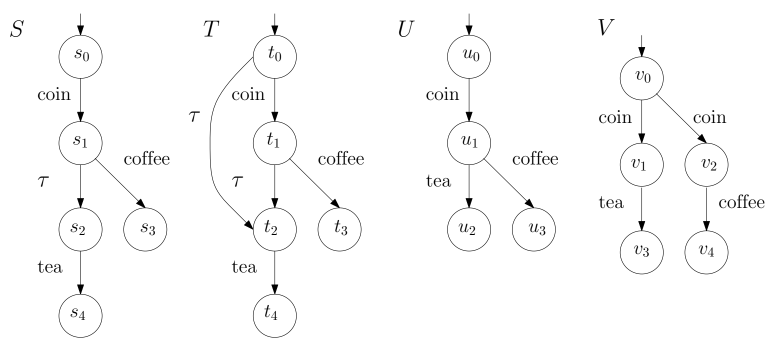
System Validation (IN4387) Resit Examination February 1, 2012, 14:00-17:00

Important Notes. It is not allowed to use study material, computers, or calculators during the examination. The examination comprises 5 question and 4 pages. Please check beforehand whether your copy is properly printed. Give complete explanation and do not confine yourself to giving the final answer. The answers may be given in Dutch or in English. **Good luck!**

Exercise 1 (20 points) Consider the following labeled transitions systems and reason whether and why each of the following equalities hold.

1. S and T are strongly bisimilar,
2. S and T are branching bisimilar,
3. T and U are branching bisimilar,
4. U and V are language equivalent.
5. U and V are strongly bisimilar.



Exercise 2 (20 points) Assume that the sort $iNatural$ of natural numbers is defined as follows:

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sort  iNatural;
cons  zero: iNatural;
      succ: iNatural → iNatural;
map   plus: iNatural × iNatural → iNatural ;
var   i, j: iNatural;
eqn   plus(zero, i)= i;                          (1)
      plus(succ(i), j)= succ(plus(i, j));        (2)
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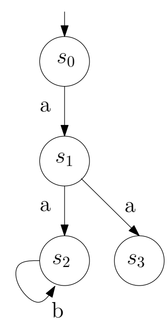
1. Prove that plus(i, zero) = i.
2. Prove that plus(i, succ(j)) = succ(plus(i, j)).
3. Prove that plus is commutative, i.e., plus(i, j) = plus(j, i).

Exercise 3 (20 points) Prove the following equations using the axioms provided in the appendix.

1. $(\alpha \mid \beta) \setminus (\beta \mid \gamma) = \alpha$
2. $x + y + ((c \vee d) \rightarrow x \diamond y) = y + x$,
3. $a.\delta \parallel (b + c) = a.(b + c).\delta + (b + c).a.\delta + (a \mid (b + c)).\delta$

Note that sequential composition binds stronger than nondeterministic choice.

Exercise 4 (20 points) Consider the following LTS.



In which states the formula $[a]\mu X.[b]X$ holds? Explain the steps towards the final answer.

Exercise 5 (20 points) Specify a track controller in mCRL2, with the following informal specification. The controller is supposed to control the entrance to a track which can allow for at most one train at a time. Thus, the trains, identified by a unique natural number, announce their arrival with an "arrive(i)" action, where i is the identifier of the train. If the track is not occupied, the train will be allowed to the track using the action "allow(i)". If the track is already occupied, the identifier of the train will be recorded in the list of waiting trains. Upon the departure of a train from the track, denoted by the action "depart", the first train in the waiting list, i.e., the one who has waited most, will be allowed into the track.

MA1	$\alpha\|\beta = \beta\|\alpha$
MA2	$(\alpha\|\beta)\|\gamma = \alpha\|(\beta\|\gamma)$
MA3	$\alpha\|\tau = \alpha$
MD1	$\tau \setminus \alpha = \tau$
MD2	$\alpha \setminus \tau = \alpha$
MD3	$\alpha \setminus (\beta\|\gamma) = (\alpha \setminus \beta) \setminus \gamma$
MD4	$(a(d)\|\alpha) \setminus a(d) = \alpha$
MD5	$(a(d)\|\alpha) \setminus b(e) = a(d)\|(\alpha \setminus b(e)) \quad$ if $a \not\equiv b$ or $d \not\approx e$

Table 1: Axioms for multi-actions

A1	$x + y = y + x$
A2	$x + (y + z) = (x + y) + z$
A3	$x + x = x$
A4	$(x + y)\cdot z = x\cdot z + y\cdot z$
A5	$(x\cdot y)\cdot z = x\cdot(y\cdot z)$
A6	$x + \delta = x$
A7	$\delta\cdot x = \delta$
Cond1	$\mathit{true}{\rightarrow}x \diamond y = x$
Cond2	$\mathit{false}{\rightarrow}x \diamond y = y$
SUM1	$\sum_{d:D} x = x$
SUM3	$\sum_{d:D} X(d) = X(e) + \sum_{d:D} X(d)$
SUM4	$\sum_{d:D}(X(d) + Y(d)) = \sum_{d:D} X(d) + \sum_{d:D} Y(d)$
SUM5	$(\sum_{d:D} X(d))\cdot y = \sum_{d:D} X(d)\cdot y$

Table 2: Axioms for the basic operators

Note that α and β range over (multi)actions and x, y and z ranger over processes.

M	$x \parallel y = x \mathbin{\Vert\mkern-9mu\lfloor} y + y \mathbin{\Vert\mkern-9mu\lfloor} x + x \mid y$
LM1	$\alpha \mathbin{\Vert\mkern-9mu\lfloor} x = \alpha \cdot x$
LM2	$\delta \mathbin{\Vert\mkern-9mu\lfloor} x = \delta$
LM3	$\alpha \cdot x \mathbin{\Vert\mkern-9mu\lfloor} y = \alpha \cdot (x \parallel y)$
LM4	$(x + y) \mathbin{\Vert\mkern-9mu\lfloor} z = x \mathbin{\Vert\mkern-9mu\lfloor} z + y \mathbin{\Vert\mkern-9mu\lfloor} z$
LM5	$(\sum_{d:D} X(d)) \mathbin{\Vert\mkern-9mu\lfloor} y = \sum_{d:D} X(d) \mathbin{\Vert\mkern-9mu\lfloor} y$
S1	$x \mid y = y \mid x$
S2	$(x \mid y) \mid z = x \mid (y \mid z)$
S3	$x \mid \tau = x$
S4	$\alpha \mid \delta = \delta$
S5	$(\alpha \cdot x) \mid \beta = \alpha \mid \beta \cdot x$
S6	$(\alpha \cdot x) \mid (\beta \cdot y) = \alpha \mid \beta \cdot (x \parallel y)$
S7	$(x + y) \mid z = x \mid z + y \mid z$
S8	$(\sum_{d:D} X(d)) \mid y = \sum_{d:D} X(d) \mid y$
TC1	$(x \mathbin{\Vert\mkern-9mu\lfloor} y) \mathbin{\Vert\mkern-9mu\lfloor} z = x \mathbin{\Vert\mkern-9mu\lfloor} (y \parallel z)$
TC2	$x \mathbin{\Vert\mkern-9mu\lfloor} \delta = x \cdot \delta$
TC3	$(x \mid y) \mathbin{\Vert\mkern-9mu\lfloor} z = x \mid (y \mathbin{\Vert\mkern-9mu\lfloor} z)$

Table 3: Axioms for the parallel composition operators