Deconvolution

(Ta3520)

Convolutional model of seismic data

Seismogram :

$$x(t) = s(t) * g(t)$$

$x(t)$ = seismogram

$s(t)$ = seismic wavelet

$g(t)$ = earth response (Green's function)

Inverse filter: deconvolution

Desired: *undo* effect of source $s(t)$:

$$f(t) * s(t) = \delta(t)$$

Apply $f(t)$ to seismogram $x(t)$:

$$\begin{aligned} f(t) * x(t) &= f(t) * s(t) * g(t) \\ &= \delta(t) * g(t) \\ &= g(t) \end{aligned}$$

Neutralizing effect of $s(t)$ is called *Deconvolution*

Inverse filter in frequency domain

In frequency domain:

$$X(f) = S(f)G(f)$$

Inverse of $S(f)$:

$$F(f) = \frac{1}{S(f)}$$

Apply to spectrum of seismogram $X(f)$:

$$\frac{X(f)}{S(f)} = G(f)$$

Deconvolution in presence of noise

Signal model in presence of noise:

$$x(t) = s(t) * g(t) + n(t)$$

In frequency domain:

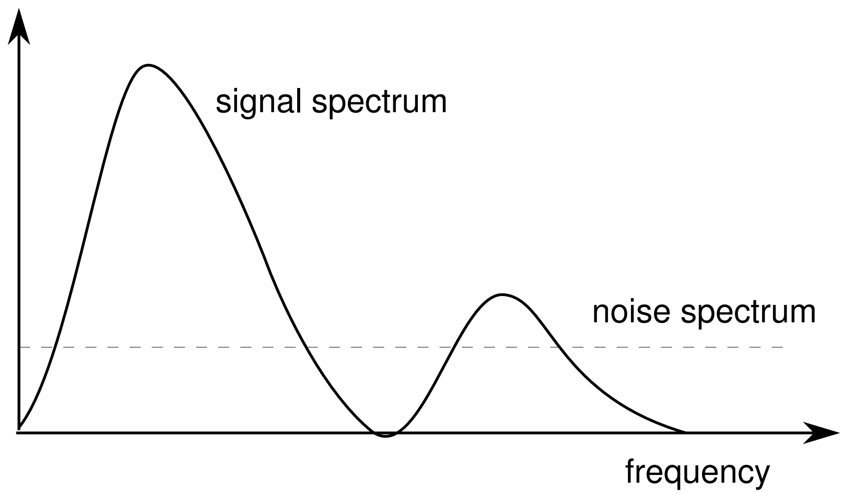
$$X(f) = S(f)G(f) + N(f)$$

Deconvolution (dividing by $S(f)$):

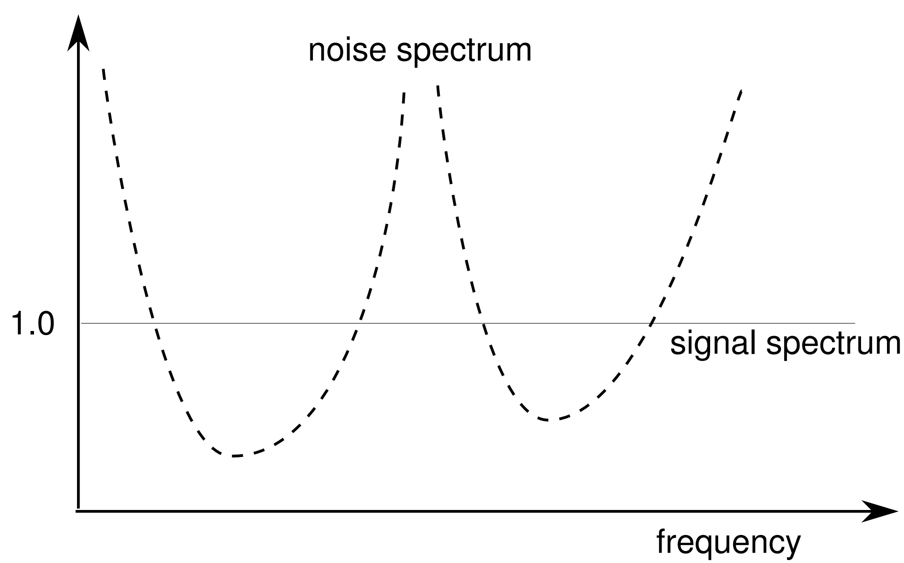
$$\frac{X(f)}{S(f)} = G(f) + \frac{N(f)}{S(f)}$$

If $S(f)$ is small for certain frequencies, this blows up the noise.

