Chapter 2

Basic principles of the seismic method

In this chapter we introduce the basic notion of seismic waves. In the earth, seismic waves can propagate as longitudinal (P) or as shear (S) waves. For free space, the one-dimensional wave equation is derived. The wave phenomena occurring at a boundary between two layers are discussed, such as Snell's Law, reflection and transmission. For seismic-exploration purposes, where measurements are taking place at the surface, the different arrivals of direct waves, reflected waves and refracted/head waves are discussed.

2.1 Introduction

The seismic method makes use of the properties of the velocity of sound. This velocity is different for different rocks and it is this difference which is exploited in the seismic method. When we create sound at or near the surface of the earth, some energy will be reflected back (bounced back). They can be characterized as echoes. From these echoes we can determine the velocities of the rocks, as well as the depths where the echoes came from. In this chapter we will discuss the basic principles behind the behaviour of sound in solid materials. When we use the seismic method, we usually discuss two types of seismic methods, depending on whether the distance from the sound source to the detector (the "ear") is large or small with respect to the depth of interest: the first is known as refraction seismics, the other as reflection seismics. Of course, there is some overlap between those two types and that will be discussed in this chapter. When features really differ, then that will be discussed in next chapter for refraction and the chapters on reflection seismics. The overlap lies in the physics behind it, so we will deal with these in this chapter. In the following chapters will deal with instrumentation, field techniques, corrections (which are not necessary for refraction data) and interpretation.

2.2 Basic physical notions of waves

Everybody knows what waves are when we are talking about waves at sea. Sound in materials has the same kind of behaviour as these waves, only they travel much faster than the waves we see at sea. Waves can occur in several ways. We will discuss two of them, namely the longitudinal and the shear waves. Longitudinal waves behave like waves in a large spring. When we push from one side of a spring, we will observe a wave going through the spring which characterizes itself by a thickening of the wires running through the spring in time (see also figure (2.1)). A property of this type of wave is that the motion of a piece of the wire is in the same direction as the wave moves. These waves are also called Push-waves, abbreviated to P-waves, or compressional waves. Another type of wave is the shear wave. A shear wave can be compared with a chord. When we push a chord upward from one side, a wave will run along the chord to the other side. The movement of the chord itself is only up- and downward: characteristic of this wave is that a piece of the chord is moving perpendicular to the direction of that of the wave (see also figure (2.1)). These types of waves are referred to as S-waves, also called dilatational waves. Characteristic of this wave is that a piece of the chord is pulling its "neighbour" upward, and this can only occur when the material can support shear strain. In fluids, one can imagine that a "neighbour" cannot be pulled upward simply because it is a fluid. Therefore, in fluids only P-waves exist, while in a solid both P- and S-waves exist.

Waves are physical phenomena and thus have a relation to basic physical laws. The two laws which are applicable are the conservation of mass and Newton's second law. These two have been used in appendix A to derive the two equations governing the wave motion due to a P-wave. There are some simplifying assumptions in the derivation, one of them being that we consider a 1-dimensional wave. When we denote p as the pressure and v_x as the particle velocity, the conservation of mass leads to:

$$\frac{1}{K}\frac{\partial p}{\partial t} = -\frac{\partial v_x}{\partial x} \tag{2.1}$$

in which K is called the bulk modulus. The other relation follows from application of Newton's law:

$$-\frac{\partial p}{\partial x} = \rho\frac{\partial v_x}{\partial t} \tag{2.2}$$

where ρ denotes the mass density. This equation is called the equation of motion. The combination of these two equations leads, for constant density ρ, to the equation which describes the behaviour of waves, namely the wave equation:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{2.3}$$

in which c can be seen as the velocity of sound, for which we have: $c = \sqrt{K/\rho}$. The

