying fluid alone, flowing at the mixture velocity, is used.

It should be stressed, however, that the above-formulated rules are considered only for Newtonian mixtures. The non-Newtonian mixtures exert in vertical pipelines frictional head losses equal to that in horizontal pipelines.

### 6.2.2 Inclined-flow model by Worster \& Denny

Worster \& Denny (1955) suggested a simple equation for the energy loss in settling slurries flowing in inclined pipelines
$\mathrm{I}_{\mathrm{mh} \omega}=\mathrm{I}_{\mathrm{f}}+\left(\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}\right) \cos \omega+\mathrm{C}_{\mathrm{Vd}}\left(\mathrm{S}_{\mathrm{S}}-1\right) \sin \omega$
$\mathrm{I}_{\mathrm{mh} \omega}$ manometric gradient in mixture flow in inclined pipe
$\mathrm{I}_{\mathrm{m}} \quad$ hydraulic gradient in the same mixture flow in horizontal pipe [-]
$\mathrm{If}_{\mathrm{f}} \quad$ hydraulic gradient in liquid flow
$\omega$ pipe inclination angle
$\mathrm{C}_{\mathrm{vd}}$ delivered volumetric solids concentration
$\mathrm{S}_{\mathrm{S}} \quad$ relative density of solids
The angle $\omega$ is considered to have positive values in an ascending pipeline and the negative values in a descending pipeline.
The head loss due to the potential energy change registered by a differential pressure transmitter is represented by the last term in Eq. 6.1. This is the hydrostatic effect on the pressure differential measured over a section of an inclined mixture pipe.


Figure 6.2. Pressure drops in inclined pipelines, after Worster \& Denny (1955).

The ratio between solids effect on the frictional head loss in the inclined pipeline and solids effect on the frictional head loss in the horizontal pipeline for the same mixture flow parameters is then given as

$$
\begin{equation*}
\frac{\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}=\cos \omega \tag{6.2}
\end{equation*}
$$

$\mathrm{I}_{\mathrm{m} \omega}$ hydraulic gradient due to friction in mixture flow in inclined pipe [-]
$\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}} \quad$ solids effect in inclined pipe
$\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}} \quad$ solids effect in horizontal pipe

### 6.2 3 Inclined-flow model by Gibert

Gibert (1960) adapted the Durand \& Condolios correlation (Eq. 4.4) to inclined pipelines by using a simple assumption that only the gravitational acceleration component perpendicular to an inclined-pipeline axis (g.cos $\omega$ ) influences the solids effect on the frictional head loss

$$
\begin{equation*}
\frac{\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{f}} \mathrm{C}_{\mathrm{vd}}}=\mathrm{K}\left(\frac{\mathrm{~V}_{\mathrm{m}}^{2}}{\mathrm{gD}} \frac{\sqrt{\mathrm{gd}}}{\mathrm{v}_{\mathrm{t}} \cos \omega}\right)^{-1.5} \tag{6.3}
\end{equation*}
$$

This gives

$$
\begin{equation*}
\frac{\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}=(\cos \omega)^{1.5} \tag{6.4}
\end{equation*}
$$

### 6.2.4 Inclined-flow model by Wilson

## FRICTIONAL HEAD LOSS

Wilson et al. (1997) proposed the following modification of the Worster \& Denny formula

$$
\begin{equation*}
\mathrm{I}_{\mathrm{mh} \omega}=\mathrm{I}_{\mathrm{f}}+\left(\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}\right) \cos \omega^{(1+\mathrm{M} \gamma)}+\mathrm{C}_{\mathrm{Vd}}\left(\mathrm{~S}_{\mathrm{S}}-1\right) \sin \omega \tag{6.5}
\end{equation*}
$$

thus

$$
\begin{equation*}
\frac{\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}=(\cos \omega)^{(1+\mathrm{M} \gamma)} \tag{6.6}
\end{equation*}
$$

The power $\gamma$ has a lower limit of 0.333 for very fine particles and, hypothetically, an upper limit of unity for very coarse particles. The power M is PSD-dependent and it gains a value 1.7 for a uniform PSD. Lower values of $M$ are obtained for a wellgraded PSD according to Eq. 4.18.