Deconvolution in presence of noise



a) Before deconvolution



Deconvolution in presence of noise

Therefore, stabilize division:

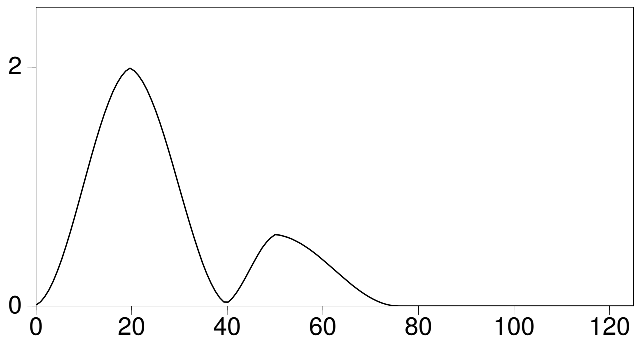
Apply filter:

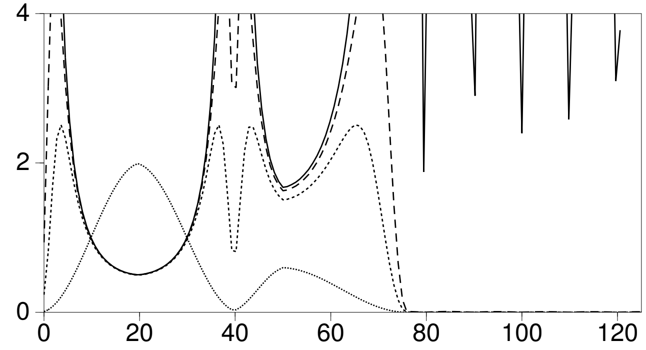
$$F(f) = \frac{S^*(f)}{S(f)S^*(f) + \epsilon^2}$$

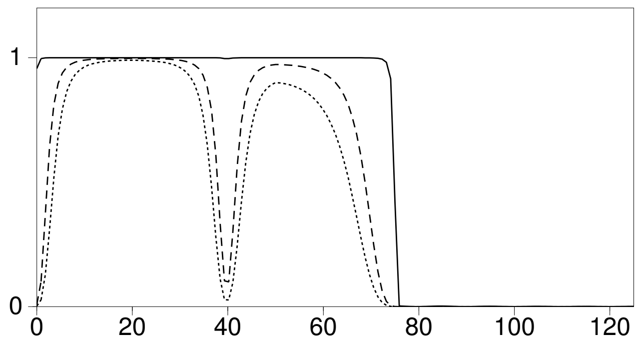
If $|S(f)| >> \epsilon : F(f) = \frac{1}{S(f)}$
If $|S(f)| << \epsilon : F(f) = \frac{S^*(f)}{\epsilon^2} \approx 0$

Choose ϵ as, e.g., : $\epsilon = \alpha \times \max(|S(f)|)$ with $\alpha \approx 0.01 - 0.1$