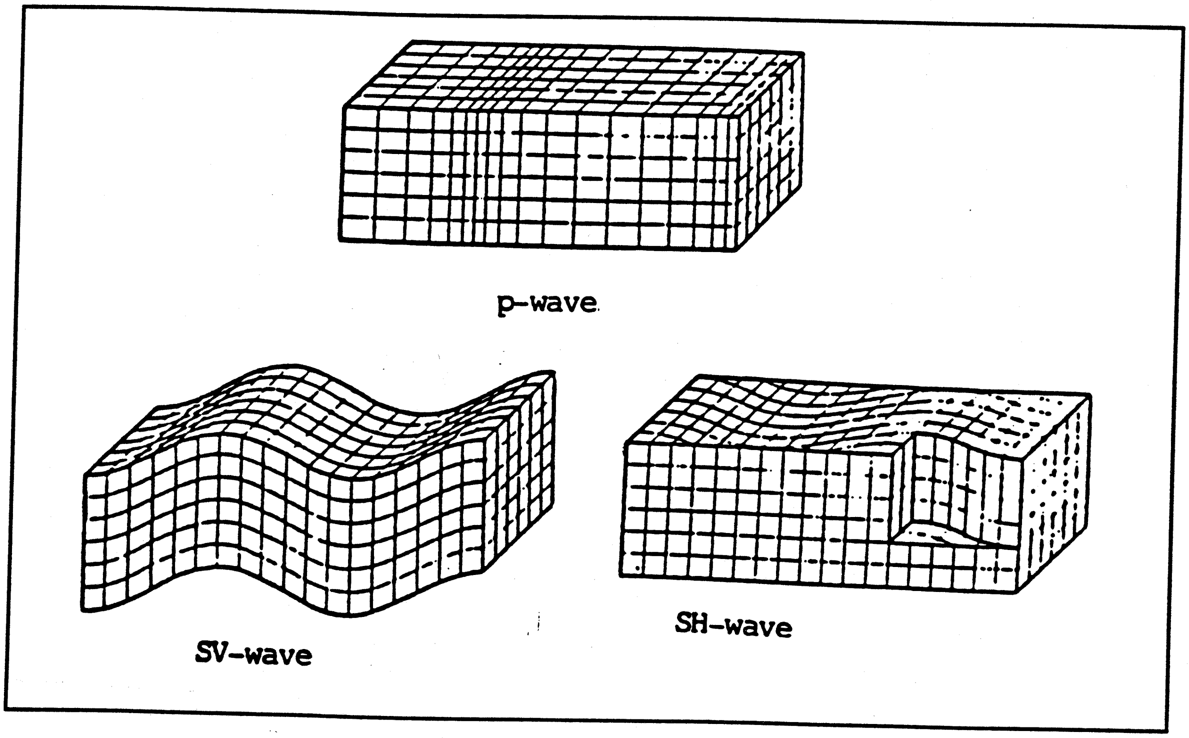

Figure 2.1: Particle motion of P and S waves

solution to this equation is:

$$p(x,t) = s(t \pm x/c) \tag{2.4}$$

where $s(t)$ is some function. Note the dependency on space and time via $(t \pm x/c)$, which denotes that it is a travelling wave. The sign in the argument is depending on which direction the wave travelling in.

Often, seismic responses are analyzed in terms of frequencies, i.e., the Fourier spectra as given in Chapter 1. The definition we use here is:

$$G(\omega) = \int_{-\infty}^{+\infty} g(t) \exp(-i\omega t)\, dt \tag{2.5}$$

which is the same as equation (1.1) from Chapter 1 but we used the radial frequency ω instead: $\omega = 2\pi f$. Using this convention, it is easy to show that a differentiation with respect to time is equivalent to multiplication with $i\omega$ in the Fourier domain. When we transform the solution of the wave equation (equation(2.4)) to the Fourier domain, we obtain:

$$P(x,\omega) = S(\omega) \exp(\pm i\omega x/c). \tag{2.6}$$

Note here that the time delay x/c (in the time domain) becomes a *linear* phase in the Fourier domain, i.e., $(\omega x/c)$.

In the above, we gave an expression for the pressure, but one can also derive the equivalent expression for the particle velocity v_x. To that purpose, we can use the equation of motion as expressed in equation (2.2), but then in its Fourier-transformed version, which is:

$$V_x(x,\omega) = -\frac{1}{i\omega\rho}\frac{\partial P(x,\omega)}{\partial x}. \tag{2.7}$$

When we subsitute the solution for the pressure from above (equation (2.6)), we get for the negative sign:

$$\begin{aligned} V_x(x,\omega) &= -\frac{1}{i\omega\rho}S(\omega)\frac{-i\omega}{c}\exp(-i\omega x/c) \\ &= S(\omega)\frac{1}{\rho c}\exp(-i\omega x/c). \end{aligned} \tag{2.8}$$

Notice that the particle velocity is a scaled version of the pressure:

$$V_x(x,\omega) = \frac{P(x,\omega)}{\rho c}. \tag{2.9}$$

The scaling factor is (ρc), being called the *seismic impedance*.