## DEPOSITION-LIMIT VELOCITY

The application of maximum deposition-limit velocity $\mathrm{V}_{\mathrm{sm}}$ by demi-McDonald can be extended to inclined pipelines by using the dimensionless deposition parameter $\Delta_{\mathrm{D}}$ sensitive to an angle of a pipeline inclination. The deposition-limit velocity in an inclined pipeline $\mathrm{V}_{\mathrm{Sm}} \omega$ is given as

$$
\begin{equation*}
\mathrm{V}_{\mathrm{sm} \omega}=\mathrm{V}_{\mathrm{sm}}+\Delta_{\mathrm{D}} \sqrt{2 \mathrm{gD}\left(\mathrm{~S}_{\mathrm{s}}-1\right)} \tag{6.10}
\end{equation*}
$$

$\mathrm{V}_{\text {Sm } \omega}$ maximum deposition-limit velocity in inclined pipe
$\mathrm{V}_{\mathrm{Sm}}$ maximum deposition-limit velocity in horizontal pipe
$\Delta_{\mathrm{D}} \quad$ deposition parameter, $\Delta_{\mathrm{D}}=\mathrm{fn}(\omega)$ from a graph on Fig. $6.3 \quad[-]$
$\omega$ pipeline inclination angle
D pipeline diameter
$\mathrm{S}_{\mathrm{S}} \quad$ relative density of solids


Figure 6.3. Effect of angle of pipe inclination on Durand deposition parameter, after Wilson \& Tse (1984).

### 6.2.5 Discussion of the empirical models

According to models of Worster \& Denny, Gibert or Wilson, the solids effect is always lower in inclined pipelines (for both the negative and positive slopes) than in horizontal pipelines. Furthermore, the friction loss is the same in pipe sections of the negative and the positive slope when the pipe inclination angle and flow parameters $\mathrm{V}_{\mathrm{m}}, \mathrm{C}_{\mathrm{vd}}, \mathrm{d}$ are identical.

The above prediction is based on an assumption that the frictional head loss due to a presence of solids is caused predominantly by solids settling tendency in a direction perpendicular to an inclined pipeline wall. A measure of a settling tendency represented by a particle settling velocity decreases with an increasing pipeline inclination angle because the component of the particle settling velocity in a direction perpendicular to a pipeline wall decreases and so decreases the possibility that the particles form a granular bed.

A physical consideration of a friction process in a stratified flow learns that the above assumption is not generally acceptable. A simplification of the flow process assumed in models described above might be acceptable for slightly stratified flows in which only a small portion of solids tends to form a bed.

The above-discussed models assume that all particles occupying the pipeline section attribute to the mixture column that produces a static head. Static pressure difference caused by pipe elevation is considered to be produced by a mixture column of concentration $\mathrm{C}_{\mathrm{vd}}$. Accepting the fact that all solids present in inclined pipe contribute to mixture column weight, the actual spatial concentration $\mathrm{C}_{\mathrm{vi}}$ should determine the solids concentration in the mixture column. Correct determination of $I_{m}$ $\omega$ from measured manometric pressure differential demands understanding of the difference between $\mathrm{C}_{\mathrm{vd}}$ and $\mathrm{C}_{\mathrm{vi}}$ in a measuring pipe section.

### 6.3 PHYSICAL MODELING OF INCLINED FLOWS

### 6.3.1 Two-layer model

Modeling of inclined flows might be successfully carried out using the empirical models if flow is only slightly stratified. Flows exerting significant flow stratification obey physical rules that are not considered in the empirical models. A two-layer model for inclined flows considering the two-layer flow pattern shown on Fig. 6.1 should handle such flows. For this model the force balance equations are formulated as follows (Matousek, 1997):
in the upper layer

$$
\begin{equation*}
\left(\Delta \mathrm{P}+\mathrm{H}_{\text {pipe }} \rho_{1} \mathrm{~g}\right) \mathrm{A}_{1}=\tau_{1} \mathrm{O}_{1} \mathrm{~L}+\tau_{12} \mathrm{O}_{12} \mathrm{~L} \tag{6.7}
\end{equation*}
$$

and in the lower layer

$$
\begin{equation*}
\left(\Delta \mathrm{P}+\mathrm{H}_{\text {pipe }} \rho_{1} \mathrm{~g}\right) \mathrm{A}_{2}-\mathrm{F}_{\mathrm{W}} \sin \omega=\tau_{2 \mathrm{f}} \mathrm{O}_{2} \mathrm{~L}+\mu_{\mathrm{s}} \mathrm{~F}_{\mathrm{N}} \cos \omega-\tau_{12} \mathrm{O}_{12} \mathrm{~L} \tag{6.8}
\end{equation*}
$$

In the whole pipe section the balance is then

$$
\begin{equation*}
\left(\Delta \mathrm{P}+\mathrm{H}_{\text {pipe }} \rho_{1} \mathrm{~g}\right) \mathrm{A}-\mathrm{F}_{\mathrm{W}} \sin \omega=\tau_{1} \mathrm{O}_{1} \mathrm{~L}+\tau_{2 \mathrm{f}} \mathrm{O}_{2} \mathrm{~L}+\mu_{\mathrm{S}} \mathrm{~F}_{\mathrm{N}} \cos \omega \tag{6.9}
\end{equation*}
$$

### 6.3.2 Discussion of the physical model

The model of this configuration for the inclined flows takes the inclination effects of different solids-support mechanisms into account. Basically, this model distinguishes between the frictional pressure differential and the static pressure differential on a basis of the physical picture of a friction process in inclined flows.