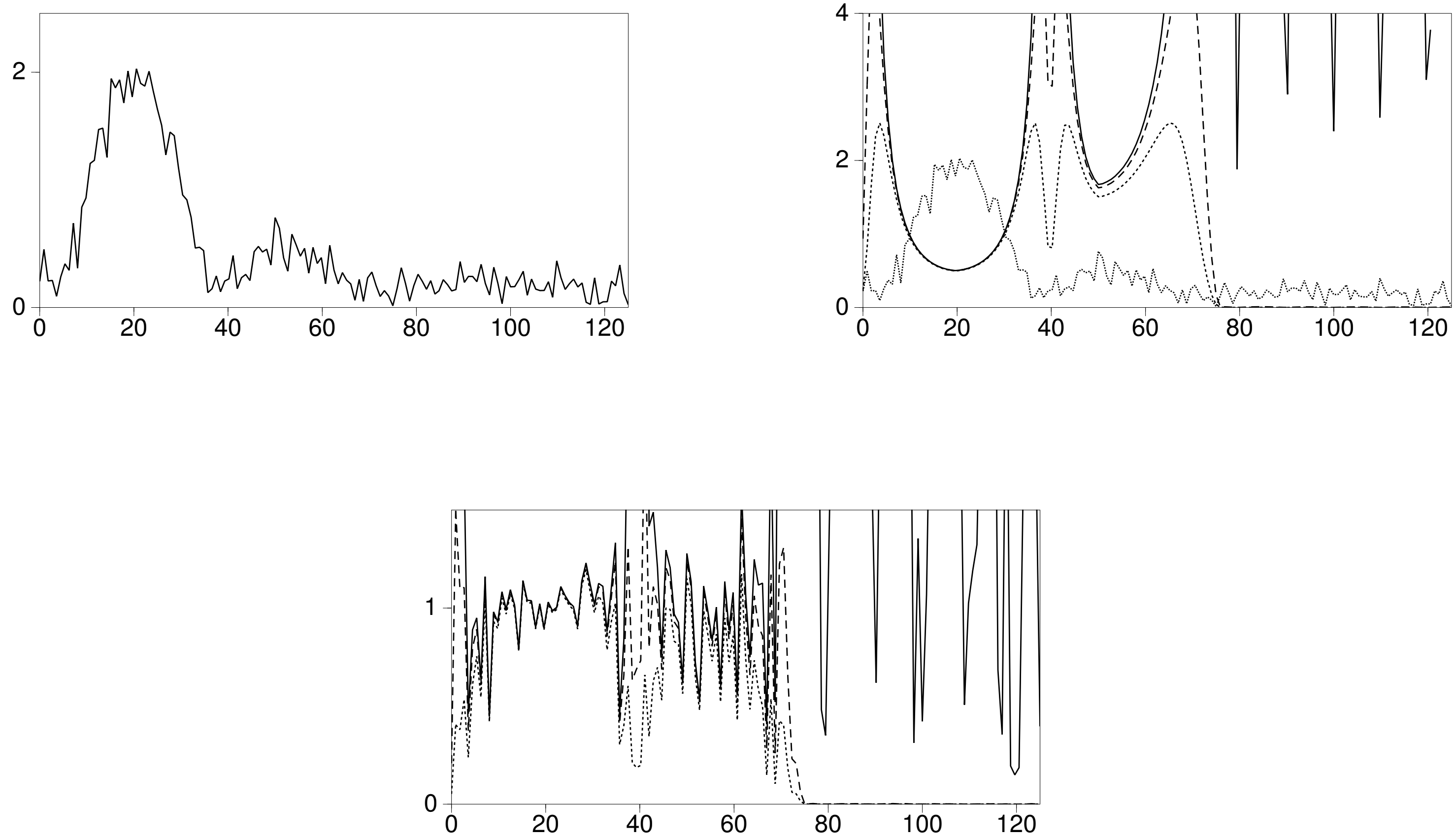
Deconvolution without noise







Deconvolution in presence of noise



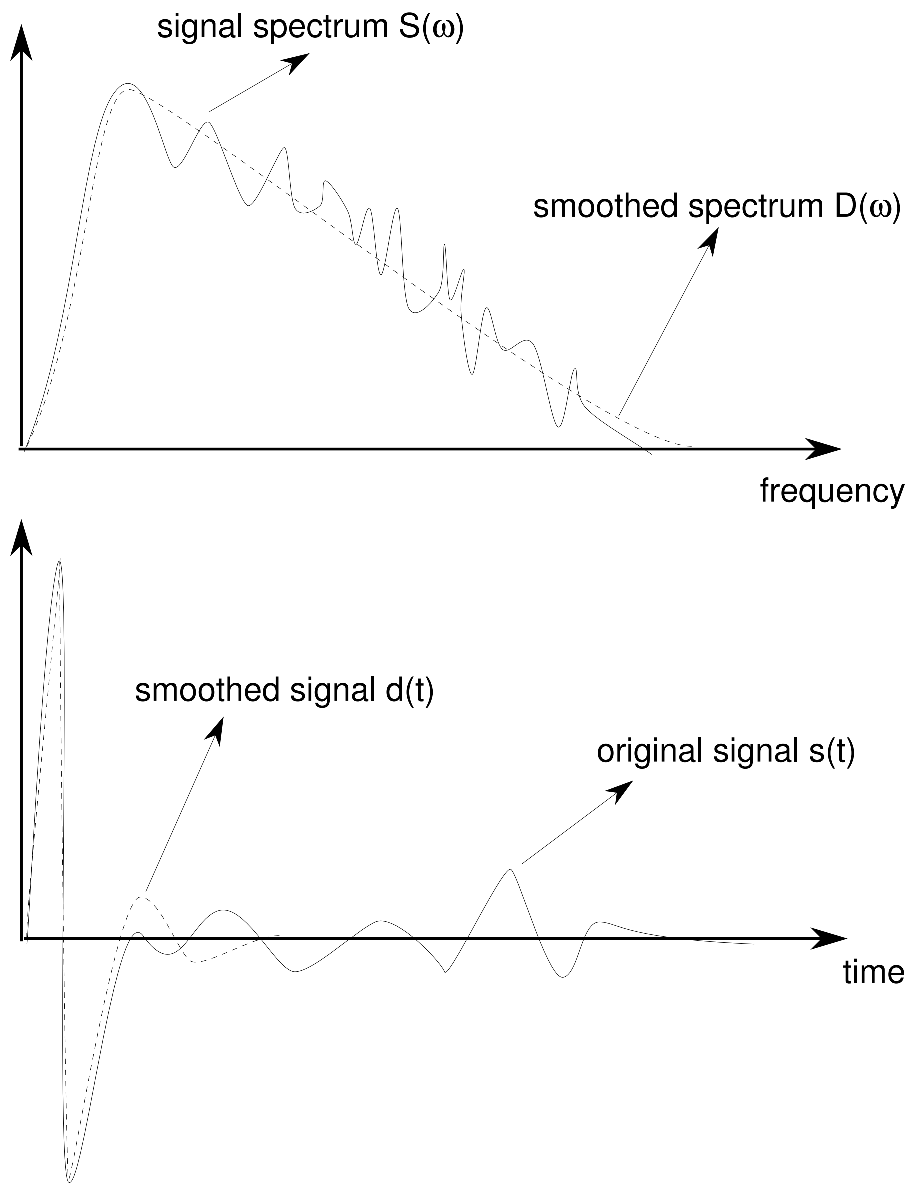
Deconvolution to desired output

Deconvolution problem:

$$f(t) * s(t) = d(t)$$

where $d(t)$ is desired output (previously $\delta(t)$)

Deconvolution to desired output