In the previous analysis, we considered one-dimensional waves. Normally in the real world, we deal with three dimensions, so a wave will spread in three directions. In a homogeneous medium (so the properties of the material are everywhere constant and the same) the wave will spread out like a sphere. The outer shell of this sphere is called the wave front. Another way of describing this wave front is in terms of the normal to the wavefront: the ray. We are used to rays in optics and we can use the same notion in the seismic method. When we were explaining the behaviour of P- and S-waves, we are already using the term "neighbour". This was an important feature otherwise the wave would not move forward. A fundamental notion included in this, is Huygens' principle. When a wave front arrives at a certain point, that point will behave also as a source for the wave, and so will all its neighbours. The new wavefront is then the envelope of all the waves which were generated by these points. This is illustrated in figure (2.2). Again, the ray can then be defined as the normal to that envelope which is also given in the figure.

So far, we only discussed the way in which the wave moves forward, but there is also another property of the wave we haven't discussed yet, namely the amplitude : how does the amplitude behave as the wave moves forward? We have already mentioned spherical spreading when the material is everywhere the same. The total energy will be spreaded out over the area over the sphere. This type of energy loss is called spherical divergence. It simply means that if we put our "ear" at a larger distance, the sound will be less loud.

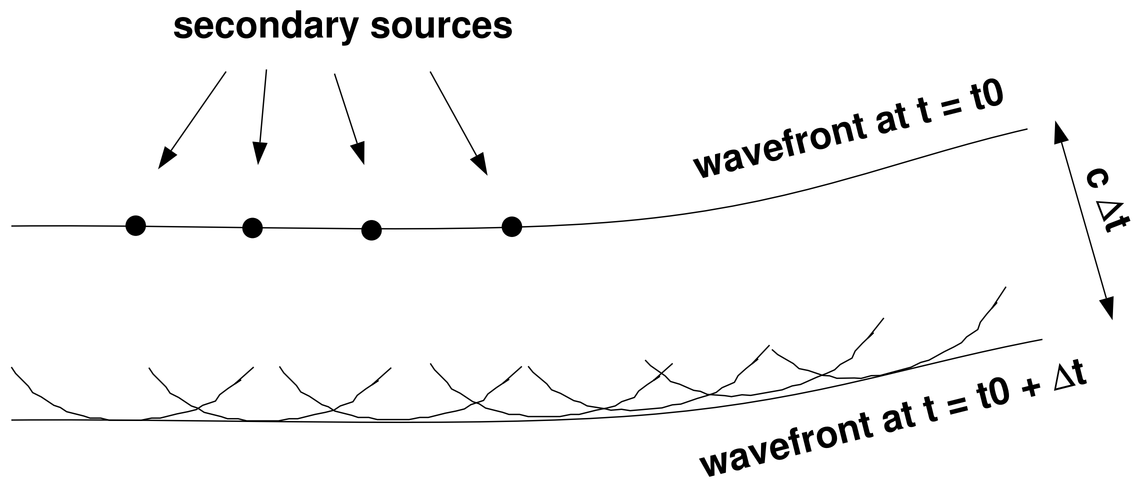

Figure 2.2: Using Huygens' principle to locate new wavefronts.

Material	velocity (m/s)	Material	velocity (m/s)
Air	330	Sandstone	2000-4500
Water	1500	Shales	3900-5500
Soil	20-300	Limestone	3400-7000
Sands	600-1850	Granite	4800-6000
Clays	1100-2500	Ultra-mafic rocks	7500-8500

Table 2.1: Seismic wave velocities for common materials and rocks.

There is also another type of energy loss, and that is due to losses within the material, which mainly consists of internal friction losses. This means that the amplitude of a wave will be extra damped because of this property. S-waves usually show higher friction losses than P-waves. Finally, we give a table of common rocks and their seismic wave velocities in table (2.1).