### 6.3.3 Observations verifying a validity of the physical model principles

Experiments carried out in Laboratory of Dredging Technology of Delft University of Technology in 1995-1996 (Matousek, 1997) provided the following conclusion:
The mixture flow behaves differently in an ascending pipe and in a descending pipe if flow tends to be stratified, i.e. if there is a bed at the bottom of the pipe. The difference was detected in measured frictional head loss, flow stratification, slip between phases in the flow and velocity of the bed. The difference diminishes at pipe-inclination angles above approximately 45 deg where the bed starts to be disintegrated.


Figure 6.4. Concentration profiles in a 150 mm pipe cross section for flow of an aqueous mixture of a $1.4-2.0 \mathrm{~mm}$ sand at velocity $3.5 \mathrm{~m} / \mathrm{s}$.
(Data from Laboratory of Dredging Technology, Delft University of Technology).
a. Flow stratification

Different degrees of flow stratification have been observed in the ascending pipe and the descending pipe for the same slurry flow conditions $\left(\mathrm{V}_{\mathrm{m}}, \mathrm{C}_{\mathrm{vd}}\right)$. The difference is
large for the coarse slurry flow and small for the relatively fine slurry flow. Thus the effect occurs in slurry flow where the turbulent suspension mechanism plays a minor role (or it is not effective at all) and the majority of particles occupy a granular bed.
A sharp flow stratification in the descending pipe and a gradual concentration change across the pipe cross section in the ascending pipe for coarse slurries (see Fig. 6.4) suggest that the small concentration gradient in the ascending pipe is the product of dispersive forces acting within the shear layer rather than of the turbulent mixing process in the liquid flow. In the ascending pipe the bed-submerged-weight component exerted against the flow direction has a resisting effect on the sliding bed and, owing to the steep velocity gradient between the sliding bed and the flow above it, a thick shear layer is developed. In the descending pipe, owing to the propelling effect of the submerged weight component in the flow direction, the velocity of the moving bed is sufficient to prevent the formation of a shear layer. Liquid turbulence alone is not able to suspend the coarse particles. Finer slurry flow at the same pipe inclinations $(+\omega,-\omega)$ demonstrates a considerably smaller difference between the shapes of concentration profiles, suggesting that here the carrier turbulence is the prevailing suspension mechanism and the shear layer is not well developed. It should be remembered that these effects are of importance primarily in flows inclined to angles not far above 35 deg. At these angles the cross-pipe component of the submerged bed weight is still important and the pipe slope is not the main cause of bed disintegration.
b. Slip ratio (transport factor) and bed velocity

The variation in the slip ratio is primarily due to a variation in the shift between layers in a stratified flow. Slip ratio is found to be strongly dependent on the shape of the concentration profile (see Fig. 6.4). The slip ratio value tends to approach unity when the flow becomes less stratified. If $\mathrm{F}_{\mathrm{W}} \sin \omega$ exceeds $\mu_{\mathrm{S}} \mathrm{F}_{\mathrm{N}} \cos \omega$ in a descending pipe the slip ratio reaches a value higher than unity. This is caused by a fact that a bed moves faster than the suspension layer above the bed under the above given condition in a descending pipeline. In an ascending pipeline the slip ratio is always lower than unity, i.e. the bed moves always slower than the suspension layer above the bed.
c. Static head

According to Bagnold's concept for the solids support in a mixture flow, the contact-load particles transfer their submerged weight to the pipe wall via the interparticle contacts. The particles are supported by the interparticle contacts. Solid particles with no interparticle contacts (suspended particles) transfer their weight to the carrying liquid and increase the density of the suspension. Thus only the solid particles whose submerged weight is not transmitted to the pipeline wall contribute to the slurry column which exerts the static pressure differential over an inclined pipeline section.
The density of the slurry column is the density of the mixture of the carrying liquid and suspended particles in an inclined pipeline section. The spatial concentration $\mathrm{C}_{\mathrm{vi}}$ in a pipeline section can be used to calculate slurry column density only when all particles are suspended. The delivered concentration $\mathrm{C}_{\mathrm{Vd}}$ determines the slurry column density only when all particles are suspended and, furthermore, the slip between phases in a pipeline section is negligible.

### 6.3.4 Comparison of empirical and physical approaches to the inclined flow modeling

Consider as an example an inclined flow of fully-stratified mixture in a descending pipeline inclined to -35 degrees. In such a flow the total pressure differential is experimentally detected as almost equal to that of carrying liquid alone. According to an empirical model this is because the frictional pressure drop produced by a presence of solids in a carrying liquid flow is reduced by a pressure gain due to the static pressure from a mixture column (containing all solid particles) in a pipeline section. According to the two-layer model, however, no pressure gradient is required to push a bed because the bed moves gravitationally (it might even move faster than the carrying liquid in the descending pipeline). Thus no extra frictional pressure differential is developed due to the presence of solids in a pipeline. In the same time, no static pressure differential occurs due to presence of solids because all solid particles occupy the bed and thus do not contribute to the column exerting a static pressure differential. Thus solid particles do not affect the total pressure differential over a descending pipeline section. The predicted sum of the frictional and the static pressure differential is similar from both the empirical and physical models.

