Reflection and Transmission at boundaries

(Ta3520)

REFLECTION / TRANSMISSION

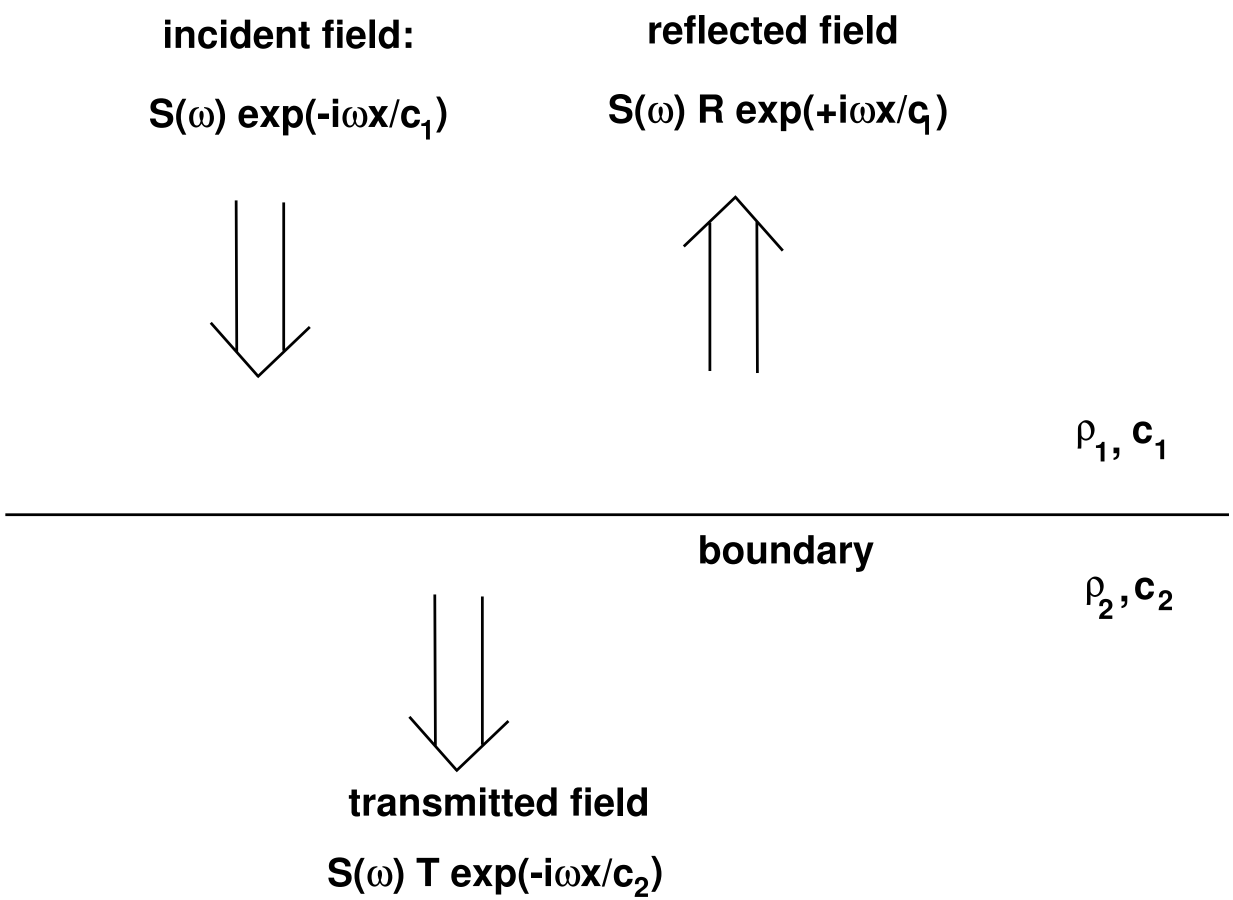
Wave equation for pressure: $\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$

Solution: $p(x, t) = s(t \pm x/c)$
where $s(t)$ is any function.

Fourier transformation: $G(\omega) = \int_{-\infty}^{+\infty} g(t) \exp(-i\omega t) dt$

Then: $P(x, \omega) = S(\omega) \exp(\pm i\omega x/c)$

Up- and Down-going wavefields



Above boundary ($x < 0$):

$$P(x,\omega) = S(\omega)\exp(-i\omega x/c_1) + RS(\omega)\exp(i\omega x/c_1)$$

Below boundary ($x > 0$):

$$P(x,\omega) = TS(\omega)\exp(-i\omega x/c_2)$$

Boundary conditions:

$$\begin{aligned}\lim_{x\uparrow 0} P(x,\omega) &= \lim_{x\downarrow 0} P(x,\omega)\\ \lim_{x\uparrow 0} V_x(x,\omega) &= \lim_{x\downarrow 0} V_x(x,\omega)\end{aligned}$$

Expression for V_x via equation of motion:

$$V_x(x,\omega) = -\frac{1}{i\omega\rho}\frac{\partial P(x,\omega)}{\partial x}$$

Working out:

$$\begin{aligned} V_x(x,\omega) &= S(\omega)\frac{1}{\rho_1 c_1}\exp(-i\omega x/c_1) \\ &\quad -RS(\omega)\frac{1}{\rho_1 c_1}\exp(i\omega x/c_1) \quad \text{for} \quad x<0 \\ V_x(x,\omega) &= TS(\omega)\frac{1}{\rho_2 c_2}\exp(-i\omega x/c_2) \quad \text{for} \quad x>0 \end{aligned}$$

Substituting expressions in boundary conditions:

$$R = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}$$

$$T = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1}$$