2.3 The interface : Snell's law, refraction and reflection

So far, we discussed waves in a material which had everywhere the same constant wave velocity. When we include a boundary between two different materials, some energy is bounced back, or reflected, and some energy is going through to the other medium. It is nice to perform Huygens' principle graphically on such a configuration to see how the wavefront moves forward (propagates), especially into the second medium. From this

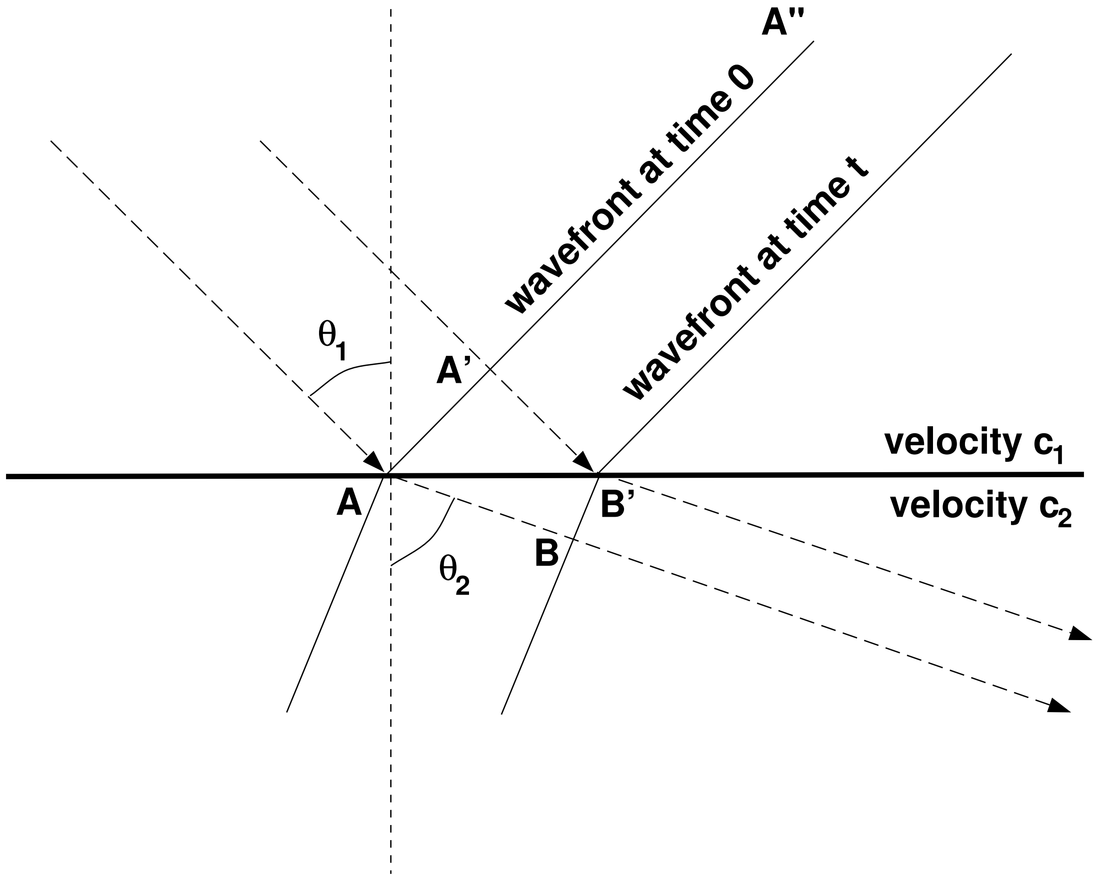

Figure 2.3: Snell's law

picture, we could also derive the ray concept. In this discussion, we will only consider the notion of rays. A basic notion in the ray concept, is Snell's law. Snell's law is a fundamental relation in the seismic method. It tells us the relation between angle of incidence of a wave and velocity in two adjacent layers (see Figure (2.3)).