## CASE STUDY 6

## Mixture flow in an inclined pipeline

An aqueous mixture of fine sand or medium gravel (see previous Case studies) is pumped from a borrowing pit to a hopper on board of a trailing suction hopper dredge. The dredging depth is 50 meter. A suction pipeline of an internal diameter 900 millimeter is inclined under the angle 45 deg.

Propose a suitable transport velocity for mixture in a pipeline and determine the required manometric head of the pump, the energy lost due to friction in a suction pipeline, the specific energy consumption and the production for mixture transport at the chosen transport velocity. The absolute roughness of a pipeline wall is 20 microns.

Remark: Consider $1.1 \mathrm{~V}_{\mathrm{dl}}$ (deposition-limit velocity) a suitable transport velocity of mixture in the inclined pipeline. For a simplification consider a narrow graded soil characterized by the median diameter only.

## Inputs:

$$
\begin{aligned}
& \mathbf{d}_{50}=\mathbf{0 . 1 2 0} \mathrm{mm} \text { of } \mathrm{d}_{50}=\mathbf{6 . 0} \mathrm{mm} \\
& \rho_{\mathbf{s}}=\mathbf{2 6 5 0} \mathrm{kg} / \mathrm{m}^{3} \\
& \rho_{\mathbf{f}}=\mathbf{1 0 0 0} \mathrm{kg} / \mathrm{m}^{3} \\
& \mathbf{v}_{\mathbf{f}}=\mathbf{0 . 0 0 0 0 0 1} \mathrm{m}^{2} / \mathrm{s} \\
& \mathrm{C}_{\mathbf{v d}}=\mathbf{0 . 2 7} \\
& \Delta \mathbf{h}_{\mathbf{d e p t h}}=\mathbf{5 0} \mathrm{m} \\
& \mathbf{\omega}=\mathbf{4 5} \mathbf{~ d e g} \\
& \mathrm{D}=\mathbf{9 0 0} \mathrm{mm} \\
& \mathrm{k}=\mathbf{0 . 0 0 0 0 2} \mathrm{m}
\end{aligned}
$$

## Solution:

## a. The deposition-limit velocity

Fine sand ( $\mathrm{d}=0.120 \mathrm{~mm}$ )
In Case study 4 the deposition-limit velocity was determined for a horizontal flow by the MTI correlation: $\quad \mathrm{V}_{\mathrm{dl}}=3.23 \mathrm{~m} / \mathrm{s}$ (Eq. 4.19).
The deposition-velocity correction for inclined flow is carried out using the Wilson method. The deposition parameter $\Delta_{\mathrm{D}}$ is found for $\omega=45 \mathrm{deg}$ from the nomograph on Fig. 6.3: $\Delta_{D}=0.33$.
The Eq. 6.10 gives
$\mathrm{V}_{\mathrm{dl} \omega}=\mathrm{V}_{\mathrm{dl}}+\Delta_{\mathrm{D}} \sqrt{2 \mathrm{gD}\left(\mathrm{S}_{\mathrm{S}}-1\right)}=3.23+0.33 \sqrt{2 \mathrm{x} 9.81 \times 0.9 \mathrm{x} 1.65}=5.01 \mathrm{~m} / \mathrm{s}$.
Medium gravel $(\mathrm{d}=6.0 \mathrm{~mm})$
In Case study 4 the deposition-limit velocity was determined for a horizontal flow by the MTI correlation:
$\mathrm{V}_{\mathrm{dl}}=7.03 \mathrm{~m} / \mathrm{s}$ (Eq. 4.19).

The deposition-velocity correction for inclined flow is carried out using the Wilson method. The deposition parameter $\Delta_{\mathrm{D}}$ is found for $\omega=45 \mathrm{deg}$ from the nomograph on Fig. 6.3: $\Delta_{\mathrm{D}}=0.33$.
The Eq. 6.10 gives
$\mathrm{V}_{\mathrm{dl} \omega}=\mathrm{V}_{\mathrm{dl}}+\Delta_{\mathrm{D}} \sqrt{2 \mathrm{gD}\left(\mathrm{S}_{\mathrm{S}}-1\right)}=7.03+0.33 \sqrt{2 \mathrm{x} 9.81 \mathrm{x} 0.9 \mathrm{x} 1.65}=8.81 \mathrm{~m} / \mathrm{s}$.
The suitable transport velocity for sand-water mixture: $\boldsymbol{V}_{\boldsymbol{m}}=1.1 \mathrm{~V}_{\mathrm{dl} \omega}=\mathbf{5 . 5 0} \mathbf{m} / \mathrm{s}$. The suitable transport velocity for gravel-water mixture: $\boldsymbol{V}_{\boldsymbol{m}}=1.1 \mathrm{~V}_{\mathrm{dl} \omega}=\mathbf{9 . 7 0} \mathbf{m} / \mathbf{s}$.

