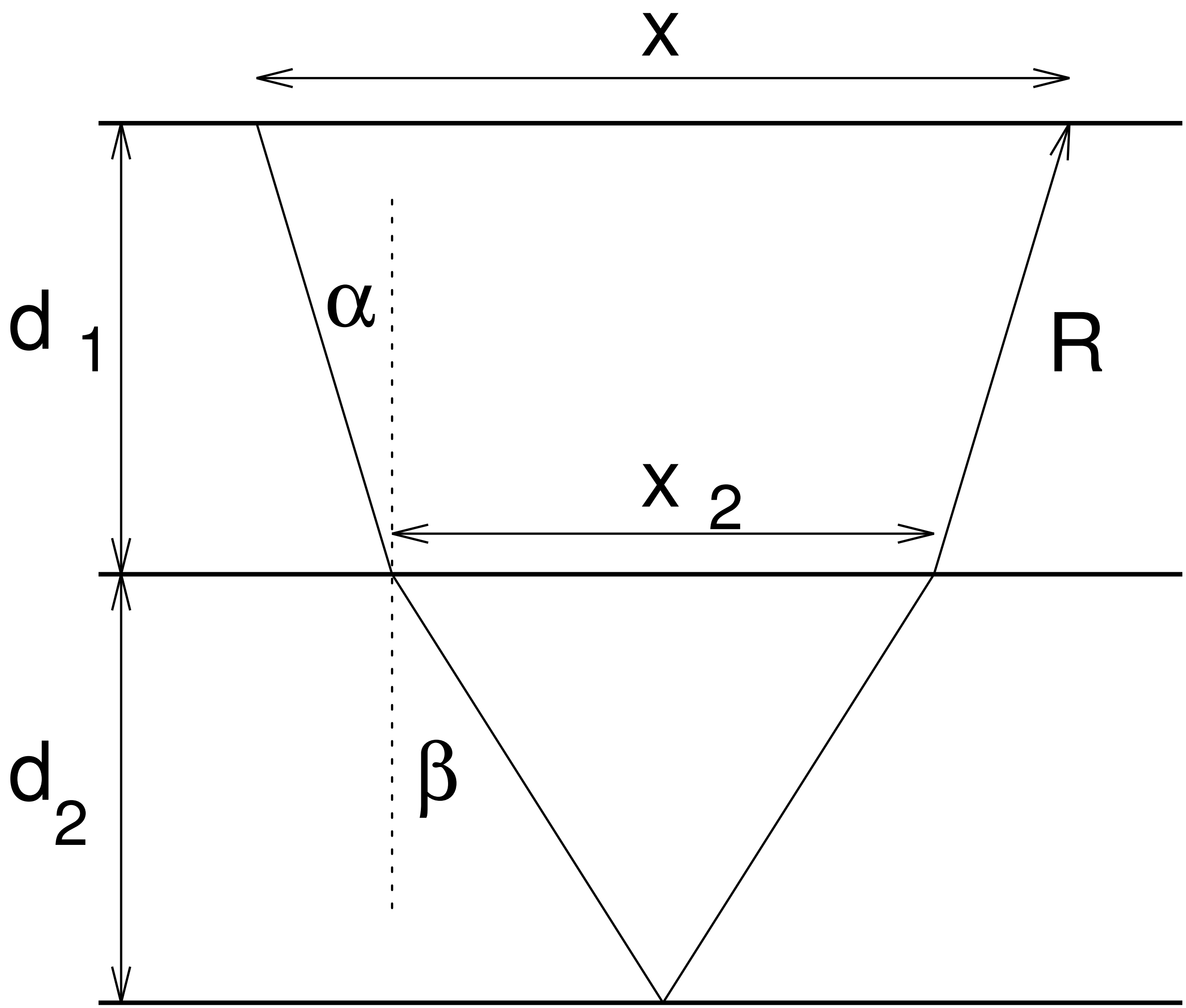
NMO for 2-layer configuration

(Ta3520)

Configuration



Normal Move-out (NMO) for more than 1 interface

Traveltime for two layers given by:

$$
\begin{aligned}
T &= \frac{\left((x - x_2)^2 + 4d_1^2)\right)^{1/2}}{c_1} + \frac{\left(x_2^2 + 4d_2^2)\right)^{1/2}}{c_2} \\
&= \frac{2d_1}{c_1}\left(1 + \frac{(x - x_2)^2}{4d_1^2}\right)^{1/2} + \frac{2d_2}{c_2}\left(1 + \frac{x_2^2}{4d_2^2}\right)^{1/2} \\
&= T_1\left(1 + \frac{x_1^2}{T_1^2 c_1^2}\right)^{1/2} + T_2\left(1 + \frac{x_2^2}{T_2^2 c_2^2}\right)^{1/2}
\end{aligned}
$$

$T_i =$ zero-offset traveltime trough layer i

Expand square root:

$$T \simeq T_1 + \frac{x_1^2}{2T_1 c_1^2} + T_2 + \frac{x_2^2}{2T_2 c_2^2}.$$

This can be rewritten as follows:

$$T^2 = (T_1 + T_2)^2 + (T_1 + T_2)\left(\frac{x_1^2}{T_1 c_1^2} + \frac{x_2^2}{T_2 c_2^2}\right) + O(x^4)$$

Approximate: $\sin\alpha \;\approx\; \tan\alpha = \dfrac{x_1}{2d_1}$

$\sin\beta \;\approx\; \tan\beta = \dfrac{x_2}{2d_2}$

Snell's Law reads:

$$\frac{\sin\alpha}{c_1} = \frac{\sin\beta}{c_2}$$

Substituting the approximation from above:

$$\frac{x_1}{2d_1c_1} = \frac{x_2}{2d_2c_2}$$

or

$$\frac{x_1}{T_1c_1^2} = \frac{x_2}{T_2c_2^2}$$

Using $x = x_1 + x_2$ and $x_2 = (T_2 c_2^2)/(T_1 c_1^2) x_1$:

$$x_1 = x \frac{T_1 c_1^2}{T_1 c_1^2 + T_2^2 c_2^2}$$

Idem for x_2:

$$x_2 = x \frac{T_2 c_2^2}{T_1 c_1^2 + T_2^2 c_2^2}$$

Using approximations and Snell's law:

$$
\begin{aligned}
T^2 &\approx (T_1 + T_2)^2 + (T_1 + T_2)x^2 \left(\frac{T_1 c_1^2 + T_2 c_2^2}{(T_1 c_1^2 + T_2 c_2^2)^2} \right) \\
&\approx (T_1 + T_2)^2 + \frac{(T_1 + T_2)}{T_1 c_1^2 + T_2 c_2^2} x^2
\end{aligned}
$$

This is of the form:

$$
T^2 = T_{tot}(0)^2 + \frac{x^2}{c_{rms}^2}
$$

This is the equation of a **hyperbola** !

Compare :

$$T^2 = T_{tot}(0)^2 + \frac{x^2}{c_{rms}^2}$$

$$T^2 \approx (T_1 + T_2)^2 + \frac{(T_1 + T_2)}{T_1 c_1^2 + T_2 c_2^2} x^2$$

c_{RMS} is given by:

$$c_{rms}^2 = \frac{1}{T_{tot}(0)} \sum_{i=1}^{N} c_i^2 T_i(0)$$

Approximation