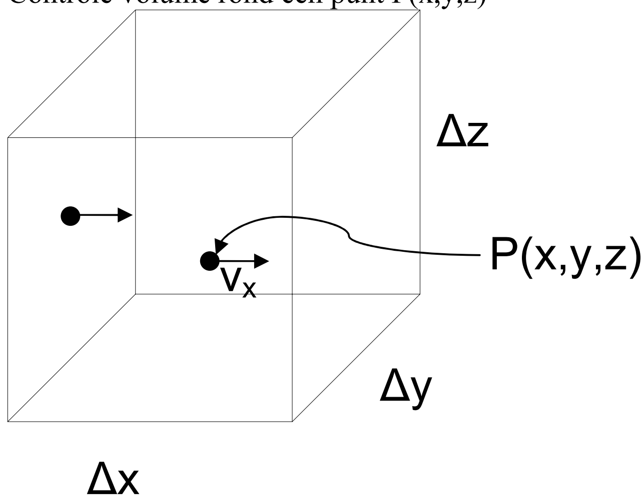
1. Laplace

Controle volume rond een punt P(x,y,z)



I: Continuiteit: Stroming door linker vlak+stroming door rechter vlak:

links in: $\left(V_x - \tfrac{1}{2}\Delta x \cdot \frac{\partial V_x}{\partial x}\right) \cdot \Delta y \Delta z$

rechts uit: $\left(V_x + \tfrac{1}{2}\Delta x \cdot \frac{\partial V_x}{\partial x}\right) \cdot \Delta y \Delta z$

verschil: $\frac{\partial V_x}{\partial x} \Delta x \cdot \Delta y \Delta z$

De som van de verschillen voor alle richtingen moet nul zijn (onsamendrukbaar):

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

II: Darcy:

$$V_x = -K_s \frac{\partial h}{\partial x}$$

III: I&II

$$\rightarrow -K_s \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right) = 0$$

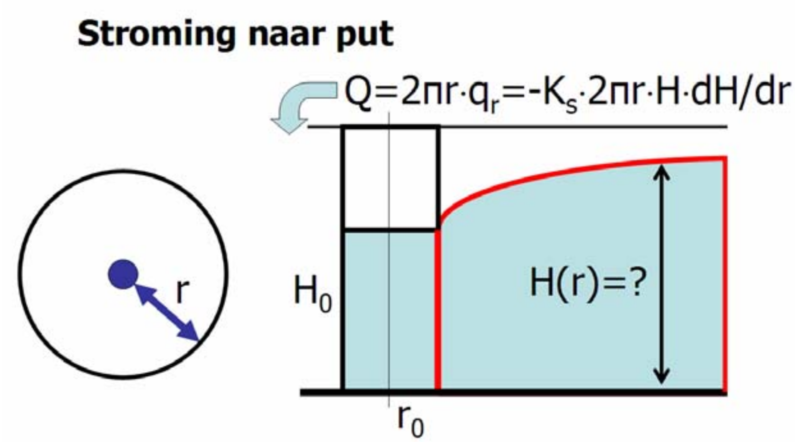
of:

$$\nabla^2 h = 0$$

2. Put

Dupuit aanname (zie ook slides College 2)

$$q = K_s H \cdot \frac{\partial H}{\partial x}$$



Integreer van r_0 en H_0 naar r en H na scheiding van variabelen:

$$Q = -K_S \cdot 2\pi r \cdot H \frac{\partial H}{\partial r}$$

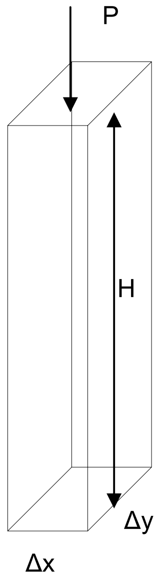
$$Q = -K_S \cdot 2\pi r \cdot H \frac{dH}{dr}$$

$$\int_{r_0}^{r} \frac{1}{r'} dr' = \int_{H_0}^{H} \frac{2\pi K_S}{Q} \cdot H' dH'$$

$$\ln\left(\frac{r}{r_0}\right) = \frac{\pi K_S}{Q}\left(H^2 - H_0^2\right)$$

3. Boussinesq

Continuiteit & Dupuit, nu voor een kolom.