## b. Energy loss due to friction \& required manometric head of the pump

## Fine sand $(d=0.120 \mathrm{~mm})$

Water flow:

$$
\begin{aligned}
& \operatorname{Re}=5.5^{*} 0.9 / 0.000001=4.95 \times 10^{6} \\
& \mathrm{k} / \mathrm{D}=0.00002 / 0.9=2.2 \times 10^{-5}(\mathrm{D} / \mathrm{k}=45000) \\
& \lambda_{\mathrm{f}}=0.0103(\text { see Moody diagram, Fig. 1.6) }
\end{aligned}
$$

Friction head loss from the Darcy-Weisbach equation (Eq. 1.20)

$$
\mathrm{I}_{\mathrm{f}}=\frac{\lambda_{\mathrm{f}}}{\mathrm{D}} \frac{\mathrm{~V}_{\mathrm{m}}^{2}}{2 \mathrm{~g}}=\frac{0.0103}{0.900} \frac{5.5^{2}}{19.62}=0.0176
$$

Mixture flow: Wilson model for heterogeneous flow
Horizontal pipeline (Eq. 4.16 and Eq. 4.17)

$$
\begin{aligned}
& \mathrm{V}_{50} \approx 3.93\left(\mathrm{~d}_{50}\right)^{0.35}\left(\frac{\mathrm{~S}_{\mathrm{S}}-1}{1.65}\right)^{0.45}=3.93(0.12)^{0.35} 1=1.87 \mathrm{~m} / \mathrm{s} . \\
& \frac{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}{\mathrm{C}_{\mathrm{Vd}}\left(\mathrm{~S}_{\mathrm{S}}-1\right)}=0.22\left(\frac{\mathrm{~V}_{\mathrm{m}}}{\mathrm{~V}_{50}}\right)^{-\mathrm{M}}=0.22\left(\frac{5.50}{1.87}\right)^{-1.7}=0.03515 \\
& \mathrm{I}_{\mathrm{m}}=0.0176+0.0352 \times 0.27 \times 1.65=0.0333[-] .
\end{aligned}
$$

Inclined pipeline
Frictional head loss (Eq. 6.6) for $\mathrm{M}=1.7$ and $\gamma=0.4$ (estimated):

$$
\begin{aligned}
& \frac{\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}=(\cos \omega)^{(1+\mathrm{M} \gamma)}=(\cos 45)^{(1+1.7 \times 0.4)}=0.5586 \\
& \boldsymbol{I}_{\boldsymbol{m} \boldsymbol{\omega}}=0.0176+0.5586(0.0333-0.0176)=\mathbf{0 . 0 2 6 4}[-] .
\end{aligned}
$$

Manometric gradient (Eq. 6.5):

$$
\mathrm{I}_{\mathrm{mh} \omega}=0.0264+0.27 \times 1.65 \times \sin (45)=0.3414[-] .
$$

The head that must be delivered by a dredge pump to lift the mixture from a borrowing pit to a hopper is the head required to overcome the friction and the difference in a geodetic position of the pit and the hopper. If the position of the pump and the hopper inlet is considered equal to the water-level position and the local losses in a suction pipeline are neglected, the required head, $\mathrm{H}_{\text {man }}$ [meter water column, mwc], is:

$$
\boldsymbol{H}_{\boldsymbol{m a n}}=\mathrm{I}_{\mathrm{mh} \omega} \times \mathrm{L}_{\mathrm{inc}}=\mathrm{I}_{\mathrm{mh} \omega} \times \Delta \mathrm{h}_{\mathrm{depth}} / \sin (\omega)=0.3414 \times 50 / \sin (45)=24.1 \boldsymbol{m} \boldsymbol{w} \boldsymbol{c} .
$$

## Medium gravel ( $\mathrm{d}=6.0 \mathrm{~mm}$ )

Water flow:

$$
\begin{aligned}
& \mathrm{Re}=9.7^{*} 0.9 / 0.000001=8.73 \times 10^{6} \\
& \mathrm{k} / \mathrm{D}=0.00002 / 0.9=2.2 \times 10^{-5}(\mathrm{D} / \mathrm{k}=45000) \\
& \lambda_{\mathrm{f}}=0.010(\text { see Moody diagram, Fig. 1.6) } \\
& \text { Friction head loss from the Darcy-Weisbach equation (Eq. 1.20) }
\end{aligned}
$$

$$
\mathrm{I}_{\mathrm{f}}=\frac{\lambda_{\mathrm{f}}}{\mathrm{D}} \frac{\mathrm{~V}_{\mathrm{m}}^{2}}{2 \mathrm{~g}}=\frac{0.010}{0.900} \frac{9.7^{2}}{19.62}=0.0533
$$