$AA'A''$ is part of a plane wave incident at angle θ_1 to a plane interface between medium 1 of velocity c_1, and medium 2 of velocity c_2. The velocities c_1 and c_2 are constant. In a time t the wave front moves to the position AB and are normals to the wave front. So the time t is given by

$$t = \frac{A'B'}{c_1} = \frac{AB}{c_2} \tag{2.10}$$

Considering the two triangles and this may be written as:

$$t = \frac{AB' \sin\theta_1}{c_1} = \frac{AB' \sin\theta_2}{c_2} \tag{2.11}$$

Hence,

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2} \tag{2.12}$$

which is Snell's law for transmission. So far, we have taken general velocities c_1 and c_2. However, in a solid, two velocities exist, namely P- and S-wave velocities. Generally, when a P-wave is incident on a boundary, it can transmit as a P-wave into the second medium, but also as a S-wave. So in the case of the latter, Snell's law reads:

$$\frac{\sin\theta_P}{c_P} = \frac{\sin\theta_S}{c_S} \tag{2.13}$$

where c_P is the P-wave velocity, and c_S the S-wave velocity. The same holds for reflection: a P-wave incident on the boundary generates a reflected P-wave and a reflected S-wave. Finally, the same holds for an incident S-wave: it generates a reflected P-wave, a reflected S-wave, a transmitted P-wave and a transmitted S-wave.

A special case of Snell's law is of interest in refraction prospecting. If the ray is refracted along the interface (that is, if $\theta_2 = 90\,\text{deg}$), we have

$$\frac{\sin\theta_c}{c_1} = \frac{1}{c_2} \tag{2.14}$$

where θ_c is known as the critical angle.

So far, we have looked at basic notions of refraction and reflection at an interface. When we measure in the field, and there would be one boundary below it, we could observe several arrivals: a direct ray, a reflected ray and a refracted ray. We will derive the arrival time of each ray as depicted in figure 2.4.

The direct ray is very simple: it is the horizontal distance divided by the velocity of the wave, i.e.,:

$$t = \frac{x}{c_1} \tag{2.15}$$

When we look at the reflected ray, we have that the angle of incidence is the same as the angle of reflection. This also follows from Snell's law: when the velocities are the same, the angles must also be the same. When we use Pythagoras' theorem, we obtain for the traveltime:

$$t = \frac{(4z^2 + x^2)^{1/2}}{c_1} \tag{2.16}$$

When we square this equation, we see that it is the equation of a hyperbola:

$$t^2 = \left(\frac{2z}{c_1}\right)^2 + \left(\frac{x}{c_1}\right)^2 \tag{2.17}$$

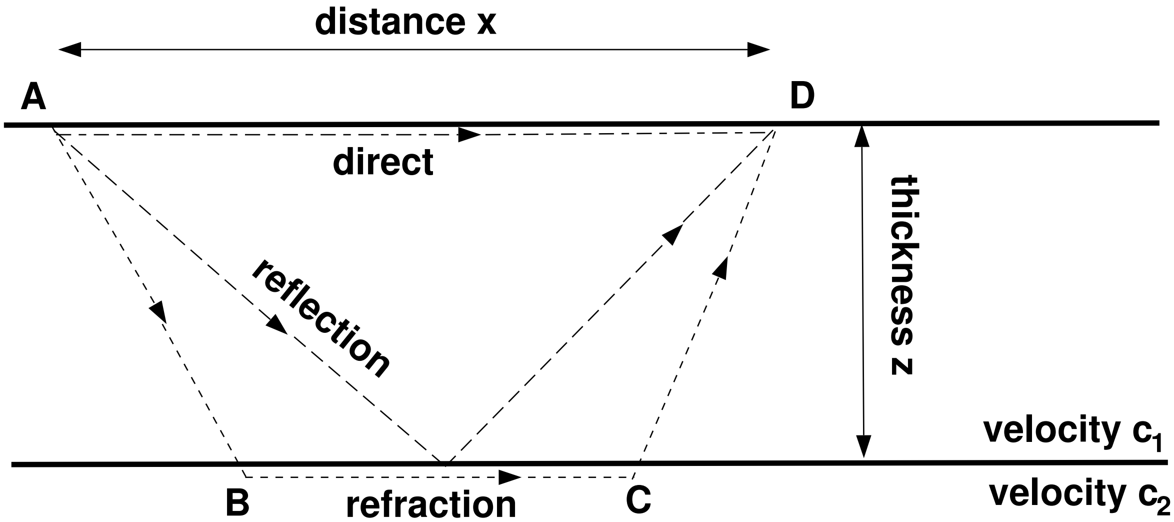

Figure 2.4: The direct, reflected and refracted ray.