Dupuit:

$$q_x = -K_S H \frac{\partial H}{\partial x}$$

Bijdrages van x&y richting:

x: $$-\frac{\partial q_x}{\partial x} \cdot \Delta x \cdot \Delta y = K_S \frac{\partial}{\partial x}\left(H \frac{\partial H}{\partial x} \right) \cdot \Delta x \cdot \Delta y$$

y: $$-\frac{\partial q_y}{\partial y} \cdot \Delta y \cdot \Delta x = K_S \frac{\partial}{\partial y}\left(H \frac{\partial H}{\partial y} \right) \cdot \Delta y \cdot \Delta x$$

Bijdrage regen:

P: $P \cdot \Delta x \Delta y$

Bijdrage verandering grondwaterstand:

H: $$\mu \frac{\partial H}{\partial t} \cdot \Delta x \Delta y$$

(μ = drainable porosity/draineerbare porositeit, hoeveelheid water die vrijkomt per ontwaterd volume)

Alle bijdrages bij elkaar opgeteld geven Boussinesq:

$$\mu \frac{\partial H}{\partial t} = P + K_S \frac{\partial}{\partial x}\left(H \frac{\partial H}{\partial x} \right) + K_S \frac{\partial}{\partial y}\left(H \frac{\partial H}{\partial y} \right)$$

oftewel:

$$\mu \frac{\partial H}{\partial t} = P + \tfrac{1}{2} K_S \left(\frac{\partial^2 H^2}{\partial x^2} + \frac{\partial^2 H^2}{\partial y^2} \right)$$

4. Donnan

Boussinesq

$$\mu \frac{\partial H}{\partial t} = P + \tfrac{1}{2} K_S \left(\frac{\partial^2 H^2}{\partial x^2} + \frac{\partial^2 H^2}{\partial y^2} \right)$$

Continue stroming en alleen in x-vlak:

$$\frac{2P}{K_S} = -\frac{d^2 H^2}{dx^2}$$

Scheiding van variabelen, tweemaal integreren:

$$\frac{2P}{K_S} x + C_1 = -\frac{dH^2}{dx}$$

$$\frac{P}{K_S} x^2 + C_1 x + C_2 = -H^2$$

Randvoorwaardes invullen:

$$x = 0 \quad \Rightarrow \quad H = H_0 \quad \Rightarrow \quad C_2 = -H_0^2$$

$$x = L \quad \Rightarrow \quad H = H_0 \quad \Rightarrow \quad C_1 = -\frac{PL}{K_S}$$

Verloop H(x):

$$\frac{P}{K_S} x^2 - \frac{PL}{K_S} \cdot x - H_0^2 = -H^2$$

Maximale grondwaterstand op x=0.5L:

$$x = \tfrac{1}{2} L \quad \Rightarrow \quad H = H_m \quad \Rightarrow \quad \tfrac{1}{4} \frac{PL^2}{K_S} - \tfrac{1}{2} \frac{PL^2}{K_S} - H_0^2 = -H_m^2$$

Oftewel:

$$H_m^2 = H_0^2 + \tfrac{1}{4} \frac{PL^2}{K_S} \qquad \text{(Donnan)}$$

Voorbeeld:

P= 300 mm/maand, 50% runoff → 5 mm/d $\approx 5 \cdot \frac{10^{-3}}{10^5} = 5 \cdot 10^{-8}$ m/s

K_s= 5.10^{-5} m/s (≈5m/dag)

→ $P/K_s = 10^{-3}$

L = 100m, H_{max} = 2m

→ $H_0^2 = 4 - ¼.10^{-3}.10^4 = 1.5$ m^2 → $H_0 \approx 1.22$ m