Deconvolution to desired output

Then, output after deconvolution:

$$F(f)X(f) = D(f)G(f)$$

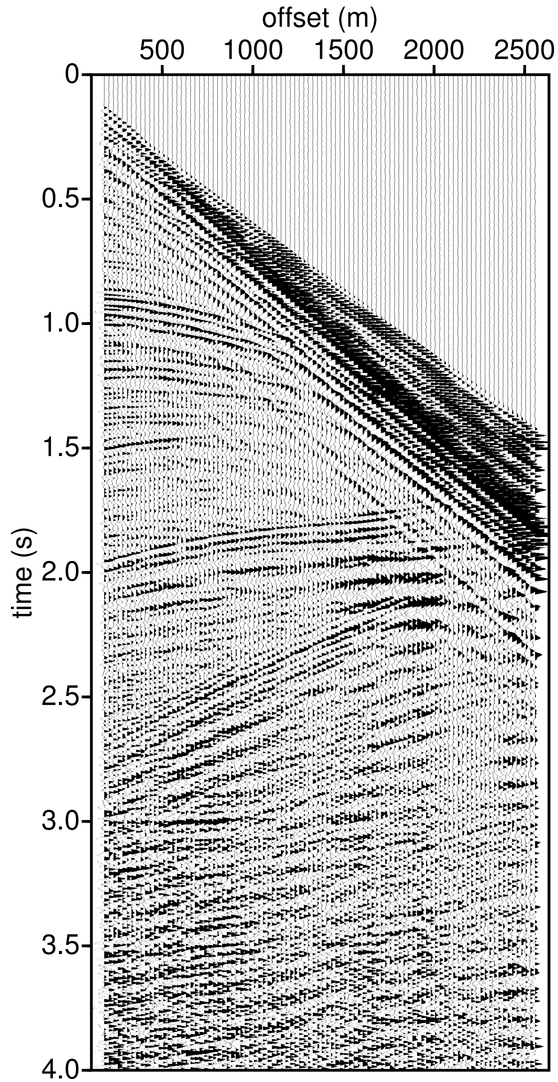
So filter:

$$F(f) = \frac{D(f)}{S(f)}$$

Or, stabilized,:

$$F(f) = \frac{D(f)S^*(f)}{S(f)S^*(f) + \epsilon^2}$$

Deconvolution: field data



Deconvolution field data