Mixture flow: Wilson model for heterogeneous flow
Horizontal pipeline (Eq. 4.16 and Eq. 4.17)

$$
\begin{aligned}
& \mathrm{V}_{50} \approx 3.93\left(\mathrm{~d}_{50}\right)^{0.35}\left(\frac{\mathrm{~S}_{\mathrm{S}}-1}{1.65}\right)^{0.45}=3.93(6.0)^{0.35} 1=7.36 \mathrm{~m} / \mathrm{s} \\
& \frac{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}{\mathrm{C}_{\mathrm{vd}}\left(\mathrm{~S}_{\mathrm{S}}-1\right)}=0.22\left(\frac{\mathrm{~V}_{\mathrm{m}}}{\mathrm{~V}_{50}}\right)^{-\mathrm{M}}=0.22\left(\frac{9.70}{7.36}\right)^{-1.7}=0.1376 \\
& \mathrm{I}_{\mathrm{m}}=0.0533+0.1376 \times 0.27 \times 1.65=0.1146[-] .
\end{aligned}
$$

Inclined pipeline
Frictional head loss (Eq. 6.6) for $\mathrm{M}=1.7$ and $\gamma=0.9$ (estimated):

$$
\begin{aligned}
& \frac{\mathrm{I}_{\mathrm{m} \omega}-\mathrm{I}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{m}}-\mathrm{I}_{\mathrm{f}}}=(\cos \omega)^{(1+\mathrm{M} \gamma)}=(\cos 45)^{(1+1.7 \times 0.9)}=0.4161 \\
& \boldsymbol{I}_{\boldsymbol{m} \boldsymbol{\omega}}=0.0533+0.4161(0.1146-0.0533)=\mathbf{0 . 0 7 8 8}[-]
\end{aligned}
$$

Manometric gradient (Eq. 6.5):
$\mathrm{I}_{\mathrm{mh} \omega}=0.0788+0.27 \times 1.65 \times \sin (45)=0.3938[-]$.
The head that must be delivered by a dredge pump to lift the mixture from a borrowing pit to a hopper is the head required to overcome the friction and the difference in a geodetic position of the pit and the hopper. If the position of the pump and the hopper inlet is considered equal to the water-level position and the local losses in a suction pipeline are neglected, the required head, $\mathrm{H}_{\mathrm{man}}$ [meter water column, mwc], is:
$\boldsymbol{H}_{\boldsymbol{m} \boldsymbol{a n}}=\mathrm{I}_{\mathrm{mh} \omega} \times \mathrm{L}_{\mathrm{inc}}=\mathrm{I}_{\mathrm{mh} \omega} \times \Delta \mathrm{h}_{\mathrm{depth}} / \sin (\omega)=0.3938 \times 50 / \sin (45)=\mathbf{2 7 . 9} \boldsymbol{m} \boldsymbol{w} \boldsymbol{c}$.

## c. Specific energy consumption (Eq. 3.6)

Fine sand ( $\mathrm{d}=0.120 \mathrm{~mm}$ )

$$
\mathrm{SEC}=2.7 \frac{\mathrm{I}_{\mathrm{m} \omega \mathrm{~h}}}{\mathrm{~S}_{\mathrm{S}} \cdot \mathrm{C}_{\mathrm{Vd}}}=2.7 \frac{0.3414}{2.65 \times 0.27}=1.288[\mathbf{k W h} /(\text { tonne. } \mathrm{km})] .
$$

Medium gravel $(\mathrm{d}=6.0 \mathrm{~mm})$

$$
\mathrm{SEC}=2.7 \frac{\mathrm{I}_{\mathrm{mh} \omega}}{\mathrm{~S}_{\mathrm{S}} \cdot \mathrm{C}_{\mathrm{Vd}}}=2.7 \frac{0.3938}{2.65 \times 0.27}=1.486[\mathbf{k W h} /(\text { tonne. } k m)] .
$$

## d. Production

## Fine sand ( $\mathrm{d}=0.120 \mathrm{~mm}$ )

Production of solids: (Eq. 3.3)
$\mathrm{Q}_{\mathrm{S}}=\frac{\pi}{4} \mathrm{D}^{2} \mathrm{~V}_{\mathrm{m}} \mathrm{C}_{\mathrm{vd}} 3600=\frac{\pi}{4} 0.9^{2} 5.5 \mathrm{x} 0.27 \times 3600=3401.0\left[\mathrm{~m}^{3} / \mathrm{hour}\right]$.
Production of in situ soil: (for porosity $\mathrm{n}=0.4$ ) (Eq. 3.4)

$$
\mathrm{Q}_{\mathrm{si}}=\frac{\pi}{4} \mathrm{D}^{2} \mathrm{~V}_{\mathrm{m}} \mathrm{C}_{\mathrm{vdsi}} 3600=\frac{\mathrm{Q}_{\mathrm{S}}}{1-\mathrm{n}}=\mathbf{5 6 6 8 . 3}\left[\boldsymbol{m}^{3} / \boldsymbol{h o u r}\right] .
$$

## Medium gravel $(\mathrm{d}=6.0 \mathrm{~mm})$

Production of solids: (Eq. 3.3)
$\mathrm{Q}_{\mathrm{S}}=\frac{\pi}{4} \mathrm{D}^{2} \mathrm{~V}_{\mathrm{m}} \mathrm{C}_{\mathrm{Vd}} 3600=\frac{\pi}{4} 0.9^{2} 9.7 \mathrm{x} 0.27 \times 3600=5998.1\left[\mathrm{~m}^{3} / \mathrm{hour}\right]$.