When we look at the refracted ray the derivation is a bit more complicated. Take each ray element, so take the paths AB, BC and CD separately. Then, for the first element, as shown in figure (2.5), we obtain the traveltime:

$$\Delta t_1 = \frac{\Delta s_1 + \Delta s_2}{c_1} = \frac{\Delta x_1 \sin\theta_c}{c_1} + \frac{z\cos\theta_c}{c_1} \tag{2.18}$$

where θ_c is the critical angle.

We can do this also for the paths BC and CD, and we obtain the total time as:

$$t = \Delta t_1 + \Delta t_2 + \Delta t_3 \tag{2.19}$$

$$= \frac{\Delta x_1 \sin\theta_c}{c_1} + \frac{z\cos\theta_c}{c_1} + \frac{\Delta x_2}{c_2} + \frac{\Delta x_3 \sin\theta_c}{c_1} + \frac{z\cos\theta_c}{c_1} \tag{2.20}$$

where $\Delta x_2 = BC$ and Δx_3 is the horizontal distance between C and D. When we use now Snell's law, i.e., $\sin\theta_c/c_1 = 1/c_2$, in the terms with Δx_1 and Δx_3, then we can add all the terms with $1/c_2$, using $x = \Delta x_1 + \Delta x_2 + \Delta x_3$ to obtain:

$$t = \frac{x}{c_2} + \frac{2z\cos\theta_c}{c_1} \tag{2.21}$$

We recognize this equation as the equation of a straight line when t is considered as a function of distance x, the line along which we do our measurements. We will use this

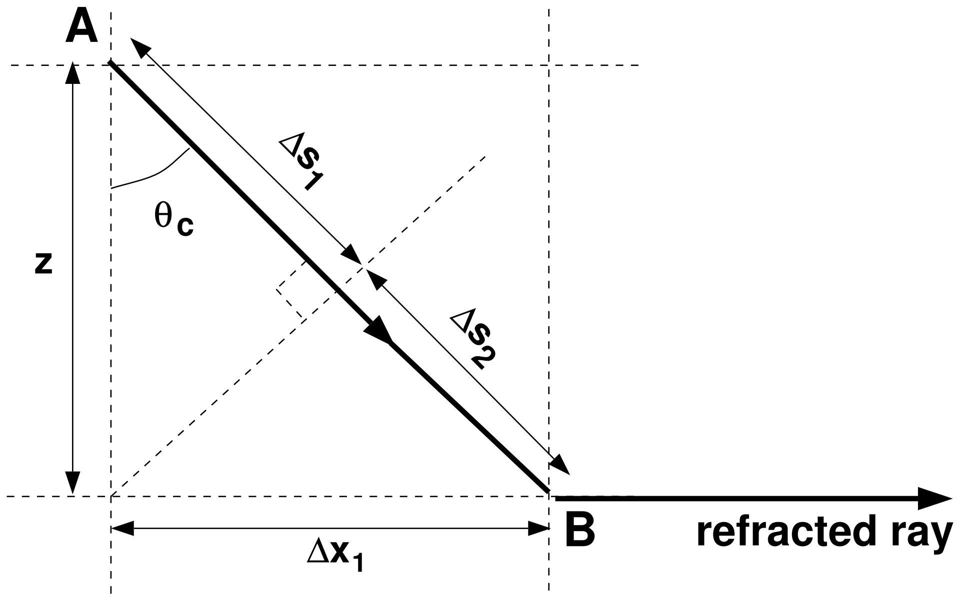

Figure 2.5: An element of the ray with critical incidence

equation later in the next chapter. We have now derived the equations for the three rays, and we can plot the times as a function of x. This is done in figure (2.6).

This picture is an important one. When we measure data in the field the characteristics in this plot can most of the time be observed.

Now we have generated this figure, we can specify better when we are performing a reflection survey, or a refraction survey. In refraction seismics we are interested in the refractions and only in the travel times. This means that we can only observe the traveltimes well if it is not masked by the reflections or the direct ray, which means that we must measure at a relatively large distance with respect to the depth of interest. This is different with reflection seismics. There, the reflections will always be masked by refractions or the direct ray, but there are ways to enhance the reflections. What we are interested in, is the arrival at relatively small offsets, thus distances of the sound source to the detector which are small with respect to the depth we are interested in.