Production of in situ soil: (for porosity $\mathrm{n}=0.4$ ) (Eq. 3.4)

$$
\mathrm{Q}_{\mathrm{si}}=\frac{\pi}{4} \mathrm{D}^{2} \mathrm{~V}_{\mathrm{m}} \mathrm{C}_{\mathrm{vdsi}} 3600=\frac{\mathrm{Q}_{\mathrm{S}}}{1-\mathrm{n}}=\boldsymbol{9 9 9 6 . 8}\left[\boldsymbol{m}^{3} / \text { hour }\right] .
$$

## Summary of the results:

Fine sand ( $\mathrm{d}=0.12 \mathrm{~mm}$ ):
suitable transport velocity:
frictional head loss:
required manometric head:
specific energy consumption:
production of in situ soil:

$$
\begin{aligned}
& V_{m}=5.50 \mathrm{~m} / \mathrm{s} \\
& I_{m \omega}=0.0264[-] \\
& H_{m a n}=24.1 \mathrm{mwc} \\
& S E C=1.288 \mathrm{kWh} /(\text { (tonne. } \mathrm{km}) \\
& Q_{s i}=5668.3 \mathrm{~m}^{3} / \text { hour } \\
& \\
& V_{m}=9.70 \mathrm{~m} / \mathrm{s} \\
& I_{m \omega}=0.0788[-] \\
& H_{m a n}=27.9 \mathrm{mwc} \\
& S E C=1.486 \mathrm{kWh} /(\text { tonne. } \mathrm{km}) \\
& Q_{s i}=9996.8 \mathrm{~m}^{3} / \mathrm{hour} \\
& \hline
\end{aligned}
$$

Medium gravel $(\mathrm{d}=6.00 \mathrm{~mm})$ :
suitable transport velocity:
frictional head loss:
required manometric head:
specific energy consumption:
production of in situ soil:

### 6.4 UNSTEADY SOLIDS FLOW

During dredging operations slurry density varies in time and space along the entire long pipeline of a conveying system. The solids flow is unsteady ( $\mathrm{Q}_{\mathrm{S}} \neq$ const $)$ even if controlled global operational parameters of the system (slurry flow rate, $\mathrm{Q}_{\mathrm{m}}$, through the conveying system and pump speed) are assumed to be maintained at an approximately constant level during the entire operational period of the system.


Figure 6.5. Process of solids aggregation along a long dredging pipeline.

### 6.4.1 Solids aggregation along a long dredging pipeline

A fluctuating density, generated at the inlet of the system, moves through a pipeline. Field measurements (see Fig. 6.5) on a dredging installation with a pipeline that is approximately 10 km long and which has three booster stations in series, show that density fluctuations in the flow of slurry containing rather broad-graded medium sand are not flattened. Whilst passing along the pipeline with pumps in series, they are transformed into long density waves with a high amplitude. The transformation of density fluctuations indicates a solids aggregation process. The influence of a pump performance on density waves transformation is negligible. An aggregation mechanism is active in the pipeline.

### 6.4.2 Description of the solids-aggregation process

When unsteady solids flow in a long pipeline is modeled by means of basic hydrodynamic equations, including transport and turbulent dispersion effects, the fluctuating slurry density entering the system is assumed to be gradually flattened and become almost constant in time and space along the long pipeline. This mechanism is effective in a short time and length scale and causes a flattening of short-time density fluctuations behind a dredge pump (compare Gr and Ja density signals of Fig. 6.5). Over a longer time and length scale (more suitable for a description of the process in a pipeline which is more than 10 km long, in which each particle needs almost one hour to reach its destination from the bottom of a lake) a different mechanism may be prevailing.

With respect to the specific flow conditions in a long slurry pipeline connected with a dredge, it is believed that a process of material aggregation is caused by the hydrodynamic interaction between the bed layer and the suspension layer in a partially-stratified flow of mixture. This interaction leads to the mass exchange between the bed layer and the suspension layer (the non-equilibrium between the settling flux and erosion flux across the interface between layers), to the variation in the bed velocity and produces variable slip in an unsteady solids flow along the long pipeline.