Before discussing any more differences between the refraction method and the reflection method, we would like to discuss amplitude effects at the boundary. Let us first introduce the acoustic impedance, which is the product of the density ρ with the wave velocity c, i.e., ρc. When a ray encounters a boundary, some energy will be reflected back, and some will be transmitted to the next layer. The amount of energy reflected back is characterized by the reflection coefficient R:

$$R = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \tag{2.22}$$

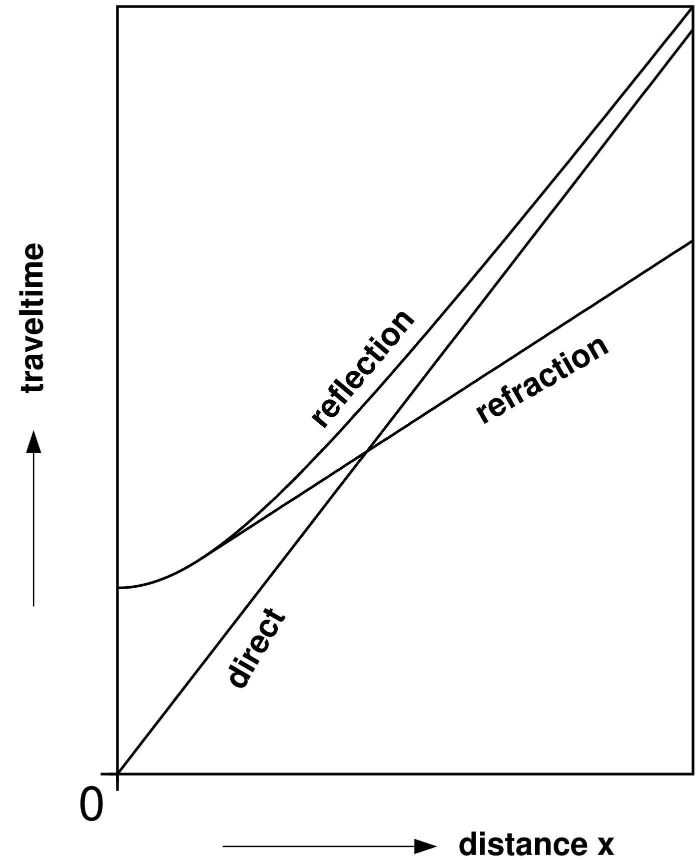

Figure 2.6: Time-distance (t, x) curve for direct, reflected and refracted ray.

That this is the case, will be derived from basic physical principles in the chapter on processing (Chapter 5). Obviously, the larger the impedance contrast between two layers, the higher the amplitude of the reflected wave will be. Notice that it is the impedance contrast which determines whether energy is reflected back or not; it may happen that the velocities and densities are different between two layers, but that the impedance is (nearly) the same. In that case we will see no reflection. We can now state another difference between refraction and reflection seismics. With refraction seismics we are only interested in traveltimes of the waves, so this means that we are interested in contrasts in velocities. This is different in reflection seismics. Then we are interested in the amplitude of the waves, and we will only measure an amplitude if there is a contrast in acoustic impedance in the subsurface.

Generally speaking the field equipment for refraction and reflection surveys have the same functionality: we need a source, detectors and recording equipment. Since reflection seismics gives us a picture of the subsurface, it is much used by the oil industry and therefore, they put high demands on the quality of the equipment. As said before, a difference between the two methods is that in reflection seismics we are interested in amplitudes as well, so this means that high-precision instruments are necessary to pick

REFRACTION SEISMICS	**REFLECTION SEISMICS**
Based on contrasts in : seismic wave speed (c)	Based on contrasts in : seismic wave impedances (ρc)
Material property determined : wave speed only	Material properties determined: wave speed and wave impedance
Only traveltimes used	Traveltimes and amplitudes used
No need to record amplitudes completely : relatively cheap instruments	Must record amplitudes correctly : relatively expensive instruments
Source-receiver distances large compared to investigation depth	Source-receiver distances small compared to investigation depth

Table 2.2: Important differences between refraction and reflection seismics.

those up accurately. Source, detectors and recording equipment will be discussed in the chapter on seismic instrumentation (chapter 3).

Finally, we tabulate the most important differences between reflection and refraction seismic in table 2.2.