The top of the granular bed is subjected to the highest shear stress if the densest suspension passes the bed. At this situation the the bed velocity and the erosion flux are the highest. When the measured signal for local solids velocity at the bottom of the pipeline cross section is compared with the measured signal for mean slurry density just passing the pipeline cross section, the reaction of bed velocity to the fluctuating slurry density is clearly seen (Fig. 6.6a, b). An exact description of the aggregation process requires an analysis of the hindered settling and hindered erosion in high concentrated mixture.

The aggregation of solids to high density waves occurs at low average velocities round and below the deposition-limit value. The interaction between layers becomes weak if mean mixture velocity grows far above the deposition-limit threshold.


Figure 6.6a. Local velocity of solids at the bottom of a pipeline for flow of medium to coarse sand mixture under fluctuating density.


Figure 6.6b. Local velocity of solids at the bottom of a pipeline for flow of medium to coarse sand mixture under fluctuating density.

### 6.4.3 Practical consequences of the solids-aggregation process

An aggregation process may have an influence on the efficiency and safety of the operation of the system. Consideration of the effects of the aggregation process on mechanical energy dissipation and granular deposit formation in a slurry pipeline may lead to a more effective control of a conveying system.

### 6.4.3.1 Consequences for pipeline operation

a. fine-to-medium sand with a small proportion of coarse sand and silt

The solids aggregation phenomenon observed in a long slurry pipeline connected with a dredge is not dangerous for pipeline operation. The formation of high density waves does not produce moving dunes or a stationary deposit at the bottom of a pipeline, nor does it increase the friction loss in the slurry flow in the pipeline.

Frictional head loss in unsteady solids flow does not increase significantly with the mean slurry density in the pipeline cross section if the transported solids are relatively broadly graded (see Fig. 6.7). As a result the specific energy consumption (SEC) in the dredging pipeline decreases rapidly when the density of transported mixture increases.


Figure 6.7. Pressure drop due to friction over a long pipeline section compared with changing mixture density in the pipeline section. Measured hydraulic gradient compared with the Durand model.

## b. coarse sand or gravel

Different phenomena may occur when coarse solids are pumped. In this case there is no impelling effect caused by the denser suspended layer, since the majority of particles occupy the bed in the flow of a mean slurry velocity not far above the deposition-limit value. The unsteady state of the solids flow causes that the thickness of the bed varies significantly along the pipeline. Further instabilities may occur owing to shear stress variation at the top of a bed of variable thickness. Instabilities may lead to the gradual development of dunes, their mutual separation and their transformation into plugs along the pipeline if the mixture flows at velocity near the deposition-limit threshold. Such plugs may block the pipeline.

### 6.4.3.2 Consequences for pump-pipeline operation

The formation of density waves in a dredging pipeline has a considerable impact on the operation of slurry pumps and drives incorporated into a conveying system. Density waves passing through the slurry pumps cause the working point of a pump-pipeline system to vary in time during the operation of the system (see Chapter 7). The situation is more complex in a system composed of a pipeline and a set of pumps. Analysis of the pump-pipeline interactions and of the impact of slurry density fluctuation on the efficiency of a conveying system is an interesting subject for further research.

### 6.4.3.3 Production measurement on board of a dredge

Slip occurs between solid and liquid phases in slurry pipelines as a result of a flow stratification. The slip must be taken into account when the solids throughput in a dredging pipeline is being determined. In horizontal pipelines occupied by the slurry exhibiting a considerable slip the solids concentration $\mathrm{C}_{\mathrm{vi}}$ (a fraction of solids actually present in the a pipeline section) is higher than the delivered concentration $\mathrm{C}_{\mathrm{vd}}$. During a dredging operation the solids throughput is usually determined on-line and displayed on the dredgemaster's control board. The solids throughput $\mathrm{Q}_{\mathrm{S}}$ is calculated as $\mathrm{Q}_{\mathrm{S}}=\mathrm{C}_{\mathrm{V}} V \pi \mathrm{D}^{2 / 4}$ from on-line signals of the measured mean liquid velocity ( $\mathrm{V}_{\mathrm{f}}$, by a magnetic flow meter) and mean spatial concentration $\left(\mathrm{C}_{\mathrm{Vi}}\right.$, by a radiometric density meter) in a pipeline cross section. The measuring instruments are often installed in a horizontal pipeline section at some distance behind a dredge pump. If flow stratification resulting in slip occurs in this pipeline section, the values of $\mathrm{Q}_{\mathrm{S}}$ obtained when using $\mathrm{C}_{\mathrm{vi}} \mathrm{V}_{\mathrm{f}} \pi \mathrm{D}^{2 / 4}$ may be too high. Using $\mathrm{C}_{\mathrm{vd}} \mathrm{V}_{\mathrm{m}} \pi \mathrm{D}^{2 / 4}$ would give the correct values. Thus the monitoring system for a dredging installation may overestimate the solids throughput in a pipeline connected with a dredge.

### 6.5 REFERENCES

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